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Adiabatic Passage of Collective Excitations in Atomic Ensembles*

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Abstract *We describe a theoretical scheme that allows for transfer of quantum states of atomic collective excitation between two macroscopic atomic ensembles localized in two spatially-separated domains. The conception is based on the occurrence of double-exciton dark states due to the collective destructive quantum interference of the emissions from the two atomic ensembles. With an adiabatically coherence manipulation for the atom-field couplings by stimulated Ramann scattering, the dark states will extrapolate from an exciton state of an ensemble to that of another. This realizes the transport of quantum information among atomic ensembles.*

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1 Introduction

In quantum information processing (QIP), the physical implementations of quantum memories and quantum carriers can be considered as one of the biggest challenges for physicists. Since photons are manifested to be ideal carriers of quantum information in many experiments, much attention has been paid to investigations on ideal storage systems of quantum information recently. Usually, quantum memories not only allow the transport of unknown quantum states among separated locations with short access time,[1] but also possess the nature of avoiding quantum decoherence, namely, one can isolate them completely from environmental interactions and control their coupling to a necessary extent. To satisfy these basic requirements, the collective excitations of symmetric internal states of an atomic ensemble[2] have been studied extensively as a very attractive candidate for this in last years.[3−6] For this quantum memory realized by atomic ensemble, despite the seemingly insurmountable difficulties (such as the quantum leakage due to the inhomogeneous coupling of external fields[7] and quantum decoherence induced by the center of mass (C.M.) motion,[8] there may be possibility to overcome them by considering that the existence of equivalence classes of collective storage states will reduce the enhanced sensitivity of the collective memory to environmental interactions.

In this paper we will work on the problem of how to transport quantum information between two atomic ensembles in direct ways other than the current aspect how to store quantum information. As an indirect way to solve this problem, a significant contribution by Duan *et al.*[9] is that, using two un-correlated linear polarized lights to couple two non-correlated atomic ensembles, the entanglement between the two atomic ensembles can be created after a non-local Bell measurement for the circular polarization mode based on the Stokes variables mixing linear polarization mode. Then, the quantum communication between two atomic ensembles can be implemented. Recent experimental success[10] clearly demonstrates the power of such an atomic ensemble based system for entangling macroscopic objects as the so-called canonical teleportation defined for arbitrary pair of canonical variables[11] beyond the usual coordinate and momentum. Our direct way to communicate quantum information between two atomic ensembles does not depend on any post-selective measurement.

The quantum memories in our protocol are also the certain collective quantum states of two atomic ensembles, which describe the collective low excitations of two clusters of atoms confined in two well-separated well potentials. It is found that, for the two excited atom systems interacting with a single mode quantized light field, there exist double-exciton dark states decoupling with light field, which extrapolate from an exciton state of an ensemble to that of another. To change the effective couplings of atomic ensembles to the light field adiabatically through a Ramann light stimulation,[12] one can transfer a single exciton state of one ensemble to another. In mathematical formulation, the double-exciton dark state is quite similar to the polariton state in the atomic ensemble storage scheme of photon Fock state,[3−6] but the physical difference is that our double-exciton dark state does not contain any variable of light photon.

This paper is organized as follows. In Sec. 2, we propose our protocol of transfer of the collective excitations of atomic ensembles and deduce its corresponding model

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Hamiltonian in a simplified way. The transfer of quantum information in terms of collective atomic states is depicted in Sec. 3 as the time evolution governed by the adiabatically manipulated Hamiltonian. In Sec. 4 the exact solution to the adiabatic dynamics is given in the macroscopic limit by defining the double-exciton dark states.

2 Model for Two Atomic Ensembles with Two Tunable Couplings

In our protocol the total system (Fig. 1) involving two atomic ensembles is localized in two far-separated places, the left and right ones containing N_l and N_r two-level atoms respectively. These two clusters of atoms are coupled to a light field with coupling coefficients g_l and g_r. To implement our scheme, it is most important for the experimental setup to require that the effective strengths g_l and g_r of light field coupling can be varied relatively or independently. This requirement shows the drawback of a naive scheme using the practical two-level atom with direct coupling between atom and light field: since the same single mode light interacts with two atomic ensembles at the same time, one cannot change the relative effective coupling strengths of field with two atomic ensembles. Thus, it is impossible to reach our goals by changing the coupling directly. In the following we will use the equivalent two-level system deduced from the practical three-level atoms stimulated by Ramann scattering.

