Effect of dipole-dipole interaction among atoms on atomic dynamics and entropy evolution of the field*

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New developments in the high techniques associated with quantum optics have made it possible to produce a high-Q cavity with an extremely small size comparable to the wavelength of atomic emission[1, 2]. In such cavities with so small size, many distinctive phenomena of atomic motion interacting with cavity field appear to be quite attractive. The Jaynes-Cummings model[3] describes the interaction of a two-level atom with a single mode field. When two atoms enter the domain of the cavity field simultaneously, and interact with each other through the dipole-dipole coupling with the exchange of visual photons, many interesting non-classical effects can be observed, such as fluorescence spectrum[4—6], emission spectra[7], emission spectra of two level atoms in an ideal cavity[8], atomic collapse-and-revival phenomenon[9], etc.

Entropy theory introduced for partially-coherent light by Gama[10] has been applied to quantum optics recently by Barnett and Phoenix[11, 12] to study evolution and correlation of the interaction between light field and matter. The concept of entropy is very useful when it is taken as a sensitive measure of interaction between the atoms and field.

In this note, we investigate the effect of dipole-dipole interactions among atoms on transitions of internal atomic state and field entropy evolution using Tavis-Cummings (T-C) model[13].

The Hamiltonian for identical two two-level atoms interacting with a single mode cavity field and including dipole-dipole interaction between atoms is

$$H=\hbar\omega a^{+}a+\frac{\hbar\omega_0}{2}\sum_{i=1}^{2}\sigma_i^z+g\sum_{i=1}^{2}(a^{+}\sigma_i^{-}+a\sigma_i^{+})+v\sum_{i\neq j}^{2}\sigma_i^{+}\sigma_j^{-}, \qquad (1)$$

where a^+ and a represent the creation and annihilation operators of the single mode cavity field with frequency ω, respectively; ω_0 is the atomic transition frequency; $\sigma_i^z=|e_i\rangle\langle e_i|-|g_i\rangle\langle g_i|(i=1,\ 2)$, $|e_i\rangle(|g_i\rangle)$ is the excited state (ground state) of i-th atom. $\sigma_i^+=|e_i\rangle\langle g_i|$, $\sigma_i^-=|g_i\rangle\langle e_i|$, g denotes the coupling constant between atom and field and v is the strength

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of the dipole-dipole interaction.

Assume $n\geqslant 2$, in an invariant subspace spanned by

$$|g,\ g,\ n+2\rangle\equiv|g_1\rangle\otimes|g_2\rangle\otimes|n+2\rangle,$$

$$|e,\ e,\ n\rangle\equiv|e_1\rangle\otimes|e_2\rangle\otimes|n\rangle,$$

$$|E,\ G,\ n+1\rangle\equiv\frac{1}{\sqrt{2}}(|g_1\rangle\otimes|e_2\rangle\otimes|n+1\rangle+|e_1\rangle\otimes|g_2\rangle\otimes|n+1\rangle).$$

The Hamiltonian takes a form

$$H=\begin{pmatrix}\hbar\omega(n+1)+v & \sqrt{2(n+2)}g & \sqrt{2(n+1)}g\\ \sqrt{2(n+2)}g & \hbar\omega(n+2)-\hbar\omega_0 & 0\\ \sqrt{2(n+1)}g & 0 & \hbar\omega n+\hbar\omega_0\end{pmatrix},\tag{2}$$

where $|n\rangle$ is Fock state of field. For simplicity, we study the case of exact resonance, i.e. $\omega=\omega_0$. Defining

$$\Omega^2=v^2+8(n+2)g^2+8(n+1)g^2,$$

$$\sqrt{2(n+2)}g=\frac{\Omega}{2}\sin\theta\cos\varphi,$$

$$\sqrt{2(n+1)}g=\frac{\Omega}{2}\sin\theta\sin\varphi.\tag{3}$$

Here $\tan\varphi=\sqrt{\dfrac{n+2}{n+1}}$, $\tan\theta=\dfrac{2g\sqrt{2(n+2)+2(n+1)}}{v}$. The Hamiltonian (2) can be rewritten as a function of θ and φ:

$$H=(n+1)\hbar\omega+\hbar\Omega\begin{pmatrix}\cos\theta & \frac{1}{2}\sin\theta\cos\varphi & \frac{1}{2}\sin\theta\sin\varphi\\ \frac{1}{2}\sin\theta\cos\varphi & 0 & 0\\ \frac{1}{2}\sin\theta\sin\varphi & 0 & 0\end{pmatrix}.\tag{4}$$

