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Dynamics of Optically Driven Exciton and Quantum Decoherence*

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Abstract *By using the normal ordering method, we study the state evolution of an optically driven excitons in a quantum well immersed in a leaky cavity, which was introduced by Yu-Xi Liu et al. [Phys. Rev.* **A63** *(2001) 033816]. The influence of the external laser field on the quantum decoherence of a mesoscopically superposed state of the excitons is investigated. Our result shows that, the classical field can compensate the energy dissipation of the excitons. Although the decoherence rate of the excitonic Schrödinger cat state does not depend on the external field, the phase of the decoherence factor can be well controlled by adjusting the amplitude of the external field as well as the detuning between the field and the transition frequency of the atom.*

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1 Introduction

One of the most profound features of quantum mechanics is the superposition principle, which plays a central role in implementing of quantum information processing (QIP),[1] such as quantum computation, quantum cryptography, and quantum teleportation. The realization of QIP has triggered intense study in various quantum systems including ion traps, cavity QED, nuclear magnetic resonance, and quantum dots.

As is well known, within the quantum information process preserving coherence is an essential requirement. However, quantum coherence is not robust enough to be exploited. In any measurement process the information's readout will lead to wave packet collapse, i.e., quantum decoherence. On the other hand, due to the influence of the environment,[2] the system's coherence information will be lost and the superposition of the system states will evolve into a statistical mixture state. In fact there are two distinct effects of the external world on the quantum system: quantum dissipation[3,4] with energy loss and quantum decoherence[5] without energy loss. Both the two effects will reduce the efficiency of quantum computation, and result in disentanglement in QIP.[6] It is obvious that to overcome the obstacle caused by decoherence and to control decoherence now become more urgent.

Some important progress has been made in the solid state system[7,8] recently. The schemes to generate maximally entangled states for excitons in coupled quantum dots have been proposed by using a classical laser field[9] or a quantum laser field.[10] Besides, a quantum superposition of macroscopically distinct states[11] in a superconducting quantum interference device (SQUID) has been demonstrated experimentally. These achievements have manifested that the possibility of implement QIP in the solid system becomes more promising than ever. In our previous works,[12] the quantum decoherence of a mesoscopically superposed state of the excitons in a quantum well placed in a leaky cavity is investigated. The results show that the coherence of the superposed states of the system will undergo oscillating decay with time evolution. Now, an immediately-following question is that how to control the dynamical evolution and to suppress the decoherence of the system.

In this paper, with the motivation to continue our previous work, we study the dynamical evolution of an optically driven exciton in the quantum well placed in a leaky cavity. The effect of the external continuous wave (c.w.) field on the state evolution and the quantum decoherence of the mesoscopically superposed states of the exciton is studied by using the normal ordering method (NOM).[13,14] Our results show that the c.w. field does not change the decoherence time. However, the phase of the decoherence factor can be controlled by adjusting the amplitude of the field and the detuning between the field and the transition frequency of the two-level atoms. Such a result seems to be important in quantum computation because in a quantum computation the phase of decoherence factor plays more crucial role.[6]

The paper is organized as follows. In Sec. 2, we give a model of the optically driven excitons in the quantum well placed in a leaky cavity. In Sec. 3, the dynamical evolution of the whole system is analytically calculated by directly solving the Schrödinger equation with the help of the normal ordering method. In addition, the influence of the external pumping field on the mean population of the exciton system is also studied. In Sec. 4, we study the decoherence behavior of the optically driven exciton system. The effect of the external field on the decoherence

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process of the exciton is investigated carefully. Finally, we will give some conclusions.

2 Model of an Optically Driven Exciton in a Leaky Cavity

In our previous work,[12,15] we considered a quantum well (or crystal slab) placed within a leaky Fabry–Perot cavity.[16] The quantum well lies in the center of the cavity. It contains an ideal cubic lattice with N lattice sites and is so thin that it has only one layer. We assume that N identical lattice two-level atoms (or molecules) distribute into these lattice sites. All these particles have equivalent mode positions, so they have the same coupling constant with the cavity modes. It is also assumed that the direction of the dipole moment for the two-level atoms and wave vectors of the cavity fields are perpendicular to the surface of the slab. In addition, when a continuous wave (c.w.) pumping field with frequency ω is applied on the quantum well, the total Hamiltonian under the rotating wave approximation becomes, with $\hbar = 1$,

