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Generalized Two-State Theory for an Atom Laser and Its Interference*

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Abstract *We present a generalized two-state theory to study the output coupler for Bose–Einstein condensate atoms and its interference. The results show that the fraction of out-coupled atoms can be adjusted between 0% and 100% by varying the amplitude of the rf. radiation, and the interference patterns of the two coherent outputs that are coupled out of a double-well trapped potential vary with time. Moreover, we study the influence of atom-atom interactions on the output. In contrast with the case without atom-atom interactions, collapses and revivals appear in the number of the output atoms.*

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I. Introduction

The recent realization of Bose–Einstein condensation (BEC) in atomic gases[1–4] provides a chance to generate a bright beam of atoms with a very narrow energy spectrum — an atom laser.[5] An atom laser would have many applications in atom optics including atom lithography and nanofabrication, as well as fundamental tests of quantum theory for describing both the interaction between atoms and matter and the interaction between atoms and the radiation field.

A number of theoretical atom laser schemes have already been proposed.[6–11] These have involved some methods of cooling atoms in an atomic cavity, a schematic model for coupling atoms to external freely traveling atomic modes and some methods for output coupling based on changing the internal atomic state using a Raman transition in a spatially localized region. For example, Wiseman[7] used dark state cooling as the mechanism in which the atoms are transferred from the source to the lasing mode irreversibly by spontaneous emission. Holland *et al.*[6] and Guzman *et al.*[8] used inelastic binary collisions to transfer atoms from the source to the lasing mode. This process is made irreversible by using evaporative cooling to rapidly remove the high energy atoms from the system.[6] Whereas in Ref. [8], the number of atoms in the lasing mode depends on the pumping and loss rates. Moy *et al.*[9] presented an atom laser scheme using a Raman transformation and two atomic cavities, one of the cavities is for the source atoms, the other has only one significantly populated mode for output atoms. The role of Raman transition is to change the state of atoms in ground mode (lasing level) in one cavity to a nontrapped state (output level) in other cavity. Hope[11] used a set of external modes to couple atoms out of trapped cavity, it provides a new formalism for describing the interaction between an atomic cavity and a continuum of external atomic modes.

In contrast with the above models, the key feature of the present paper is to develop a two-state theory for an atom coupler. The paper is organized as follows. In Sec. II, we derive the Hartree–Fock equations, which describe the spatial distribution of trapped and untrapped atoms. In Sec. III, ignoring the atom-atom interactions, we introduce the usual Bogoliubov transformation to discuss the properties of out-coupled atoms and its interference. Considering

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the atom-atom interactions, the coherence loss is discussed in Sec. IV. In Sec. V, we discuss the influence of the atom-atom interactions on the out-coupled atoms. Finally, in Sec. VI, we summarize our conclusions and discuss some limitations of our work.

II. The Hartree–Fock Equations

In this section, we derive the Hartree–Fock (HF) equations within the framework of the variational principle. The solutions of these equations represent the amplitude of trapped and untrapped atoms' spatial distributions.

The ground-state free energy is the expectation value of $\hat{H}-\mu\hat{N}$, where $\hat{H}$ is the many-body Hamiltonian of the boson system, $\hat{N}$ the number operator, and μ the chemical potential,

$$\hat{H}-\mu\hat{N}=\int \mathrm{d}^3r\hat{\psi}^\dagger(r)H_0\hat{\psi}(r)+\frac{1}{2}\int \mathrm{d}^3r\mathrm{d}^3r'\hat{\psi}^\dagger(r)\hat{\psi}^\dagger(r')V(r-r')\hat{\psi}(r')\hat{\psi}(r)\,, \tag{1}$$

where $\hat{\psi}^\dagger(r)$ creates an atom, $\hat{\psi}(r)$ annihilates one at position r, $V(r-r')$ stands for the interatomic potential, $H_0(r)$ is the one-body part of the free energy

$$H_0(r)=-\frac{\nabla^2}{2m}+V_{\rm ex}(r)-\mu\,, \tag{2}$$

where $V_{\rm ex}(r)$ is the external potential. To find equations for single-particle orbitals, a variational principle is applied to the ground-state free energy. Using the pseudopotential approximation and writing $\hat{\psi}(r)=\sum_i \hat{a}_i\phi_i(r)$ the atom-atom interaction $V_{\rm aa}$ in free energy becomes

$$V_{\rm aa}=\frac{g}{2}\sum_{ijkl}\int \mathrm{d}^3r\phi_i^*(r)\phi_j^*(r)\phi_k(r)\phi_l(r)\langle\hat{a}_i^\dagger\hat{a}_j^\dagger\hat{a}_k\hat{a}_l\rangle\,, \tag{3}$$