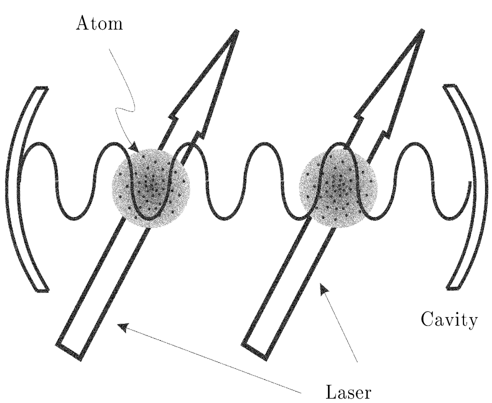

Fig. 1 Illustration of the system: two 3-level atomic ensembles consisting of the identical atoms are located in a quantized cavity and coupled to two classical laser fields, respectively.

For conceptual simplicity we here assume that each atom used in practice only has three energy levels as shown in Fig. 2. There are two meta-stable lower states ($|\,g\rangle$ and $|e\rangle$) and an auxiliary state $|a\rangle$. The transition $|a\rangle \to |\,g\rangle$ of each of these atoms in both wells is coupled to a same quantized radiation mode with Rabi-frequency Ω and frequency ω while the transitions $|a\rangle \to |e\rangle$ of the atoms in the left (right) well are driven by a classical control field of Rabi-frequency Ω_l (Ω_r). Moreover, we also assume that the detuning between $|\,g\rangle$ and $|a\rangle$ with respect to the quantized light is the same as that between $|e\rangle$ and $|a\rangle$ with respect to the classical light. Then, we can write down the single atom Hamiltonians in the interaction picture,

$$H_{Is} = [\Omega|\,g(s)\rangle a^\dagger + \Omega_s|e(s)\rangle]\langle a(s)|\,\mathrm{e}^{-\mathrm{i}\Delta t} + \text{h.c.}\,, \qquad (1)$$

for $s = l, r$. Here, $a^\dagger$ is the creation operator of the quantized field. Due to the stimulated Ramann effect for large detuning Δ, the effective Hamiltonians can be obtained as

$$\begin{aligned} H_s = & -\frac{|\Omega|^2}{\Delta}a^\dagger a|\,g(s)\rangle\langle g(s)| - \frac{|\Omega_s|^2}{\Delta}|e(s)\rangle\langle e(s)| \\ & - \Big[\frac{\Omega\Omega_s^*}{\Delta}|\,g(s)\rangle\langle e(s)|a^\dagger + \text{h.c.}\Big] \end{aligned} \qquad (2)$$

by an adiabatic elimination of the upper level $|a\rangle$. This just realizes an equivalent two-level atom system with the excited state $|e\rangle$ and ground state $|\,g\rangle$ with effective coupling

$$g_s = -\frac{\Omega\Omega_s^*}{\Delta}\,, \qquad s = r, l\,. \qquad (3)$$

g_s can be set real by adding a proper phase to $|\,g(s)\rangle$ or $\langle e(s)|$. In this sense, the effective coupling strengths g_l and g_r and their ratio (relative effective coupling strength) can be well controlled independently. If the level difference between $|e\rangle$ and $|\,g\rangle$ is ω_a, the effective level differences of the equivalent two-level atoms in the left and right wells are

$$\omega_s = \omega_s(a^\dagger a) = \omega_a + \frac{|\Omega|^2}{\Delta}I - \frac{|\Omega_s|^2}{\Delta}\,,$$

where $s = r, l$ and $I = a^\dagger a$ is regarded as the density of the quantized light. In most cases we can neglect the Stark shifts $|\Omega|^2/\Delta$ and $|\Omega_s|^2/\Delta$.

