

Thermodynamic witness of quantum probing

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The thermodynamic influence of quantum probing on an object is studied. Here, quantum probing is understood to be a pre-measurement based on a non-demolition interaction, which records some information of the probed object but does not change its energy state when both the probing apparatus and the probed object are isolated from the environment. It is argued that when the probing apparatus and the probed object are immersed in the same equilibrium environment, the probing can affect the effective temperature of the object or induce a quantum isothermal process for the object to transfer its energy. This thermodynamic feature can be regarded as a witness of quantum probing.

quantum measurement, quantum thermalization, fidelity

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The Landauer principle that the erasure of one bit of information requires a minimum heat generation of $k_B T \ln 2$, which is based on the second law of thermodynamics [1,2], underlies the thermodynamics of information processing. This principle eventually resolves Maxwell's demon paradox: why a demon can assist a binary thermal medium to do extra work [2]. Landauer's principle is undoubtedly crucial to the science and technology of quantum information because it gives a physical limitation of the spatial and temporal scales of logical devices on chips.

However, in the arguments for and against Landauer's principle, the conventional question in the thermodynamics of information processing of whether the measurement process requires a cost of heat generation has never been convincingly answered. Certainly, earlier works have touched on this problem, but it has not been clarified because the measurement process has not been defined properly. In particular, in the quantum approach of measurement, there is no consensus on fundamental problems such as whether there is wave function collapse [3]. To clarify the situation, we need to unambiguously answer two subtle

questions: when can we say a system is performing a measurement on another system and what kind of measurement can dissipate information?

In this paper, we use the term quantum probing to refer to a pre-measurement [4,5] on the basis of non-demolition coupling of the probed system S to the apparatus A , which is simply a unitary process that entangles S and A and does not concern subtle, seemingly philosophical arguments, such as wave function collapse. Generally speaking, when the energy state of the measured system S is not influenced by coupling to the measuring apparatus A , but A can record information of the system S , the measurement performed by A on S is called a pre-measurement. The so-called non-demolition (pre-) measurement [4] is an ideal measurement under some circumstances. In the case of a non-demolition measurement, the system-apparatus coupling V_{SA} commutes with the Hamiltonian H_S of the measured system but does not commute with the Hamiltonian H_A of the measuring apparatus. When studying a non-demolition measurement, it is usually assumed that both A and S are isolated from the environment. In this paper, we investigate the effect of the environment on the measured system in a non-demolition measurement. In the following discussion, we use the term

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quantum probing or just probing when regarding such non-demolition measurements. In the presence of an environment, the practical thermalization [6,7] of the total system $S+A$ will thermodynamically affect S . Such effects can be regarded as the thermodynamic witness of quantum probing.

When the existence of an environment E is considered, quantum probing is understood in two steps (illustrated in Figure 1).

(1) The extremely weak coupling between $S+A$ and E can be neglected within the dephasing time τ_2 of the system; the dephasing is the result of the interaction between S and A . At the end of this step, the initial factorized state of $S+A$ becomes a state assuming the form of an ideal Schmidt decomposition $\sum c_n |n\rangle |D_n\rangle$ [5], with $\langle D_m | D_n \rangle = \delta_{mn}$.

(2) If the probing apparatus continues to probe the state of the system S , the non-demolition coupling should hold for a longer time, and the communication with the environment will result in the thermalization of the total system $S+A$. In this step, the information of the initial state should be erased totally, but there needs to be correlation between A and S for there to be a witness of quantum probing.

As shown in the following, it is the common environment of the probed system and the probing apparatus that select a special set of ideal entanglement states of $S+A$ to be thermalized, and thus these witnesses of quantum probing are thermodynamic witnesses and are observable at the classical or macroscopic level.

1 Universal setup for a quantum probing apparatus

Let A be a general apparatus weakly coupled to the system S to be probed. We require that the energy spectrum of A be denser than that of S . A heuristic argument may help justify this requirement: to measure the spatial scale of an object, the ruler should have a graduation much finer than the size of the object.