Then, the eigenvalues and the corresponding eigenstates are given by

$$E_0=(n+1)\hbar\omega,$$

$$E_\pm=\hbar\Omega\frac{\cos\theta\pm1}{2}+\hbar\omega(n+1),\tag{5}$$

and

$$|d\rangle=\sin\varphi|g,\ g,\ n+2\rangle-\cos\varphi|e,\ e,\ n\rangle,$$

$$|\varphi_+\rangle=\sin\frac{\theta}{2}(\cos\varphi|g,\ g,\ n+2\rangle+\sin\varphi|e,\ e,\ n\rangle)+\cos\frac{\theta}{2}|E,\ G,\ n+1\rangle,$$

$$|\varphi_-\rangle = -\cos\frac{\theta}{2}(\cos\varphi|g,\ g,\ n+2\rangle + \sin\varphi|e,\ e,\ n\rangle) + \sin\frac{\theta}{2}|E,\ G,\ n+1\rangle, \tag{6}$$

respectively. The above solution shows that the system of two two-level atoms with special case $\omega_0=\omega$ is a dark system with dark state $|d\rangle$ decoupling with the state $|E,\ G,\ n+1\rangle$. In the process of adiabatic evolution the probability of transition form $|d\rangle$ to $|E,\ G,\ n+1\rangle$ is zero when the system is initially in state $|d\rangle$. Exact Fock state of cavity field had been produced by dark atom in single-atom-field system[14]. Momentum transfer between the atom and the field had also been achieved throuth dark atom.

In order to study the evolution of field entropy, we must select one of the initial conditions since different initial conditions cause different results. Let us consider the case that the two atoms are initially in their excited states and the field in superposition of Fock state, i.e. $\rho(0)=\sum_n p(n)|e,\ e,\ n\rangle\langle e,\ e,\ n|$. At t, the density operator of this system is given by

$$\begin{aligned}\rho(t) = \Bigg\{ \sum_{n,\ n'} p(n)\Bigg[\Bigg(& \frac{1}{4}\sin2\varphi\sin\theta e^{-\frac{i}{\hbar}E_+t} + \frac{1}{2}\sin2\varphi\cos^2\frac{\theta}{2}e^{-\frac{i}{\hbar}E_-t} \\ & - \frac{1}{2}\sin2\varphi e^{-\frac{i}{\hbar}E_0t}\Bigg)|g,\ g,\ n+2\rangle + \Bigg(\sin^2\varphi\sin^2\frac{\theta}{2}e^{-\frac{i}{\hbar}E_+t} + \cos^2\varphi e^{-\frac{i}{\hbar}E_0t} \\ & + \sin^2\varphi\cos^2\frac{\theta}{2}e^{-\frac{i}{\hbar}E_-t}\Bigg)|e,\ e,\ n\rangle + \sin\varphi\sin\theta\sin\frac{\hbar\Omega t}{2}|E,\ G,\ n+1\rangle\Bigg]\Bigg\} \\ & \cdot \Bigg\{p(n')\Bigg[\Bigg(\frac{1}{4}\sin2\varphi\sin\theta e^{\frac{i}{\hbar}E_+t} + \frac{1}{2}\sin2\varphi\cos^2\frac{\theta}{2}e^{\frac{i}{\hbar}E_-t} - \frac{1}{2}\sin2\varphi e^{\frac{i}{\hbar}E_0t}\Bigg)\langle g,\ g,\ n'+2| \\ & + \Bigg(\sin^2\varphi\sin^2\frac{\theta}{2}e^{\frac{i}{\hbar}E_+t} + \cos^2\varphi e^{\frac{i}{\hbar}E_0t} + \sin^2\varphi\cos^2\frac{\theta}{2}e^{\frac{i}{\hbar}E_-t}\Bigg)\langle e,\ e,\ n'| \\ & + \sin\varphi\sin\theta\sin\frac{\hbar\Omega t}{2}\langle E,\ G,\ n'+1|\Bigg]\Bigg\}.\end{aligned} \tag{7}$$