Generalization to multi-mode field and multi-level atoms is straightforward. Let the left and right wells contain N_l and N_r atoms respectively. Then, the many-atom Hamiltonian is given by

$$\begin{aligned} H = & \frac{1}{2}\omega_l\sum_{j=1}^{N_l}\sigma_z^{[j]}(l) + \frac{1}{2}\omega_r\sum_{j=1}^{N_r}\sigma_z^{[j]}(r) \\ & + \omega_a a^\dagger a + \Big\{a\Big[\sum_{j=1}^{N_l} g_l\sigma_+^{[j]}(l) + \sum_{j=1}^{N_r} g_r\sigma_+^{[j]}(r)\Big] + \text{h.c.}\Big\}\,. \end{aligned} \qquad (4)$$

Here, we have defined the quasi-spin ladder operators $\sigma_+^{[j]}(s) = |e(s)\rangle_{jj}\langle g(s)|$ and $\sigma_-^{[j]}(s) = [\sigma_+^{[j]}(s)]^\dagger$ and the population inversion operator $\sigma_z^{[j]}(s) = |e(s)\rangle_{jj}\langle e(s)| - |\,g(s)\rangle_{jj}\langle g(s)|$ in terms of the excited and ground states $|e(s)\rangle_j$ and $|\,g(s)\rangle_j$ ($s = l, r$) in the two wells. It is noticed that, for simplicity, the coupling constants Ω, Ω_l, and Ω_r

between the atoms and the fields are assumed to be equal for all atoms in the well.

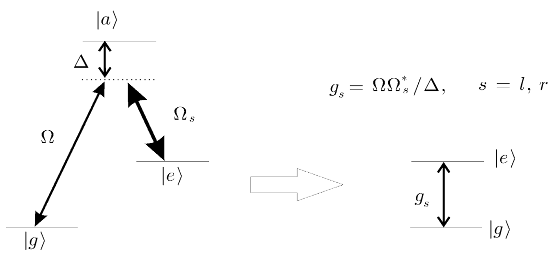

Fig. 2 Illustration of the scheme for adiabatic transport of collective excitations. The right figure shows the equivalent two-level atom induced by Ramann scattering of a three-level atom.

When all atoms are prepared initially in the symmetric ground state $|g_l\rangle = |\, g_1(l), g_2(l), \ldots, g_{N_l}(l)\rangle$ and $|g_r\rangle = |g_1(r), g_2(r), \ldots, g_{Nr}(r)\rangle$, the n-th excited state of the system can be described by symmetrizing the effects of n flips of different atoms from $|\, g_j(s)\rangle \rightarrow |e_j(s)\rangle$. In fact, they are the totally symmetric Dicke-states,

$$
\begin{aligned}
|\boldsymbol{e}(s)\rangle &= \frac{1}{\sqrt{N_s}} \sum_{j=1}^{N_s} |\, g_1(s), \ldots, e_j(s), \ldots\rangle\,, \\
|\boldsymbol{e}^2(s)\rangle &= \frac{1}{\sqrt{2N_s(N_s-1)}} \\
&\quad \times \sum_{i\neq j=1}^{N_s} |\, g_1(s), \ldots, e_i(s), \ldots, e_j(s), \ldots,\rangle\,, \\
&\cdots
\end{aligned} \tag{5}
$$

3 Adiabatic Transfer of Collective Excitations

To get the basic idea for our quantum state transferring mechanism via adiabatic manipulation, we first consider the special case of the above model, where there is only one atom in each well. For such a system of two atoms interacting with a single mode light field, there exists an asymmetric two-atom state, i.e. the dark state,

$$
|d(g_l, g_r)\rangle \sim \cos(\theta)|e(l), g(r)\rangle - \sin(\theta)|\, g(l), e(r)\rangle\,, \tag{6}
$$

decoupling from the light vacuum field, which is a special eigenstate of the atom-field system.[13] Here, the mixing angle is defined by $\tan\theta = g_l/g_r$. Due to the destructive quantum interference between the two transitions from the two atoms in this state, the so-called emission trapping occurs, namely, no emission transmits from such a state. Furthermore, by changing the coupling strengths g_l and g_r sufficiently slowly, the mixing angle θ can be rotated from 0 to $\pi/2$. According to the quantum adiabatic theorem, if the total system is initially prepared in the dark state $|d(g_l, g_r)\rangle$, the adiabatic following will determine the quantum state transfer from $|e(l), g(r)\rangle$ to $-|\, g(l), e(r)\rangle$. This mechanism for adiabatic transfer is the basis of our scheme to transport quantum states from an atomic ensemble to another.