where the bracket $\langle\cdots\rangle$ denotes a thermal expectation value and a short-range interaction $V(r-r')=g\delta(r-r')$ is used. Using the generalized Wick's theorem, this expectation value is expressed in terms of all possible contractions

$$\begin{aligned}\langle\hat{a}_i^\dagger\hat{a}_j^\dagger\hat{a}_k\hat{a}_l\rangle&=\langle\hat{a}_i^\dagger\hat{a}_j^\dagger\rangle\langle\hat{a}_k\hat{a}_l\rangle+\langle\hat{a}_i^\dagger\hat{a}_k\rangle\langle\hat{a}_j^\dagger\hat{a}_l\rangle+\langle\hat{a}_i^\dagger\hat{a}_l\rangle\langle\hat{a}_j^\dagger\hat{a}_k\rangle\\&=\langle\hat{a}_i^\dagger\hat{a}_j^\dagger\rangle\langle\hat{a}_k\hat{a}_l\rangle+N_i\delta_{ik}N_j\delta_{jl}+N_i\delta_{il}N_j\delta_{jk}\,.\end{aligned} \tag{4}$$

With this notation, the mean-field expression for the ground-state energy reads

$$F=\langle\hat{H}-\mu\hat{N}\rangle=h_1+v_{\rm dir}+v_{\rm exch}+v_{\rm pair}\,, \tag{5}$$

where $h_1=\sum_i N_i\phi_i^*(r)H_0\phi_i(r)$ is the one-body contribution to the ground state energy;

$$v_{\rm dir}=\frac{g}{2}\int \mathrm{d}^3r\sum_{ij}\phi_i^*(r)\phi_j^*(r)\phi_i(r)\phi_j(r)N_iN_j$$

denotes the direct energy contribution to the energy in analogy with the Hartree–Fock theory;

$$v_{\rm exch}=\frac{g}{2}\int \mathrm{d}^3r\sum_{ij}\phi_i^*(r)\phi_j^*(r)\phi_j(r)\phi_i(r)N_iN_j$$

stands for the exchange energy in comparison with the Hartree–Fock theory;

$$v_{\rm pair}=\frac{g}{2}\int \mathrm{d}^3r\sum_{ijkl}\phi_i^*(r)\phi_j^*(r)\phi_k(r)\phi_l(r)\langle\hat{a}_i^\dagger\hat{a}_j^\dagger\rangle\langle\hat{a}_k\hat{a}_l\rangle$$

describes the pair energy, which is absence for the Hartree–Fock expressions. The extremum of F with respect to N_i leads to an eigenvalue equation for ϕ_i,

$$H_0\phi_i+2gN_i|\phi_i|^2\phi_i+g\sum_{j\neq i}N_j|\phi_j|^2\phi_i=0\,. \tag{6}$$

The scheme of an atom laser is most easily discussed using the following two-level model:[10] N bosons can occupy only two one-particle states $\phi_0(r)$ and $\phi_1(r)$ taken orthonormally, $\int \mathrm{d}^3r\phi_i^*\phi_j=\delta_{ij}$. According to Eq. (6), $\phi_0(r)$ and $\phi_1(r)$ are determined by

$$H_0\phi_0+2gN_0|\phi_0|^2\phi_0+gN_1|\phi_1|^2\phi_0=0\,,\qquad H_0\phi_1+2gN_1|\phi_1|^2\phi_1+gN_0|\phi_0|^2\phi_1=0\,. \tag{7}$$

This is a set of coupled HF equations (Each of them is time-independent Gross–Pitaevskii equation). In order to get the solutions of the HF equations, one must solve the coupled HF equations self-consistently, we refer the detailed method of solving the HF equations to

Ref. [13]. Strictly, the temporal coordinate should be involved in Eqs (7), this can be done by converting H_0 into $H_0 - \mathrm{i}\hbar(\partial/\partial t)$.[12]

III. Fraction of Out-Coupled Atoms and Its Interference

In MIT experiment, the untrapped level couples with the trapped level through a resonant rf. pulse. The trapped state evolves into a superposition of the trapped and untrapped states, which makes a coherent beam of atoms out of trapped level.