From the above equation, the transition probability from initial state to $|E,\ G,\ n+1\rangle$ is obtained as

$$P_{e\to E} = \sum_n p^2(n)\sin^2\varphi\sin^2\theta\sin^2\frac{\hbar\Omega}{2}t. \tag{8}$$

If the field is initially in a coherent state $|\alpha\rangle$, i.e. $p^2(n)=e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}$, the numerical results of eq. (8) are illustrated in Fig. 1 with fixed photon number. (For clarity, the scale of the vertical axes are not specified.) Fig. 1(a)—(d) corresponds to different v, (a) $v=1$, (b) $v=5$, (c) $v=10$, (d) $v=15$, respectively; other parameters are $|\alpha|^2=3$, $g=1$. Fig. 1 shows that the dipole-dipole interaction affects the period of the transition among the internal states of two-atom system. The larger the strength of dipole-dipole interaction, the longer the period of transition. Effect of the dipole-dipole interaction on the transition probability is not evident when the mean photon number $|\alpha|^2$ of the field is very large ($|\alpha|^2 \gg 20$) and the dipole-dipole interaction is very small ($v \leqslant 1$), which is in agreement with the result of ref. [9]. Fig. 2 shows the effect of mean photon number on the period of transition probability with fixed v. The larger the mean photon number the longer the period.

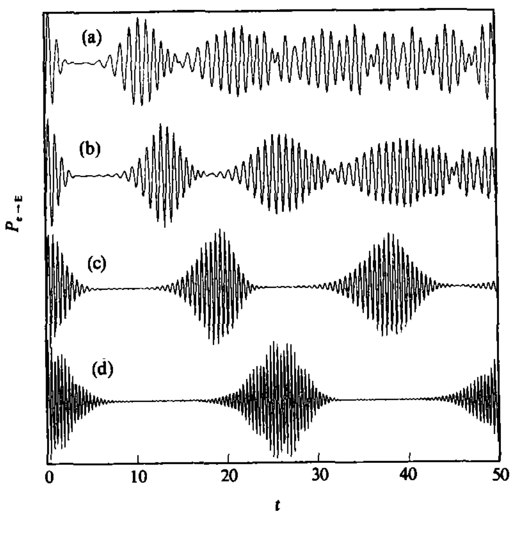

Fig. 1. The transition probability $P_{e\to E}$ with difinite mean photon number as a function of time. Fig. 1(a)—(d), corresponds to different v, (a) $v=1$, (b) $v=5$, (c) $v=10$, (d) $v=15$. Other parameters are $|\alpha|^2=3$, $g=1$.

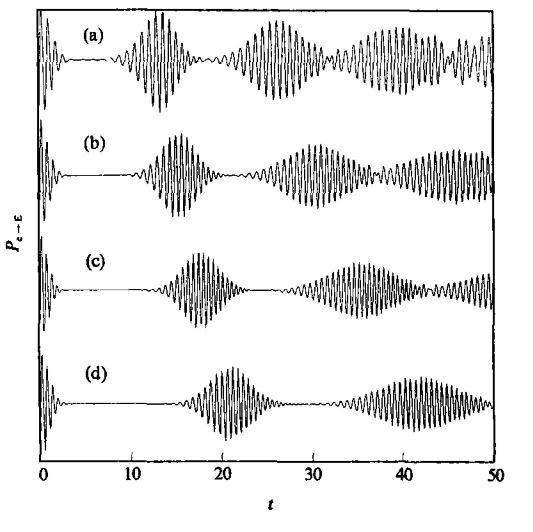

Fig. 2. The transition probability $P_{e\to E}$ with definite v as a function of time. Fig. 2 (a)—(d) corresponds to different $|\alpha|^2$, (a) $|\alpha|^2=3$, (b) $|\alpha|^2=6$, (c) $|\alpha|^2=10$, (d) $|\alpha|^2=12$, and $v=5$, $g=1$.