$$H = \Omega S_z + \sum_j \omega_j b_j^\dagger b_j + \sum_j g_j (b_j S_+ + b_j^\dagger S_-) + \kappa(\mathrm{e}^{-\mathrm{i}\omega t} S_+ + \mathrm{e}^{\mathrm{i}\omega t} S_-) \tag{1}$$

with the collective operators

$$S_Z = \sum_{n=1}^{N} s_z(n), \quad S_\pm = \sum_{n=1}^{N} s_\pm(n)\,, \tag{2}$$

where $s_z(n) = (1/2)(|e_n\rangle\langle e_n| - |g_n\rangle\langle g_n|)$, $s_+(n) = |e_n\rangle\langle g_n|$ and $s_-(n) = |g_n\rangle\langle e_n|$ are quasi-spin operators of the n-th atom. Here $|e_n\rangle$ and $|g_n\rangle$ denote the excited state and the ground state of the n-th atom, and Ω is a transition frequency of the isolated atom. Operators $b_j^\dagger(b_j)$ are creation (annihilation) operators of the field modes, which are labeled by continuous index j with mode frequency ω_j. The Rabi frequency κ denotes the coupling between the atoms and the c.w. pumping field. The coupling constant g_j between the molecules and the cavity fields takes a simple form which is proportional to a Lorentzian

$$g_j = \frac{\eta\Gamma}{\sqrt{(\omega_j - \Omega)^2 + \Gamma^2}}\,, \tag{3}$$

where η depends on the atomic dipole and Γ is the decay rate of a quasi-mode of the cavity. In this paper we restrict our investigation to the Jaynes–Cummings situation, where only one quasi-mode of the cavity is involved, and it is resonant with the transition frequency of the isolated atom Ω.[17] In the case of low density of the excitation with the attractive exciton-exciton collisions due to the bi-exciton effect[15] neglected, the collective behavior of the atoms can be described by a bosonic exciton.[18] With this so-called bosonic approximation we can make the replacement: $a = S_-/\sqrt{N}$ and $a^\dagger = S_+/\sqrt{N}$ with $[a, a^\dagger] = 1$. Then the Hamiltonian (1) becomes

$$H = \Omega a^\dagger a + \sum_j \omega_j b_j^\dagger b_j + \sum_j g(\omega_j)(b_j^\dagger a + a^\dagger b_j) + \mathcal{R}(\mathrm{e}^{-\mathrm{i}\omega t} a^\dagger + \mathrm{e}^{\mathrm{i}\omega t} a) \tag{4}$$

with $g(\omega_j) = \sqrt{N} g_j$ and $\mathcal{R} = \sqrt{N}\kappa$. We note that our model of the optically driven excitons plus the cavity fields now becomes a standard driven damped oscillator system. We will solve the time evolution of the coupled system described by the Hamiltonian of Eq. (4) by directly solving the Schrödinger equation with the help of the NOM.

3 Exact Solution in Terms of NOM

In this section we calculate the state evolution of the driven excitons immersed in a lossy cavity by using NOM, which was first introduced by Louisell[13] to study the dynamic evolution of a driven oscillator as well as that of two weakly coupled oscillators without dissipation. Here we follow our previous works[14] to study the dynamical evolution of the optically driven exciton immersed in a leaky cavity.

The state vector of the whole system obeys the Schrödinger equation which has a solution of the form $|\psi(t)\rangle = U(t)|\psi(0)\rangle$, where the time-evolution operator $U(t)$ satisfies $\mathrm{i}\partial_t U(t) = HU(t)$ with the initial condition $U(0) = 1$. We assume that the evolution operator has its normal order form $U(t) = U^{(n)}(t)$. Because the normal form of any operator is unique, one can establish the one-to-one corresponding relationship between the normal ordered evolution operator $U^{(n)}(t)$ and an ordinary function $\bar{U}^{(n)}(t)$, with $\bar{U}^{(n)}(t) = \langle \alpha, \{\beta_j\}|U^{(n)}(t)|\{\beta_j\}, \alpha\rangle$. Here $|\{\beta_j\}\rangle = \prod_j |\beta_j\rangle$ denotes multimode coherent state of the radiation field. Such a corresponding relation defines a map $\mathcal{N}^{-1}$,

$$\mathcal{N}^{-1} : U^{(n)}(t) \to \bar{U}^{(n)}(t)\,. \tag{5}$$

On the other hand, we can also define the inverse transformation $\mathcal{N}$,

$$\mathcal{N} : \bar{U}^{(n)}(t) \to U^{(n)}(t) = U(t)\,. \tag{6}$$

Therefore, one can write down the Schrödinger equation of $U(t)$ in the normal ordering form. Then implementing the operator $\mathcal{N}^{-1}$, one can get a c-number equation of $\bar{U}^{(n)}$,