In order to calculate the fraction of the out-coupled atoms, we use the second quantized Hamiltonian

$$\hat{H} - \mu\hat{N} = e_{00}a_0^\dagger a_0 + e_{11}a_1^\dagger a_1 + [(e_{10}+v)a_1^\dagger a_0 + \mathrm{h.c.}] + \frac{1}{2}\sum_{ijkl} u_{ijkl}a_i^\dagger a_j^\dagger a_k a_l\,, \tag{8}$$

which is easily derived from Eq. (1) by using $\hat{\psi}(r) = a_0\phi_0 + a_1\phi_1$. Here

$$e_{ij} = \int \mathrm{d}^3 r\phi_i^* H_0\phi_j\,, \qquad u_{ijkl} = g\int \mathrm{d}^3 r\phi_i^*\phi_j^*\phi_k\phi_l\,.$$

We have introduced the term $v = v(k,F) = F\int \mathrm{d}^3 r\phi_1^*\,\mathrm{e}^{\mathrm{i}kr}\phi_0$ to describe the interaction between the atoms and the rf. radiation, which varies with the amplitude F of rf. field.

For clarification, we drop the interaction among the atoms, it is given that

$$\hat{H} - \mu\hat{N} = e_{00}a_0^\dagger a_0 + e_{11}a_1^\dagger a_1 + [(e_{10}+v)a_1^\dagger a_0 + \mathrm{h.c.}]\,. \tag{9}$$

Through the usual Bogoliubov transformation

$$a_0 = -\cos\theta A + \sin\theta B\,, \qquad a_1 = \sin\theta A + \cos\theta B\,, \tag{10}$$

the Hamiltonian can be diagonalized

$$\hat{H} - \mu\hat{N} = E_A A^\dagger A + E_B B^\dagger B\,, \tag{11}$$

where θ satisfies

$$\sin\theta\cos\theta(e_{11}-e_{00}) - \cos^2\theta(e_{10}+v) + \sin^2\theta(e_{01}+v^*) = 0 \tag{12}$$

and the energy of quasiparticles is

$$E_A = e_{00}\cos^2\theta + e_{11}\sin^2\theta - \mathrm{Re}\,(e_{10}+v)\sin\theta\cos\theta\,,$$

$$E_B = e_{00}\sin^2\theta + e_{11}\cos^2\theta + \mathrm{Re}\,(e_{10}+v)\sin\theta\cos\theta\,. \tag{13}$$

The transformations enable us to establish a relation between the quasiparticle (QP) state and the resulting particle (RP) state. First, the QP vacuum state $|0\rangle_{\mathrm{QP}}$ defined by $A|0\rangle_{\mathrm{QP}} = B|0\rangle_{\mathrm{QP}} = 0$ is just the RP vacuum state, i.e., $|0\rangle_{\mathrm{RP}} = |0\rangle_{\mathrm{QP}}$. Second, the coherent states of the quasiparticles are related to the coherent state of resulting particles. The connection between the parameters of both the coherent states are

$$\alpha_{a_0} = -\cos\theta\alpha_A + \sin\theta\alpha_B\,, \qquad \alpha_{a_1} = \sin\theta\alpha_A + \cos\theta\alpha_B\,, \tag{14}$$

where α_i $(i = a_0, a_1, A, B)$ stands for the coherent state of i particle.

In Ref. [10], the authors achieved a Bose condensate of atoms in trapped state, then turned on a resonant rf. radiation that couples the trapped and untrapped states during time τ approximated 5 μs, the output pulse of atoms in the "repelled" state was observed after the rf. interaction. To compare with the experiment, we assume that the atoms are initially in condensate, in other words, the initial condition is

$$|\varphi(0)\rangle = |0\rangle_1 \otimes |\alpha_0\rangle_0\,, \tag{15}$$

which satisfies $a_0|\varphi(0)\rangle = \alpha_0|\varphi(0)\rangle$, $a_1|\varphi(0)\rangle = 0$. The interference between two freely expanding Bose–Einstein condensations has confirmed that the Bose condensed atoms are coherent and show long-range correlations.[5] These enable us to take Eq. (15) as the initial condition. At time t, the state evolves to

$$|\varphi(t)\rangle = |\alpha_0\sin^2\theta\,\mathrm{e}^{-\mathrm{i}E_B t} + \alpha_0\cos^2\theta\,\mathrm{e}^{-\mathrm{i}E_A t}\rangle_0$$

$$\otimes\,|-\alpha_0\sin\theta\cos\theta\,\mathrm{e}^{-\mathrm{i}E_A t} + \alpha_0\sin\theta\cos\theta\,\mathrm{e}^{-\mathrm{i}E_B t}\rangle_1\,, \tag{16}$$

where $|\cdots\rangle_0$ and $|\cdots\rangle_1$ denote the coherent trapped and untrapped states, respectively. The number of atoms coupled out of the condensate reads

$$N_1 = \langle\varphi(t)|a_1^\dagger a_1|\varphi(t)\rangle = \frac{|\alpha_0|^2}{2}\sin^2 2\theta[1-\cos(E_A-E_B)t]\,. \tag{17}$$