Motivated by the above conceptions for the two-atom case, the immediately following problem is to find the dark state for the many-atom case. We write down the collective quasi-spin operators:

$$
S_\beta(s) = \sum_{j=1}^{N_s} \sigma_\beta^{[j]}(s) \quad (\beta = \pm, z;\;\; s = l, r)\,. \tag{7}
$$

Correspondingly the mixing collective operators

$$
\varphi = \frac{\cos\alpha}{\sqrt{N_l}} S_-(l) - \frac{\sin\alpha}{\sqrt{N_r}} S_-(r) \tag{8}
$$

and

$$
\phi = \frac{\sin\alpha}{\sqrt{N_l}} S_-(l) + \frac{\cos\alpha}{\sqrt{N_r}} S_-(r) \tag{9}
$$

can be defined to depict the double excitation in two atomic ensembles, where the mixing angle α is defined by $\tan\alpha = g_l\sqrt{N_l}/g_r\sqrt{N_r}$. In terms of these mixing collective operators, we can define the double excitation of the two atomic ensembles by

$$
|D(n)\rangle = |D(n, g_l, g_r)\rangle = \frac{1}{\sqrt{n!}} \varphi^{\dagger n} |g_l, g_r\rangle\,, \tag{10}
$$

where the ground state $|g_l, \boldsymbol{g}_r\rangle = |g_l\rangle \otimes |g_r\rangle \otimes |0\rangle_p$ is the total ground state, ($|0\rangle_p$ shows that there is 0 photon in the light vacuum field) and

$$
|\boldsymbol{e}_l^m, \boldsymbol{e}_r^n\rangle = \frac{1}{\sqrt{N_l^m N_r^n m! n!}} S_+^m(l) S_+^n(r) |g_l, g_r\rangle \tag{11}
$$

represents the m atom excitations in the left ensemble and n atom excitations in the right one (with 0 photon in the light vacuum field) with $|\boldsymbol{e}_l^m, \boldsymbol{e}_r^n\rangle = |\boldsymbol{e}^m(l)\rangle \otimes |\boldsymbol{e}^n(r)\rangle \otimes |0\rangle_p$. Now we can show the central result that $|D(n)\rangle$ is approximately a dark many-atom state, namely, it is cancelled by the interaction Hamiltonian

$$
H_I = g(a\phi^\dagger + a^\dagger\phi)\,,
$$

or

$$
H_I|D(n)\rangle \simeq 0\,,
$$

where $g = \sqrt{g_l^2 N_l + g_r^2 N_r}$. In fact, it is easy to check the commutation relation

$$
[\varphi^\dagger, \phi] = \frac{1}{2}\sin(2\alpha)\Big[\frac{S_z(l)}{N_l} - \frac{S_z(r)}{N_r}\Big],
$$

and the limit behaviors

$$
\frac{S_z(l)}{N_l} \rightarrow -1\,, \qquad \frac{S_z(r)}{N_r} \rightarrow -1
$$

for very large N_l and N_r in the low excitation case that most atoms occupy the ground state. Then, $[\varphi^\dagger, \phi] \rightarrow 0$ and thus

$$
H_I|D(n)\rangle \sim \phi\varphi^{\dagger n}|g_l, g_r\rangle \sim \varphi^{\dagger n}\phi|g_l, g_r\rangle = 0\,.
$$

With the above observations, we can describe our scheme about the transport of quantum information carried by the quantum state of atomic ensemble in quite a similar way to the well-known scheme for the storage of photon information by atomic ensemble. For the resonance case in the interaction picture, as the eigenstates of H_I with zero eigenvalue, the dark states $|D(n)\rangle$ are degenerate for $n = 0, 1, 2, \ldots$ Adiabatic changes of the two classical light fields through $\Omega_{l,r}$ can induce the independent variances of the effective coupling constants g_l and g_r. This can result in the asymptotic behavior of the dark states by varying the mixing angle α from 0 to $\pi/2$, which physically corresponds to two limit cases:

$$g_r\sqrt{N_r} \gg g_l\sqrt{N_l}\,, \qquad g_l\sqrt{N_l} \gg g_r\sqrt{N_r}\,.$$

In this process, the number of excitons in the dark state $|D(n)\rangle$ is an adiabatic invariant. Thus, by adiabatically changing the parameters, the collective quantum state $|D(n)\rangle$ will vary from

$$|D(n, g_l = 0, g_r = 1)\rangle = |\boldsymbol{e}_l^m, \boldsymbol{e}_r^0\rangle = |\boldsymbol{e}^n(l)\rangle \otimes |0\rangle_p$$

to

$$|D(n, g_l = 1, g_r = 0)\rangle = |\boldsymbol{e}_l^0, \boldsymbol{e}_r^n\rangle = |\boldsymbol{e}^n(r)\rangle \otimes |0\rangle_p\,.$$

If the initial quantum information stored in the left ensemble is described by the density matrix $\hat{\rho}_l = \sum_{n,m} \rho_{nm}\,|\boldsymbol{e}^n(l)\rangle\langle \boldsymbol{e}^m(l)|$, the adiabatic transfer process generates a same quantum state of collective excitations in the right atomic ensemble $\hat{\rho}_r = \sum_{n,m} \rho_{nm}\,|\boldsymbol{e}^n(r)\rangle\langle \boldsymbol{e}^m(r)|$. In fact, the adiabatic evolution of the total state

$$\hat{\rho}(g_l, g_r) = \sum_{n,m} \rho_{nm} |D(n, g_l, g_r)\rangle\langle D(m, g_l, g_r)| \tag{12}$$

in the interaction picture just extrapolates between two factorized states

$$\hat{\rho}(g_l = 0, g_r = 1) = \hat{\rho}_l \otimes |\boldsymbol{g}_r\rangle\langle g_r| \otimes |0\rangle_{pp}\langle 0| \tag{13}$$

and

$$\hat{\rho}(g_l = 1, g_r = 0) = |g_l\rangle\langle \boldsymbol{g}_l| \otimes \hat{\rho}_r \otimes |0\rangle_{pp}\langle 0|\,. \tag{14}$$

Since $\hat{\rho}_l$ and $\hat{\rho}_r$ possess the same single exciton density matrix, one may say the quantum information has been transported from the left atomic ensemble to the right one.

Like the problem in the scheme of atomic ensemble storage of photon quantum information,[14] there still exist two problems, quantum decoherence due to the interaction with environment and quantum leakage due to inhomogeneous coupling of the classical (and quantum) fields.[7,8] The former can be partially solved by considering that there are equivalence classes of collective storage states besides the non-degenerate symmetric state with maximum $J_s = N_s/2$ and the transitions to them do not affect the reading-out of collective quantum state of exciton in right ensemble to which the quantum information is transferred. The fact was found recently by Fleischhauer *et al.* as a key element to overcome decoherence in atomic ensemble quantum information processing.[4] Our present model can be used to illustrate this interesting idea by the introduction of non-symmetric single excitation states

$$|\boldsymbol{e}_k(s)\rangle = \frac{1}{\sqrt{N_s}} \sum_{j=1}^{N_s} \times\, \mathrm{e}^{2\mathrm{i}\pi jk/N_s} |\, g_1(s), \ldots, e_j(s), \ldots, g_{N_s}(s)\rangle \tag{15}$$

for $k = 0, 1, 2, \ldots, N_s - 1$ and the corresponding collective excitation operators

$$S_{\beta k}(s) = \frac{1}{\sqrt{N_s}} \sum_{j=0}^{N_s-1} \mathrm{e}^{2i\pi jk/N_s} \sigma_\beta^{[j]}(s) \tag{16}$$

for $k = 0, 1, 2, \ldots, N_s - 1$ ($\beta = \pm, z; s = l, r$). For very large N_s, we have a complete set of boson operators,

$$\Big\{ b_k(s) = \lim_{N_s\to\infty} \frac{1}{\sqrt{N_s}} S_{-k}(s) | k = 0, 1, \ldots, N_s - 1 \Big\}\,. \tag{17}$$