$$\mathrm{i}\partial_t \bar{U}^{(n)} = \Big[\Omega\alpha^*(\alpha + \partial_{\alpha^*}) + \sum_j \omega_j \beta_j^*(\beta_j + \partial_{\beta_j^*}) + \sum_j g(\omega_j)\beta_j^*(\alpha + \partial_{\alpha^*}) + \sum_j g(\omega_j)\alpha^*(\beta_j + \partial_{\beta_j^*}) + \mathcal{R}\,\mathrm{e}^{-\mathrm{i}\omega t}\alpha^* + \mathcal{R}\,\mathrm{e}^{\mathrm{i}\omega t}(\alpha + \partial_{\alpha^*})\Big]\bar{U}^{(n)}\,, \tag{7}$$

where $\partial_{\alpha^*} = \partial/\partial\alpha^*$, and $\partial_{\beta_j^*} = \partial/\partial\beta_j^*$. We assume that $\bar{U}^{(n)}$ takes the form

$$\bar{U}^{(n)} = \exp\Big[A + B\alpha + C\alpha^* + D\alpha^*\alpha + \sum_j B_j \beta_j^*\beta_j + {\sum_{j,j'}}' B_{j,j'}\beta_j^*\beta_{j'}$$

$$+\sum_j C_j\beta_j^*\alpha+\sum_j D_j\alpha^*\beta_j+\sum_j E_j\beta_j^*+\sum_j F_j\beta_j\Big]\,. \tag{8}$$

Here, the prime in $\sum'$ denotes sum over index "j" and "j" with the condition $j \neq j'$. By substituting Eq. (8) into Eq. (7), we get equations which the time-dependent coefficients obey (for the details of calculations, please see Appendix). In this paper we restrict our study to zero temperature situation for the fields where no background radiation is involved in our consideration. At zero temperature the radiation field of the present model is in its pure vacuum state $|\{0_j\}\rangle=\prod_j|0_j\rangle$. Thus only the coefficients A, B, C, D, C_j, and E_j contribute to the state evolution of the whole system. We can write the explicit expressions of these coefficients as

$$D(t)=u(t)\,\mathrm{e}^{-\mathrm{i}\Omega t}-1\,, \tag{9a}$$

$$C(t)=B(t)\,\mathrm{e}^{-\mathrm{i}\omega t}=w(t)\,\mathrm{e}^{-\mathrm{i}\Omega t}\,, \tag{9b}$$

$$A(t)=-\mathrm{i}\mathcal{R}\int_0^t \mathrm{d}t' w(t')\,\mathrm{e}^{\mathrm{i}\delta t'}\,, \tag{9c}$$

$$C_j(t)=u_j(t)\,\mathrm{e}^{-\mathrm{i}\Omega t}\,, \tag{9d}$$

$$E_j(t)=v_j(t)\,\mathrm{e}^{-\mathrm{i}\Omega t}\,, \tag{9e}$$

where $\delta=\omega-\Omega$ is the detuning between the c.w. field and the transition frequency of the two-level atom. The time-dependent functions in the above equations are

$$u(t)=\Big[\cos(\Theta t)+\frac{\Gamma}{2\Theta}\sin(\Theta t)\Big]\,\mathrm{e}^{-(\Gamma/2)t}\,, \tag{10}$$

$$u_j(t)=-\frac{g(\omega_j)}{2}\Big(1-\frac{\mathrm{i}\Gamma}{2\Theta}\Big)\frac{\mathrm{e}^{\mathrm{i}\Theta t}\,\mathrm{e}^{-\Gamma t/2}-\mathrm{e}^{-\mathrm{i}(\omega_j-\Omega)t}}{\omega_j-\Omega+\Theta+\mathrm{i}\Gamma/2}-\frac{g(\omega_j)}{2}\Big(1+\frac{\mathrm{i}\Gamma}{2\Theta}\Big)\frac{\mathrm{e}^{-\mathrm{i}\Theta t}\,\mathrm{e}^{-\Gamma t/2}-\mathrm{e}^{-\mathrm{i}(\omega_j-\Omega)t}}{\omega_j-\Omega-\Theta+\mathrm{i}\Gamma/2}\,, \tag{11}$$