It is obvious from Eqs (16) and (17) that at time t the N-atom wavefunction evolves to a superposition of both the coherent states for trapped and untrapped atoms. Moreover, the total N-body wavefunction factorizes into the condensate and the untrapped pulse of atoms. After the interaction of the rf. pulse of duration τ, the fraction of atoms coupled out of the condensate oscillates with the amplitude of rf. pulse through θ as

$$\frac{N_1}{N} = \frac{N - N_0}{N} = \frac{1}{2}\sin^2 2\theta[1 - \cos(E_A - E_B)\tau]\,. \tag{18}$$

The fraction of out-coupled atoms, as showed in Eq. (18), can be adjusted between 0% and 100% by varying the amplitude of the rf. radiation. As illustrated in Fig. 1 the results are in good agreement with the experiment. In a recent interference experiment reported in Ref. [5], using Ioffe trap, 5×10^6 sodium atoms are collected in a cigar-shaped trap volume which is divided into two halves along the longitudinal direction by a sheet of far-blue detuned laser light, the condensate expansion is started by switching off the trapped field and eventually leads to an overlap of the two clouds and to an expanding interference pattern. Based on the NLSE, A. Röhrl *et al.*[14] have shown the excellent agreement between this experiment and the theoretical prediction. In contrast with this experiment and theoretical predictions, we predict that for double-well trap two atom beams (one coupled out of the left well and another of the right one) will overlap and lead to an interference pattern. This intuition arises from the coherence of the atoms coupled out of the condensate.

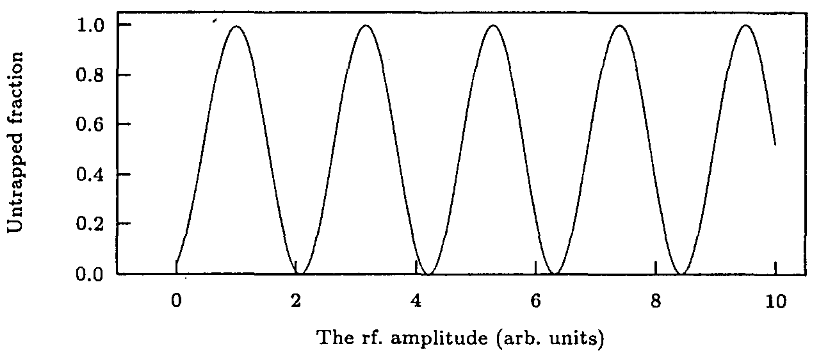

Fig. 1. The fraction of the out-coupled atoms versus the amplitude of the rf. pulse. All parameters are taken in arbitrary units. $e_{00} = 1.0$, $e_{11} = 3.0$, $e_{10} = e_{01} = 2.0$, $g = 1$.

To show the interference pattern, we shall compute the space-dependent wavefunction for the atoms coupled out of the condensate. Using Eq. (16), the wavefunction of those atoms reads

$$|\varphi_1(r,t)\rangle = \langle\varphi(t)|a_1\,\mathrm{e}^{\mathrm{i}kr}\phi_1(r)|\varphi(t)\rangle = \mathrm{e}^{\mathrm{i}kr}\phi_1(r)f\,\mathrm{e}^{\mathrm{i}Et}\,, \tag{19}$$

where

$$f = f(t,\theta,\alpha_0,g) = \sum_{n=0}^{\infty}\frac{|\beta|^{2n}}{\sqrt{n!(n-1)!}\,\alpha_0\sin\theta\cos\theta\,\mathrm{Mod}(\mathrm{e}^{-\mathrm{i}E_Bt} - \mathrm{e}^{-\mathrm{i}E_At})}\,\mathrm{e}^{-|\beta|^2}\,,$$

$$E = \frac{\mathrm{e}^{-\mathrm{i}E_Bt} - \mathrm{e}^{-\mathrm{i}E_At}}{\mathrm{Mod}(\mathrm{e}^{-\mathrm{i}E_Bt} - \mathrm{e}^{-\mathrm{i}E_At})}\,, \qquad \beta = \alpha_0\sin\theta\cos\theta(\mathrm{e}^{-\mathrm{i}E_Bt} - \mathrm{e}^{-\mathrm{i}E_At})\,,$$