There are N_s fault-tolerant equivalence classes C_n

$$\Big\{ \prod_{k=0}^{N_s-1} b_k^{\dagger m_k}(s) \varphi^{\dagger n} |0\rangle | m_k = 0, 1, \ldots, N_s - 1 \Big\}\,, \tag{18}$$

which contain $N_s^2 - 1$ elements besides the non-degenerate symmetric state with maximum J_s. The transporting process of the quantum state does not distinguish among those states. A physical explanation of these equivalence classes is given in terms of quasi-particle excitations. Only excitations of double dark-state exciton modes with specific wave vectors couple to the quantum field. Excitations of other excitons by $b_k^{\dagger m_k}$ do not affect the quantum state transporting. Hence, from the point of view of quantum memory, all collective atomic states with the same number of $\varphi^\dagger$ excitations are equivalent. Then, taking this equivalence into account, one can see that the equivalence class will compensate the $\sqrt{N_s}$ enhancement of decoherence.[7,8]

4 Quantum Dynamics in Macroscopic Limit

In the macroscopic limit with very large atomic numbers N_l and N_r, the two central results in this paper will be given in this section: (i) φ and ϕ define bosonic excitations cooperating between the two atomic ensembles; (ii) The collective Fock state $|D(n)\rangle$ becomes a dark state decoupling from the quantized light field.

To illustrate them, we first consider the general observation about the classical limit of the quantum angular momentum operators $\boldsymbol{J} = (J_1, J_2, J_3)$. It was already noticed that, when the total angular momentum J approaches infinite, algebra SU(2) gives a representation of the usual bosonic Heisenberg algebra by defining the boson operators according to $b = \lim_{J\to\infty}(J_1 - \mathrm{i}J_2)/\sqrt{2J}$, $b^\dagger = \lim_{J\to\infty}(J_1 + \mathrm{i}J_2)/\sqrt{2J}$. The above limit is taken under the low excitation condition that $J - J_3$ is finite. The detailed argument was given recently for the exciton-polariton problem in semiconductor microcavity.[15] Considering that $S_\beta(s) = \sum_{j=1}^{N_s} \sigma_\beta^{[j]}(s)$ gives a spinor representation of angular momentum with $J_s = N_s/2$, the classical limit of angular momentum can be realized as the macroscopic limit with $N_s \to \infty$. In this limit, we have the independent excitations described by bosonic operators,

$$b_l = \lim_{N_l \to \infty} \frac{S_-(l)}{\sqrt{N_l}}\,, \qquad b_r = \lim_{N_r \to \infty} \frac{S_-(r)}{\sqrt{N_r}}\,. \tag{19}$$

Then the above defined double collective excitation can be described by the canonical transformations

$$\varphi = b_l \cos\alpha - b_r \sin\alpha\,, \qquad \phi = b_l \sin\alpha + b_r \cos\alpha \tag{20}$$

of two boson modes b_l and b_r. The two quasi-boson operators satisfy the independent bosonic commutation relations

$$[\varphi, \varphi^\dagger] = 1\,, \quad [\phi, \phi^\dagger] = 1\,, \quad [\varphi, \phi] = [\varphi^\dagger, \phi^\dagger] = 0$$

for very large N_l and N_r. This is an approximation up to the order of $O(1/N_s)$ $(s = l, r)$. It is noticed that the ground state $|\boldsymbol{g}_l, \boldsymbol{g}_r\rangle$ can become the vacuum $|0\rangle$ for b_l and b_r, $(|\boldsymbol{g}_l, \boldsymbol{g}_r\rangle = |g_l\rangle \otimes |\boldsymbol{g}_r\rangle \otimes |0\rangle_p \equiv |0\rangle_l \otimes |0\rangle_r \otimes |0\rangle_p = |0\rangle)$, and also for φ and ϕ equivalently.