$$w(t)=-\frac{\mathcal{R}}{2}\Big(1-\frac{\mathrm{i}\Gamma}{2\Theta}\Big)\frac{\mathrm{e}^{\mathrm{i}\Theta t}\,\mathrm{e}^{-\Gamma t/2}-\mathrm{e}^{-\mathrm{i}\delta t}}{\delta+\Theta+\mathrm{i}\Gamma/2}-\frac{\mathcal{R}}{2}\Big(1+\frac{\mathrm{i}\Gamma}{2\Theta}\Big)\frac{\mathrm{e}^{-\mathrm{i}\Theta t}\,\mathrm{e}^{-\Gamma t/2}-\mathrm{e}^{-\mathrm{i}\delta t}}{\delta-\Theta+\mathrm{i}\Gamma/2}\,, \tag{12}$$

where $\Theta=\sqrt{M\Gamma-(\Gamma/2)^2}$. The coefficients $A(t)$, and $v_j(t)$ can also be solved. Here we do not give the explicit expressions, for brevity.

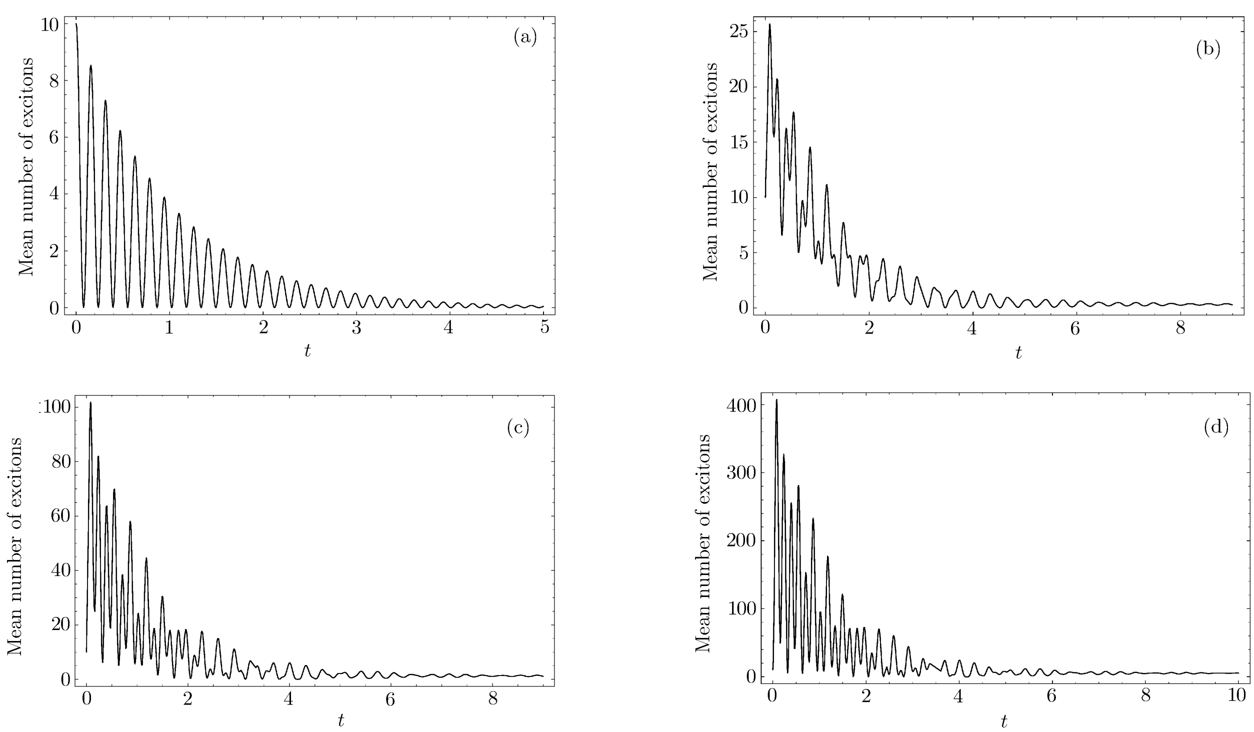

Fig. 1 The mean number of excitons as the function of time with a given set of parameters: $\hbar\Gamma=0.05$ meV, $|\alpha|^2=10$, $\hbar M=20$ meV, $\hbar\delta=0.1$ meV. (a) No pumping case: $\hbar\mathcal{R}=0$ meV, (b) $\hbar\mathcal{R}=5$ meV, (c) $\hbar\mathcal{R}=10$ meV, (d) $\hbar\mathcal{R}=20$ meV.