and $\mathrm{Mod}(\cdots)$ denotes the module of the terms in brackets. The terms $\mathrm{e}^{\mathrm{i}kr}$ in Eq. (19) arise from the atom-rf. pulse interaction and the gravity acceleration, the rf. radiation excited an atom from the trapped state to untrapped state with a momentum transfer, whereas the gravity accelerated downward the output atoms. On the analogy of Eq. (19), we introduce $|\varphi_{R(L)}(r,t)\rangle = \mathrm{e}^{\mathrm{i}k_{R(L)}(r+(-)d)}\phi_{R(L)}(r)f_{R(L)}\,\mathrm{e}^{\mathrm{i}E_{R(L)}t}$ to denote the wavefunction of the atoms coupled out of the right- (left-)trapped well. With this notation, the interference pattern is

$$N(r,t) = \sum_{i=R,L}|\phi_i|^2|f_i|^2 + 2\phi_R\phi_Lf_Rf_L\cos[(K_R - K_L)r + (K_L + K_R)d + (E_R - E_L)t]\,. \tag{20}$$

In general, the difference between the two wave vectors $|K_R - K_L|$ is so small that the term $(K_R - K_L)r$ in Eq. (20) can be neglected. Thus the fringe spacing of the interference pattern is mainly dependent on the separation of the double wells. Considering classically the wave vector as $K_{R(L)} = (r/t)m$, our result is that the fringe spacing is directly proportional to time and inversely proportional to the separation d. Neglecting the interactions among the atoms, both ϕ_R and ϕ_L are determined by Eq. (7) with $g = 0$ to be a Gaussian packet. Thus the first two terms in Eq. (20) are Gaussian, but the third term is more complicated. Figure 2 displays a theoretical interference pattern for $g = 0$ in x direction, from which we note that the contrast of the interference is reduced in the central region and vanishes rapidly outside the center. This is very similar to the theoretical and experimental results for two independent condensates. The phase difference between the beams which varies only between different runs of the experiment has been dropped here, it will result in a shift of the interference pattern.

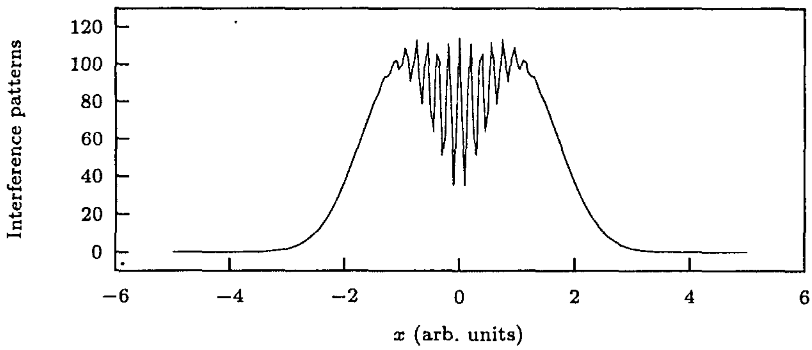

Fig. 2. Interference patterns in x direction after the interaction of an rf. pulse with duration $\tau = 5$ μs. Here we neglect the interaction among the trapped and untrapped atoms.

In the case of considering atom-atom interaction, the fringe spacing of the interference patterns is very similar to what we stated above. On the analogy of the results stated in Refs [14] ~ [16], the fringe spacing in position space increases with a velocity proportional to $\hbar\pi/m\tilde{d}$, where $2\tilde{d}$ is the mean space separation of the atoms in the double well.

IV. Coherence Loss

As mentioned in Sec. III, all conclusions derived from the Hamiltonian (9) are based on the assumption that the interactions among the trapped and untrapped atoms can be dropped. However, thermalizing collisions will play a crucial role for long evolution time. Unfortunately, no successful theory has been proposed for the treatment of the dynamics of inhomogeneous condensates in such a nonequilibrium situation. In order to obtain a preliminary estimate for the influence of atom-atom interactions on the coherence, we resort to the following consideration, which is based on the short-time perturbative expansion.[17]

For two independent Bose condensates, the theory of correlation functions showed that[15] a visible interference between the two condensates requires conditions

$$\langle a_1^\dagger a_1\rangle = \langle a_2^\dagger a_2\rangle \sim \frac{N}{2}\,, \qquad \frac{|\langle a_i^\dagger(t)a_i(t+\tau)\rangle|}{\langle a_i^\dagger a_i\rangle} \simeq 1\,.$$

The first condition implies that the occupation of the two condensate modes ($i = 1, 2$) has to be comparable to the total number of atoms N, whereas the second condition requires that the phase coherence has to be large during the observation time τ, in which the spatial pattern is recorded. In other words, visibility of these fringes requires a negligible decay of the first order correlation function for the experimental observation time τ. In this section, we do not deal with the problem of visibility of these fringes discussed in Sec. III, but shed some light on the coherence of the atoms coupled out of the condensate.