To show the second observation that $|D(n)\rangle$ is a dark state with double excitons, we write the effective Hamiltonian as

$$\begin{aligned} H = {} & \omega_a a^\dagger a + \omega_l b_l^\dagger b_l + \omega_r b_r^\dagger b_r \\ & + a(g_l \sqrt{N_l} b_l^\dagger + g_r \sqrt{Nr} b_r^\dagger) + \text{h.c.} \end{aligned} \tag{21}$$

in the macroscopic limit by ignoring the constant form. When the Stark shifts are neglected, we have $\omega_l = \omega_r = \epsilon$ and

$$H = \omega_a a^\dagger a + \epsilon(\phi^\dagger \phi + \varphi^\dagger \varphi) + g a \phi^\dagger + \text{h.c.} \tag{22}$$

In this case, the excitation φ decouples with the quantized field and the complementary excitation ϕ. It is obvious that double-exciton dark state $|D(n)\rangle = (1/\sqrt{n!}\ )\varphi^{\dagger n}|0\rangle$ can be cancelled by the interaction part $H_I = g a \phi^\dagger + \text{h.c.}$

In this limit, we can exactly solve the quantum dynamics for entangling the collective excitations of atomic ensembles with the quantized light field. To this end, we regard this entanglement as a reading-out process of quantum information stored in the atomic ensemble. To describe the coherent transfer of the quantum information from atomic ensembles to the quantum light field in our scheme, we consider the dressed excitation by the bosonic polariton operators

$$\begin{aligned} A &= a \cos\Big(\frac{\vartheta}{2}\Big) + \phi \sin\Big(\frac{\vartheta}{2}\Big)\,, \\ B &= -a \sin\Big(\frac{\vartheta}{2}\Big) + \phi \cos\Big(\frac{\vartheta}{2}\Big)\,, \end{aligned} \tag{23}$$

and rewrite the total Hamiltonian in terms of the canonical modes of polariton as

$$H = \Theta \hat{N} + \Xi(A^\dagger A - B^\dagger B) + \epsilon \varphi^\dagger \varphi\,. \tag{24}$$

Here

$$\begin{aligned} \hat{N} &= A^\dagger A + B^\dagger B\,, \\ \Xi &= \sqrt{\Big(\frac{\omega - \epsilon}{2}\Big)^2 + g^2}\,, \\ \Theta &= \frac{\omega + \epsilon}{2}\,, \qquad \tan\vartheta = \frac{2g}{\omega - \epsilon}\,. \end{aligned} \tag{25}$$

We consider the case that the coupling parameters of the total system do not change. Suppose there be no photons in the quantized mode $|0\rangle_p$ and all the atoms be coherently prepared in the left well with the initial state

$$|S(\eta)\rangle_l = \mathrm{e}^{\mathrm{i}\eta S_y(l)/\sqrt{N_l}} |g_l, g_r\rangle \otimes |0\rangle_p \tag{26}$$

by a collective Rabi rotation. Here, we have defined $S_y(l) = \sum_{j=1}^{N_l} \sigma_y^{[j]}(s)$. The atom number $N_l = \sqrt{\eta}$ possesses a minimum uncertainty. It can be approximated by a coherent state

$$|\eta\rangle_l = \exp[\eta(b_l^\dagger - b_l)]|0\rangle \tag{27}$$

in the macroscopic limit, where $|0\rangle = |0\rangle_l \otimes |0\rangle_r \otimes |0\rangle_p$.

In order to calculate the evolution of the state $|\eta\rangle_l$, we first get the Heisenberg equations of motion of operators from the Hamiltonian Eq. (14),

$$\begin{aligned} \dot{A}(t) &= -\mathrm{i}(\Theta + \Xi) A(t)\,, \\ \dot{B}(t) &= -\mathrm{i}(\Theta - \Xi) B(t)\,, \\ \dot{\varphi}(t) &= -\mathrm{i}\epsilon \varphi(t)\,, \end{aligned} \tag{28}$$

which give

$$\begin{aligned} A(t) &= A(0) \exp[-\mathrm{i}(\Theta + \Xi)t]\,, \\ B(t) &= B(0) \exp[-\mathrm{i}(\Theta - \Xi)t]\,, \\ \varphi(t) &= \varphi(0) \exp(-\mathrm{i}\epsilon t)\,. \end{aligned} \tag{29}$$