If, as an example, the initial state of the total system is $|\psi(0)\rangle=|\alpha\rangle\otimes|\{0_j\}\rangle$, then at any time t the total system will evolve into

$$|\psi(t)\rangle = |\alpha u(t)\,\mathrm{e}^{-\mathrm{i}\Omega t} + w(t)\,\mathrm{e}^{-\mathrm{i}\Omega t}\rangle \otimes |\{\alpha u_j(t)\,\mathrm{e}^{-\mathrm{i}\Omega t} + v_j(t)\,\mathrm{e}^{-\mathrm{i}\Omega t}\}\rangle\,, \tag{13}$$

from which we can determine the mean number of excitons at time t as

$$N(t) = |\alpha|^2|u(t)|^2 + |w(t)|^2 + \alpha u(t)w^*(t) + \text{c.c.}\,, \tag{14}$$

where $|\alpha|^2$ is the initial mean number of excitons. In Fig. 1, we plot the mean number of excitons as a function of time. From Fig. 1(a) we find that, without pumping, the mean number of excitons in the cavity oscillates periodically, and decays to zero. However, after applying the pump field (Figs. 1(b) $\sim$ 1(d)), the oscillation amplitude of the mean number becomes more and more larger with the increase of the Rabi frequency, and tends to a non-zero value. We find that the c.w. field compensates the dissipation of excitons due to the damping of cavity.

4 Quantum Decoherence of the Optically Driven Exciton System

If we consider a superposition of distinct coherent states as Schrödinger's cat, i.e., the excitons are initially in the state $C_1|\alpha_1\rangle + C_2|\alpha_2\rangle$, where $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are coherent states of the excitons, and the cavity fields are in the vacuum states $|\{0_j\}\rangle$. Then the state vector at any time t is

$$\begin{aligned}|\Psi(t)\rangle &= C_1\exp[(A + B\alpha_1)/2 - \text{c.c.}]|\alpha_1 u\,\mathrm{e}^{-\mathrm{i}\Omega t} + w\,\mathrm{e}^{-\mathrm{i}\Omega t}\rangle \otimes |\{\alpha_1 u_j\,\mathrm{e}^{-\mathrm{i}\Omega t} + v_j\,\mathrm{e}^{-\mathrm{i}\Omega t}\}\rangle \\ &\quad + C_2\exp[(A + B\alpha_2)/2 - \text{c.c.}]|\alpha_2 u\,\mathrm{e}^{-\mathrm{i}\Omega t} + w\,\mathrm{e}^{-\mathrm{i}\Omega t}\rangle \otimes |\{\alpha_2 u_j\,\mathrm{e}^{-\mathrm{i}\Omega t} + v_j\,\mathrm{e}^{-\mathrm{i}\Omega t}\}\rangle\,,\end{aligned} \tag{15}$$

where we have used the following "sum rules",

$$|\alpha_1|^2 = \sum_j |\alpha_1 u_j + v_j|^2 + |\alpha_1 u + w|^2 + (A + B\alpha_1 + \text{c.c.})\,, \tag{16a}$$

$$|\alpha_2|^2 = \sum_j |\alpha_2 u_j + v_j|^2 + |\alpha_2 u + w|^2 + (A + B\alpha_2 + \text{c.c.})\,, \tag{16b}$$

and

$$\begin{aligned}\langle\alpha_1|\alpha_2\rangle &= \exp[-(B\alpha_1 - \text{c.c.})/2]\exp[(B\alpha_2 - \text{c.c.})/2]\langle\{\alpha_1 u_j\,\mathrm{e}^{-\mathrm{i}\Omega t} + v_j\,\mathrm{e}^{-\Omega t}\}|\{\alpha_2 u_j\,\mathrm{e}^{-\mathrm{i}\Omega t} + v_j\,\mathrm{e}^{-\mathrm{i}\Omega t}\}\rangle \\ &\quad\times\langle\alpha_1 u\,\mathrm{e}^{-\mathrm{i}\Omega t} + w\,\mathrm{e}^{-\mathrm{i}\Omega t}|\alpha_2 u\,\mathrm{e}^{-\mathrm{i}\Omega t} + w\,\mathrm{e}^{-\mathrm{i}\Omega t}\rangle\,,\end{aligned} \tag{16c}$$

under the consideration of normalization condition of the wave function, equation (15) is one of the main results of our study, from which we see that, due to the field's fluctuation and the back-action of system on the fields, the state vector evolved from factorized initial state becomes fully entangled. However under certain conditions the total state vector can be partially factorized.[19] In the following context of this paper we will investigate the effect of the external c.w. field on the quantum decoherence of the superposition of the excitons.