The short-time perturbative expansion, reported in Ref. [17], can be summarized as follows. Considering two interacting subsystems, the total Hamiltonian can be written as

$$H = H_{01} + H_{02} + H_{\rm int}\,, \tag{21}$$

where H_{01} (H_{02}) describes the subsystem 1 (subsystem 2), $H_{\rm int}$ represents the interaction between the subsystems. According to the quantum coherence theory, $\delta_i(t) = {\rm Tr}_i\{\rho_i(t) - \rho^2(t)\}$ describes the decoherence of the subsystem i, where $\rho_i(t)$ is the reduced density operator of subsystem i and ${\rm Tr}_i$ denotes a trace over subsystem i. In terms of a short-time power series, the decoherence $\delta_2(t)$ becomes

$$\delta_2(t) = 1 - {\rm Tr}_2\Big(\rho_2^2(0) + t\frac{{\rm d}\rho_2^2}{{\rm d}t}(0) + \frac{t^2}{2}\frac{{\rm d}^2\rho_2^2}{{\rm d}t^2}(0) + \cdots\Big) = \delta_0 - \frac{t}{\tau_1} - \frac{t^2}{\tau_2^2} - \cdots \tag{22}$$

with definitions

$$\begin{aligned} &\delta_0 = 1 - {\rm Tr}_2(\rho_2^2(0))\,, \quad 1/\tau_1 = 2{\rm i}{\rm Tr}_2\{\rho_2(0){\rm Tr}_1[\rho(0), H]\}\,, \\ &1/\tau_2^2 = -{\rm Tr}_2\{({\rm Tr}_1[\rho(0), H])^2\} - \langle {\rm Tr}_1[[\rho(0), H], H]\rangle_{\rho_2(0)}\,, \end{aligned} \tag{23}$$

where the symbol $\langle\cdots\rangle_{\rho_2(0)}$ denotes an average value over the subsystem 2, $\rho(0)$ stands for the total density operator at $t = 0$. In the present case the subsystems 1 and 2 are the trapped and untrapped atoms, respectively. As showed in Sec. III, the atoms coupled out of the condensate are coherent in the situation of ignoring the interactions among the atoms. However, the interactions will play a crucial role for long evolution time. In order to study the influence of atom-atom interactions on the coherence of the out-coupled atoms clearly, we reduce the Hamiltonian (8) to the energy conserving terms[6]

$$\begin{aligned} H = {} & e_{00}a_0^\dagger a_0 + e_{11}a_1^\dagger a_1 + (e_{10} + v)a_1^\dagger a_0 + (e_{01} + v^*)a_0^\dagger a_1 \\ & + \tfrac{1}{2}gu_{0000}a_0^\dagger a_0^\dagger a_0 a_0 + \tfrac{1}{2}gu_{1111}a_1^\dagger a_1^\dagger a_1 a_1 + 2gu_{1010}a_0^\dagger a_0 a_1^\dagger a_1\,. \end{aligned} \tag{24}$$

This approximation does not consider those processes that are energetically unfavored, such as the annihilation of the two ground-state atoms producing two atoms in high energy. These unfavored terms will be discussed further.

In accordance with the experiment in MIT, we take

$$|\varphi(0)\rangle = |\alpha_0\rangle_0 \otimes |\alpha_1\rangle_1 \tag{25}$$

as the initial condition, in other words, the trapped atoms are initially in a coherent state $|\alpha_0\rangle_0$, while the untrapped atoms are in a coherent state $|\alpha_1\rangle_1$. Starting from this initial condition, we obtain

$$\delta_0 = 0\,, \qquad 1/\tau_1 = 0\,, \qquad 1/\tau_2^2 = -4g^2u_{0101}^2|\alpha_1|^2|\alpha_0|^2\,. \tag{26}$$

These results imply that the decoherence time depends on the coupling constant g, spatial distribution of the trapped and untrapped atoms and the initial conditions. In experiment,[10] the atoms are prepared in condensate at $t = 0$, in this case, $\delta_0 = 1/\tau_1 = 1/\tau_2^2 = 0$. Therefore, we conclude that the atom-atom interactions could not destroy the coherence of output coupling in MIT experimental conditions. The interaction term $u_{1111}a_1^\dagger a_1^\dagger a_1 a_1$ will lead to destroying the coherence after the atoms being released from the condensate. We refer an alternative discussion on this problem to Ref. [16].