Together with Eqs. (20) and (23), one can write down the operator

$$\begin{aligned} b_l(t) = {} & \varphi(0) \exp(-\mathrm{i}\epsilon t) \cos\alpha + A(0) \\ & \times \exp[-\mathrm{i}(\Theta + \Xi)t] \sin\Big(\frac{\vartheta}{2}\Big) \sin\alpha \\ & + B(0) \exp[-\mathrm{i}(\Theta - \Xi)t] \cos\Big(\frac{\vartheta}{2}\Big) \sin\alpha\,. \end{aligned} \tag{30}$$

Since the Hamiltonian of the system is bilinear, we have $U(t)|0\rangle = \exp(-\mathrm{i}Ht)|0\rangle = |0\rangle$. Therefore, driven by the classical lights, the total system with the initial state $|\eta\rangle_l$ will evolve into a factorized state with three approximate components of coherent state,

$$\begin{aligned} |\eta, t\rangle = U(t)|\eta\rangle_l &= \exp\{\eta[b_l^\dagger(-t) - b_l(-t)]\}|0\rangle \\ &= |\eta f(t)\rangle_l \otimes |\eta g(t)\rangle_r \otimes |\eta h(t)\rangle_p\,, \end{aligned} \tag{31}$$

where

$$\begin{aligned} f(t) = {} & \cos^2(\alpha) \exp(-\mathrm{i}\epsilon t) \\ & + \sin^2\Big(\frac{\vartheta}{2}\Big) \sin^2(\alpha) \exp[-\mathrm{i}(\Theta + \Xi)t] \\ & + \cos^2\Big(\frac{\vartheta}{2}\Big) \sin^2(\alpha) \exp[-\mathrm{i}(\Theta - \Xi)t]\,, \end{aligned}$$

$$g(t) = \Big\{ -\exp(-\mathrm{i}\epsilon t) + \sin^2\Big(\frac{\vartheta}{2}\Big)\exp[-\mathrm{i}(\Theta+\Xi)t] + \cos^2\Big(\frac{\vartheta}{2}\Big)\exp[-\mathrm{i}(\Theta-\Xi)t]\Big\}\frac{\sin(2\alpha)}{2}\,,$$
$$h(t) = -\,\mathrm{i}\sin(\vartheta)\sin(\alpha)\exp(-\mathrm{i}\Theta t)\sin(\Xi t)\,. \tag{32}$$

While the second component corresponds to the coherence tunneling of the collective internal excitation from the left ensemble to the right one, the third term presents the evolution of the quantized light field entangled with the collective excitation of atomic ensembles. As a consequence, the effective increasing $\eta^2|\alpha(\varphi,t)|^2$ of photons in the quantized light mode can record the information concerning the transfer of collective excitations in atomic ensembles.

5 Summary

In conclusion we have discussed the idea of the double-exciton dark states for two-atomic-ensemble system dressed by a single mode light field. We also considered the auxiliary role of a quantized light field. The independent adiabatic manipulation of two classical light fields interacting with two atomic ensembles is the key technology for the realistic realization.

Obviously, our protocol based on this conception to transfer information between quantum memories mainly depends on a many-atom enhancement mechanism to realize a convenient manipulation for the effective coupling strengths (the two effective coupling strengths can be changed independently by the stimulated Ramann adiabatic passage). This is in fact, not quite surprising, that earlier cavity QED experiments have relied on the enhanced dipole interaction of a collection of many atoms.[16−18] In free space, the phenomena of superfluorescence or super-radiance[16] constitutes another example of collective state dynamics. Notice that the electric dipole couplings $g_l \sim 1/\sqrt{V_l}$ and $g_r \sim 1/\sqrt{V_r}$, where V_l and V_r are the effective model volume of fields interacting with left and right ensembles. So the effective coupling strengths $g_l\sqrt{N_l}$ and $g_r\sqrt{N_r}$ are proportional to the square-root of effective densities $n_s = N_s/V_s$ $(s = l, r)$ of field-atom interaction. Usually, similar to the thermodynamics limit, one can consider n_s as a constant since both N_s and V_s approach infinity. In this sense, the role of many-atom enhancement in manipulation of the effective coupling strengths is limited by the density of atomic gas. As mentioned in Sec. 3, the imperfections of our scheme are due to the inhomogeneous interaction and out-control of atoms, which may cause the decoherence of order $\sqrt{N_s}$. This shows that the technique is not extremely robust.

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