In the following we calculate the entropy evolution of the field. According to the definition of quatum entropy $s=-\mathrm{Tr}(\rho\ln\rho)$, the entropy is zero when the system is in a pure state. The entropy is not zero when it is in a mixed state. The quantized light field is an open sub-system of a closed system formed by atom plus field. Thus its entropy changes frequently. We can understand the dynamic behaviour of the field through analysis of the evolution of the field entropy. According to ref. [15] and eq. (7), the entropy of field at time t is given by

$$S(t)=-\sum_{n=0}^{\infty} p^2(n)P_f(n)\ln P_f(n), \tag{9}$$

where

$$P_f(n)=\langle n|\rho_f(t)|n\rangle, \tag{10}$$

$$\rho_f(t)=\mathrm{Tr}_a(\rho(t)). \tag{11}$$

Substituting eq. (6) into eq. (11), we find

$$P_f(n)=\sin^2\varphi_n\sin^2\theta_n\sin^2\frac{\hbar\Omega(n)}{2}t+\frac{1}{4}\sin^2 2\varphi_{n+2}\left|\frac{1}{2}\sin\theta_{n+2}e^{-\frac{i}{\hbar}E_+(n+2)t}\right.$$

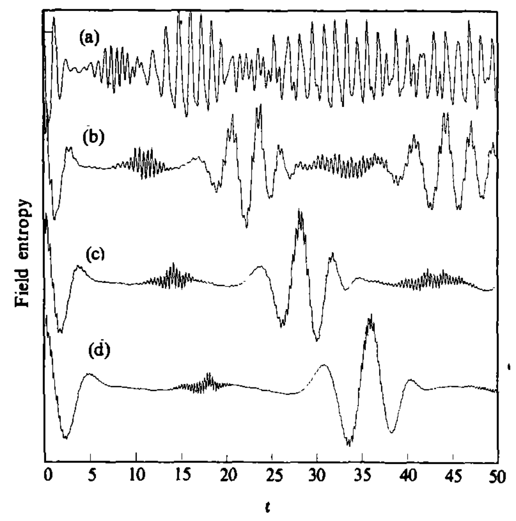

Fig. 3. Evolution of the field entropy with the field is initially in a coherent state $|\alpha\rangle$ where $g=1$, $\omega=2$, $|\alpha|^2=5$. Fig. 3 (a)— (d) corresponds to different v, (a) $v\to 0$, (b) $v=10$, (c) $v=15$, (d) $v=20$.

$$+\cos^2\frac{\theta_{n+2}}{2}e^{-\frac{i}{\hbar}E_-(n+2)t}-e^{-\frac{i}{\hbar}E_0(n+2)t}\Big|^2+\Big|\sin^2\varphi_{n+1}\Big(\sin^2\frac{\theta_{n+1}}{2}e^{-\frac{i}{\hbar}E_+(n+1)t}$$

$$+\cos^2\frac{\theta_{n+1}}{2}e^{-\frac{i}{\hbar}E_-(n+1)t}\Big)+\cos^2\varphi_{n+1}e^{-\frac{i}{\hbar}E_0(n+1)t}\Big|^2, \tag{12}$$

where $\varphi_n=\varphi(n)$, $\theta_n=\theta(n)$. For the field is initially in a coherent state $|\alpha\rangle$, the numerical results are illustrated in fig. 3 with parameters: $g=1$, $\omega=2$, $|\alpha|^2=5$. Fig. 3(a)—(d) correspond to different v, (a) $v\to 0$, (b) $v=10$, (c) $v=15$, d $v=20$. From fig. 3 we can see that the dipole-dipole interaction affects the period of the evolution of the field entropy. The larger the strength of the interaction is, the longer the period of entropy evolution is. This means that the dipole-dipole interaction is correlated with the entanglement between the atoms and the field.

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