V. The Influence of Atom-Atom Interactions on the Fraction of Out-Coupled Atoms

As shown in the last section, the interactions among the atoms do not destroy the coherence of the output coupling. This conclusion is available in the regime in which (i) rotating wave approximation (RWA) is valid and (ii) all the atoms are initially in condensate. A question then arises naturally: How does the atom-atom interaction effect the out-coupled atoms when the two conditions (i) and (ii) are held? To answer this question, we start with the Hamiltonian given in Eq. (24). We introduce $N = a_0^\dagger a_0 + a_1^\dagger a_1$ to denote the number operator of the system, it is obvious that $[H, N] = 0$, namely, the total number of the atoms is invariant in the evolution process. If the total number N_t of the atoms is fixed, the eigenfunction of the system can be expanded as

$$|\varphi\rangle = \sum_n B_n |n, N_t - n\rangle\,, \tag{27}$$

where $|n, N_t - n\rangle = |n\rangle_0 \otimes |N_t - n\rangle_1$, there are n trapped atoms and $N_t - n$ untrapped atoms in the system. Substituting Eq. (27) into steady Schrödinger equation $H|\varphi\rangle = E|\varphi\rangle$ yields

$$\begin{pmatrix} B_{n+1} \\ B_n \end{pmatrix} = M_n \begin{pmatrix} B_n \\ B_{n-1} \end{pmatrix}, \tag{28}$$

where

$$\begin{aligned}
M_n &= \begin{pmatrix} -(H_{n,n} - E)/H_{n,n+1} & -H_{n,n-1}/H_{n,n+1} \\ 1 & 0 \end{pmatrix}, \\
H_{n,n-1} &= (e_{01} + v^*)\sqrt{n(N_t - n + 1)}\,, \\
H_{n,n} &= e_{00}n + e_{11}(N_t - n) + \frac{1}{2}u_{0000}n\sqrt{(n-1)(n+1)} \\
&\quad + 2u_{0101}n(N_t - n) + \frac{1}{2}u_{1111}(N_t - n)\sqrt{(N_t - n - 1)(N_t - n + 1)}\,, \\
H_{n,n+1} &= (e_{10} + v)\sqrt{(n+1)(N_t - n)}\,.
\end{aligned}$$

The maximum of n is N_t, this gives a condition of steady solution for the system

$$\Big(\prod_{n=0}^{N_t} M_n\Big)_{11} = 0\,. \tag{29}$$

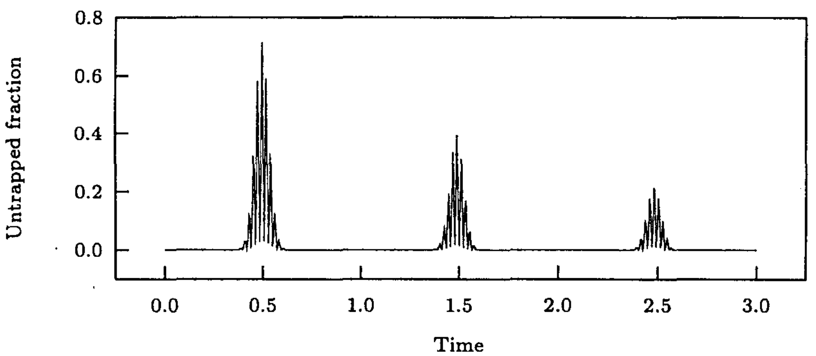

Fig. 3. The influence of the atom-atom interactions on the number of the out-coupled atoms. The figure shows the number of the out-coupled atoms versus time. The parameters are taken as $u_{00} = 0.3$, $u_{11} = 0.1$, $u_{01} = u_{10} = 0.2$. The other parameters are the same as those in Fig. 1.

To compare with the experiment, we take $|0\rangle_1 \otimes |\alpha_0\rangle_0$ as the initial condition. The time-dependent Schrödinger equation for the many-body wavefunction in the Fock representation

$$|\varphi(t)\rangle = \sum_{n=0}^{N_t} B_n(t)|n, N_t - n\rangle \tag{30}$$

gives

$$\mathrm{i}\hbar(\partial B_n(t)/\partial t) = H_{n,n}B_n(t) + H_{n,n-1}B_{n-1}(t) + H_{n,n+1}B_{n+1}(t)\,. \tag{31}$$

At time t, the number N_1 of atoms coupled out of condensate and the number N_0 of atoms in condensate are given as $N_1 = |\alpha_0|^2 - N_0$ and $N_0 = \sum_{n=0}^{N_t} n|B_n(t)|^2$. Equation (31) is similar to Eq. (5) in Ref. [18], which was derived to study the quantum evolution of a collective mode of a Bose–Einstein condensate. Here we do not deal with the detailed numerical calculations, but give the main results. In comparison with the results given in Eq. (18), the number of atoms coupled out of condensate varies with time to show collapse and revivals. The collapse time depends on the coupling constant and the parameter α_0, which is illustrated in Fig. 3. The results indicate that it needs long duration of the rf. pulse to couple out the atoms from a condensate. The dependence of the fraction on the amplitude is more complicated in this situation.