We can calculate the reduced density matrix of the exciton system $\rho(t) = \mathrm{Tr}_R\{|\Psi(t)\rangle\langle\Psi(t)|\}$. Substituting Eq. (15) into $\rho(t)$, we get the decoherence factor, which is defined as the coefficient of the off-diagonal element of the reduced density matrix,

$$F(t) = \exp\Big[\Big(-\frac{1}{2}|\alpha_1|^2 - \frac{1}{2}|\alpha_2|^2 + \alpha_1^*\alpha_2\Big)(1 - |u(t)|^2)\Big]\,\mathrm{e}^{(\alpha_1-\alpha_2)u(t)w^*(t)/2-\text{c.c.}}\,, \tag{17}$$

where we have used Eq. (16c) in deriving Eq. (17). The explicit expressions of the time-dependent functions $u(t)$ and $w(t)$ are given in Eq. (10), and Eq. (12), respectively. We consider that $\alpha_1 = \alpha$, and $\alpha_2 = \alpha\,\mathrm{e}^{\mathrm{i}\Delta\varphi}$, where $\Delta\varphi$ is the phase shift of the initial superposed states. The characteristic time τ_d of the decoherence of the superposition state is determined by the short time behavior of $|F(t)|$, that is $\Gamma t, \Theta t \ll 1$. Within this time scale the norm of decoherence factor can be simplified as

$$|F(t)| = \exp[-2|\alpha|^2\sin^2(\Delta\varphi/2)\Gamma t]\,. \tag{18}$$

Then the characteristic time is determined as

$$\tau_d^{-1} = 2|\alpha|^2\Gamma\sin^2(\Delta\varphi/2)\,, \tag{19}$$

where $|\alpha|^2$ is the mean number of the excitons. We can define the "distance" $D = |\alpha_1 - \alpha_2| = 2|\alpha|\sin(\Delta\varphi/2)$ between the two superposed states of the exciton. Substituting D into Eq. (19), we get the characteristic time of decoherence $\tau_d = 2\tau_p/D^2$, where $\tau_p = 1/\Gamma$ is the life time of the quasimode. Our result shows that the decoherence time of the exciton is determined by the distance of their initial superposed states and the decay rate of the quasimode. The external c.w. laser field does not change the decoherence time of the exciton. One can suppress the decoherence speed of the exciton by adjusting the distance of the initial superposition of the exciton.[12]

For a special case $\Delta\varphi = \pi$, i.e., when the system is prepared initially in odd or even Schrödinger cat states[20−23] of the exciton. Then the decoherence factor is

$$F(t) = \exp[-|\alpha|^2(1 - |u(t)|^2)]\,\mathrm{e}^{\mathrm{i}\phi(t)}\,, \tag{20}$$

where we have introduced the phase of the decoherence factor in Eq. (20),

$$\phi(t) = \mathrm{Im}\,[\alpha u(t)w(t)^* - \text{c.c.}]\,, \tag{21}$$

where Im [...] represents to take imaginary part. We find that the phase of the decoherence factor depends on the external field and can be controlled artificially by adjusting field detuning δ and Rabi frequency $\mathcal{R}$. However, as mentioned above, the decoherence rate is determined by the norm of $F(t)$. The time evolution of $|F(t)| = \exp[-|\alpha|^2(1-|u(t)|^2)]$ is plotted in Fig. 2. We find that the decoherence rate strongly depends on the initial mean number of excitons. For the case $|\alpha|^2 = 3.1$ (solid line), the coherent information of the Schrödinger cat state decays more rapidly than that of small mean number case $|\alpha|^2 = 0.01$ (dotted line).