VI. Conclusion

In this work we have generalized the two-level model pioneered by Ketterle *et al.* as an atom laser model to study the fraction of output, its interference and the coherence loss. The space-dependent wavefunction of the out-coupled atoms was derived to be the solution of NLSE. Neglecting the atom-atom interaction, this wavefunction is a Gaussian packet. To learn more information about this wavefunction, we have to solve the NLSE consistently. The fraction of the atoms coupled out of the condensate can be adjusted between 0% and 100% by varying the amplitude of rf. radiation with duration τ. The output atoms can be well described using a coherent state. A criticism that has recently been made for a number of atom laser schemes is the lack of consideration on the atom-atom interactions (collisions), which is important obviously in a number of models[6,8] for an atom laser to provide the transitions to the lasing mode. Under the RWA, we studied the influence of atom-atom interaction on the coherence and the number of output atoms. In the condition of experiment held in MIT, the atom-atom interactions do not lead to destroying the coherence of the output atoms, but they change the fraction of the output and the dependence of the output on time, the longer duration of the rf. pulse is needed to couple atoms out of condensate than in the case of no interactions. For a double-well trapped potential, two beams of atoms which are coupled out of the right well and of the left well, respectively, will overlap and show inference patterns, and the fringe spacing depends mainly on the separation of double wells and time. If the separation of double wells disappears, there are not interference patterns, but an enhanced beam of atoms.

This work removes from consideration of the finite temperature and the dissipative effects. The atom interactions we considered here are only the energetically favored term, i.e., the terms under RWA. The other terms of the atom-atom interactions have also effects on an atom laser and the coherence loss. These need further investigations.

References

[1] M.H. Anderson *et al.*, Science **269** (1995) 198.
[2] C.C. Bradley *et al.*, Phys. Rev. Lett. **75** (1995) 1687.
[3] K.B. Davis *et al.*, Phys. Rev. Lett. **75** (1995) 3969.
[4] M.-O. Mewes *et al.*, Phys. Rev. Lett. **77** (1997) 988.
[5] M.R. Andrews *et al.*, Science **275** (1997) 637.
[6] M. Holland, K. Burnett, C. Gardiner, J.Z. Cirac and P. Zoller, Phys. Rev. **A54** (1996) R1757.
[7] H.M. Wiseman and M.J. Collett, Phys. Lett. **A202** (1995) 246.
[8] A.M. Guzman, M. Moore and P. Meystre, Phys. Rev. **A53** (1996) 997.
[9] G.M. Moy, J.J. Hope and C.M. Savage, Phys. Rev. **A55** (1997) 3631.
[10] M.-O. Mewes, M.R. Andrews, D.M. Kurn, D.S. Durfee, C.G. Townsend and W. Ketterle, Phys. Rev. Lett. **78** (1997) 582; J. Javanainen and M. Wilkens, Phys. Rev. Lett. **78** (1997) 4675.
[11] J.J. Hope, Phys. Rev. **A55** (1997) R2531.
[12] R.J. Ballagh, K. Burnett and T.F. Scott, Phys. Rev. Lett. **78** (1997) 1607.
[13] D.A.W. Hutchinson, E. Zaremba and A. Griffin, Phys. Rev. Lett. **78** (1997) 1842.
[14] A. Röhrl, M. Naraschewski, A. Schenzle and H. Wallis, Phys. Rev. Lett. **78** (1997) 4143.
[15] M. Naraschewski, H. Wallis, A. Schenzle, J.Z. Cirac and P. Zoller, Phys. Rev. **A54** (1996) 2185.
[16] H. Wallis, A. Röhrl, M. Naraschewski and A. Schenzle, Phys. Rev. **A55** (1997) 2109.
[17] Ji Il Kim, M.C. Nemes, A.F.R. de Toledo Piza and H.E. Borges, Phys. Rev. Lett. **77** (1996) 207.
[18] A.B. Kuklov *et al.*, Phys. Rev. **A55** (1997) R3307.