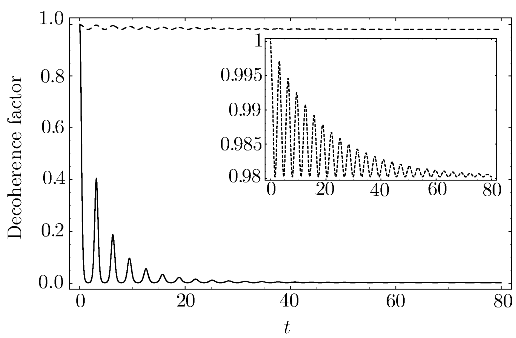

Fig. 2 Time evolution of $|F(t)|$ when the excitons are initially in the even-odd Schrödinger cat state. $\hbar\Gamma = 0.05$ meV, $\hbar M = 20$ meV. Solid line: $|\alpha|^2 = 3.1$; dotted line: $|\alpha|^2 = 0.01$. Inset: detail for $|\alpha|^2 = 0.01$.[12]

5 Conclusions

We have studied the dynamical evolution of the superposition of the mesoscopically distinct quantum state in a system of an optically-driven exciton in a quasimode cavity. By utilizing normal ordering technique, the explicit expression of state vector at any time is obtained in the case of no background radiation. The influence of c.w. field on the mean number of excitons in the lossy cavity is also studied. We find that the field compensates the loss of population of excitons. By adjusting the external parameters we can obtain different value of mean number of excitons at the long-time limit.

By solving the explicit form of the decoherence factor, we investigate the decoherence behavior of the exciton system and find that the decoherence rate of the exciton does not depend on the c.w. laser field. However the phase of the decoherence rate can be well controlled by adjusting the amplitude of the external field as well as the detuning between the field and the transition frequency of the atom.

Acknowledgments

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Appendix: Normal Ordering Method

In this appendix, we will give the equations of the time-dependent coefficients and the formal solution of the equations. By substituting Eq. (8) into Eq. (7), we have

$$\dot{A} = -\mathrm{i}\mathcal{R}\,\mathrm{e}^{\mathrm{i}\omega t}C\,, \tag{A1a}$$

$$\dot{B} = -\mathrm{i}\mathcal{R}\,\mathrm{e}^{\mathrm{i}\omega t}(1+D)\,, \tag{A1b}$$

$$\dot{C} = -\mathrm{i}\Omega C - \mathrm{i}\sum_j g(\omega_j)E_j - \mathrm{i}\mathcal{R}\,\mathrm{e}^{-\mathrm{i}\omega t}\,, \tag{A1c}$$

$$\dot{D} = -\mathrm{i}\Omega(1+D) - \mathrm{i}\sum_j g(\omega_j)C_j\,, \tag{A1d}$$

$$\dot{E}_j = -\mathrm{i}\omega_j E_j - \mathrm{i}g(\omega_j)C\,, \tag{A1e}$$

$$\dot{C}_j = -\mathrm{i}\omega_j C_j - \mathrm{i}g(\omega_j)(1+D)\,. \tag{A1f}$$

By solving Eqs. (A1), we get formal solutions of these time-dependent coefficients as Eqs. (9). We have introduced four new functions

$$u(t) = \mathcal{L}^{-1}\{\tilde{u}[s]\}\,, \tag{A2a}$$

$$u_j(t) = \mathcal{L}^{-1}\Big\{\frac{-\mathrm{i}g(\omega_j)\tilde{u}[s]}{s+\mathrm{i}(\omega_j-\Omega)}\Big\}\,, \tag{A2b}$$

$$w(t) = \mathcal{L}^{-1}\Big\{\frac{-\mathrm{i}\mathcal{R}\tilde{u}[s]}{s+\mathrm{i}\delta}\Big\}\,, \tag{A2c}$$

$$v_j(t) = \mathcal{L}^{-1}\Big\{\frac{-\mathrm{i}g(\omega_j)\tilde{w}[s]}{s+\mathrm{i}(\omega_j-\Omega)}\Big\}\,, \tag{A2d}$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transformation, and $\tilde{u}[s] = 1/(s+\tilde{K}[s])$. Now we need to determine the explicit form of the kernel function $K(t-t') = \sum_j |g(\omega_j)|^2\,\mathrm{e}^{-\mathrm{i}(\omega_j-\Omega)(t-t')}$, and further $\tilde{K}[s]$, the Laplace transformation of $K(t)$. As the standard treatment[12] we firstly change the sum $\sum_j$ in $K(t-t')$ into the integration $(L/\pi c)\int_0^\infty \mathrm{d}\omega_j$, where L is the length of the cavity and c is the speed of the light in the vacuum,[24] i.e.,

$$K(t-t') = \frac{\eta^2\Gamma^2 NL}{\pi c}\int_0^\infty \frac{\mathrm{e}^{-\mathrm{i}(\omega_j-\Omega)(t-t')}}{(\omega_j-\Omega)^2+\Gamma^2}\mathrm{d}\omega_j\,. \tag{A3}$$

If we assume that Ω is much larger than all other quantities of the dimension of frequency and Γ is small quantity, then we may adopt to the standard approximation of extending the lower limit of the integral Eq. (A3) to $-\infty$. By integrating Eq. (A3) we get

$$K(t-t') = M\Gamma\,\mathrm{e}^{-\Gamma|t-t'|} \tag{A4}$$

with $M = N\eta^2 L/c$. As long as the kernel function is determined, one can solve time-dependent functions defined in Eqs. (A2) with the help of the Laplace transformation of $K(t-t')$.

References

[1] D. Bouwmeester, A.K. Ekert, and A. Zeilinger (eds), *The Physics of Quantum Information*, Springer-Verlag, Berlin, Heidelberg, New York (2000).

[2] W.H. Zurek, Phys. Today **44** (1991) 36.

[3] A.O. Caldeira and A.J. Leggett, Ann. Phys. (N.Y.) **149** (1983) 374; A.J. Leggett, S. Chakravarty, A.T. Dosey, M.P.A. Fisher, and W. Zwerger, Rev. Mod. Phys. **59** (1987) 1.

[4] L.H. Yu and C.P. Sun, Phys. Rev. **A49** (1994) 592; C.P. Sun and L.H. Yu, ibid. **A51** (1995) 1845.

[5] W.H. Zurek, Philos. Trans. R. Soc. London, Ser. **A356** (1998) 1793.

[6] C.P. Sun, H. Zhan, and X.F. Liu, Phys. Rev. **A58** (1998) 1810.

[7] Gang Chen, N.H. Bonadeo, D.G. Stell, D. Gammon, D.S. Katzer, D. Park, and L.J. Sham, Science **289** (2000) 1906.

[8] M. Bayer, P. Hawrylak, K. Hinzer, S. Fafard, M. Korkusinski, Z.R. Wasilewski, O. Stern, and A. Forchel, Science **291** (2001) 451.

[9] L. Quiroga and N.F. Johnson, Phys. Rev. Lett. **83** (1999) 2270.

[10] X.X. Yi, G.R. Jin, and D.L. Zhou, Phys. Rev. **A63** (2001) 062307.

[11] Jonathan R. Friedman, Vijav Patel, W. Chen, S.K. Tolpygo, and J.E. Lukens, Nature **406** (2000) 43; Caspar H. van der Wal, A.C.J. ter Haar, F.K. Wilhelm, R.N. Schouten, C.P.M. Harmans, T.P. Orlando, Seth Lloyd, and J.E. Mooij, Science **290** (2000) 773.

[12] Yu-Xi Liu, C.P. Sun, and S.X. Yu, Phys. Rev. **A63** (2000) 033816.

[13] W.H. Louisell, *Quantum Statistical Properties of Radiation*, John Wiley & Sons, New York (1990).

[14] G.R. Jin, D.L. Zhou, Yu-Xi Liu, X.X. Yi, and C.P. Sun, Commun. Theor. Phys. (Beijing, China) **36** (2001) 331.

[15] Yu-Xi Liu, C.P. Sun, S.X. Yu, and D.L. Zhou, Phys. Rev. **A63** (2000) 023802.

[16] M.O. Scully and M.S. Zubairy, *Quantum Optics*, Cambridge University Press, England (1997).

[17] C.K. Law, T.W. Chen, and P.T. Leung, Phys. Rev. **A61** (2000) 023808.

[18] H. Haken, *Quantum Field Theory of Solids*, North-Holland Publishing Company, Amsterdam (1976).

[19] C.P. Sun, Y.B. Gao, H.F. Dong, and S.R. Zhao, Phys. Rev. **E57** (1998) 3900.

[20] J.F. Poyatos, J.I. Cirac, and P. Zoller, Phys. Rev. Lett. **77** (1996) 4728.

[21] K.M. Fonseca Romero, M.C. Nemes, J.G. Peixoto de Faria, A.N. Salgueiro, and A.F.R. de Toledo Piza, Phys. Rev. **A58** (1998) 3205.

[22] M.C. de Oliveira and W.J. Munro, Phys. Rev. **A61** (2000) 42309.

[23] M.C. de Oliveira, M.H. Moussa, and S.S. Mizrahi, Phys. Rev. **A61** (2000) 63809.

[24] Roy Lang, M.O. Scully, and Jr. Willis E. Lamb, Phys. Rev. **A7** (1973) 1788.