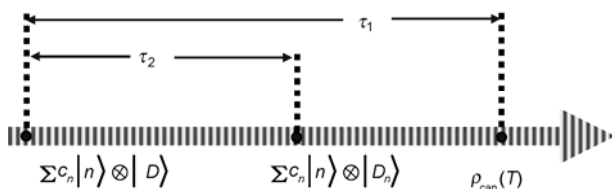


Figure 1 Time scale of non-demolition pre-measurement and thermalization. (1) In the time interval $(0, \tau_2)$, the apparatus makes the pre-measurement $\sum c_n |n\rangle \otimes |D\rangle \rightarrow \sum c_n |n\rangle \otimes |D_n(t)\rangle$ and the environment does not have a role because its coupling to $S+A$ is weaker than that between S and A . (2) In the time interval (τ_2, τ_1) , the effect of thermalization becomes prominent because of the environment. The total system $S+A$ is finally thermalized in the canonical state $\rho_{\text{can}}(T)$ with temperature T .

To be precise, let $H_A = \sum_k \epsilon_k |k\rangle \langle k|$ be the spectrum decomposition of the Hamiltonian of A and $H_S = \sum_n E_n |n\rangle \langle n|$ the spectrum decomposition of the Hamiltonian of S . The requirement can then be expressed as $\min\{|E_n - E_{n+M}|\} \gg \max\{|\epsilon_k - \epsilon_{k+1}|\}$. Here, $|k\rangle$ is the eigenvector of H_A corresponding to the eigenvalue ϵ_k , and $|n\rangle$ is the eigenvector of H_S corresponding to the eigenvalue E_n . Let V_{AS} be weak coupling between S and A . The weakness of V_{AS} means that its effect on the dynamics of the total system $S+A$ can be well studied employing the perturbation method [8,9].

To investigate the behavior of the total system $S+A$ immersed in an environment, we consider the partition function

$$Z = \text{Tr}(e^{-\beta H}) = \text{Tr}(W^+ e^{-\beta H} W). \tag{1}$$

Here, as a trick, we have introduced a unitary transformation $W = \exp(-S)$, defined by an anti-Hermitian operator S , which is a perturbation quantity of the same order as V_{AS} . If

$$V_{AS} + [H_A + H_S, S] = 0, \tag{2}$$

then the partition function can be approximated as $Z = \text{Tr} \exp(-\beta H_{\text{eff}})$, where the effective Hamiltonian

$$H_{\text{eff}} = H_A + H_S + \frac{1}{2}[V_{AS}, S], \tag{3}$$

is just the Frohlich-Nakajima Hamiltonian from solid state physics [8,9]. Here, we re-derive the Hamiltonian to justify its applicability in thermodynamics.

Because the minimal energy level spacing of the system is much larger than the energy level spacing of the apparatus, the effective interaction V_{eff} can be obtained as $V_{\text{eff}} = \sum H(n) |n\rangle \langle n|$, where

$$H(n) = \sum_{k,k'} G_{kk'}(n) |k\rangle \langle k'| \tag{4}$$

is a branched effective Hamiltonian of the apparatus corresponding to the situation that the system is prepared in the state $|n\rangle$ with the coupling

$$G_{kk'}(n) = \sum_q \frac{\langle nk | V_{AS} | nq \rangle \langle nk' | V_{AS} | nk' \rangle}{\epsilon_k - \epsilon_{k'}}. \tag{5}$$

The above obtained effective interaction V_{eff} satisfies $[H_S, V_{\text{eff}}] = 0$. Thus, the total effective Hamiltonian $H_{\text{eff}} = H_S + H_A + V_{\text{eff}}$ describes a non-demolition measurement without the presence of an environment. In this case, the factorized initial state $|\varphi(0)\rangle = \sum_n c_n |n\rangle \otimes |D\rangle$ will evolve into $|\varphi(t)\rangle = \sum_n c_n |n\rangle \otimes |D_n(t)\rangle$, where $|D_n(t)\rangle$ takes the form

$|D_n(t)\rangle = \exp[-iG(n)t]|D\rangle$. If $\{|D_n(t)\rangle\}$ becomes an orthogonal set as t approaches infinity, then the time evolution $|\varphi(t)\rangle$ represents a process of ideal pre-measurement.

2 Thermodynamic effects of measurements

We now study the change in the thermodynamic features of the system caused by the apparatus. We assume that both the probed system and the measuring apparatus are immersed in the same thermal bath with temperature T or inverse temperature $\beta=1/k_B T$. After or during the measuring process, the total system $S+A$ will reach the state with the same temperature T if the total system is non-degenerate. We can then calculate the reduced density matrix $\rho_S = \text{Tr}_A [\exp(-\beta(H_S + H_A + V_{\text{eff}}))]$, obtaining

$$\rho_S = \frac{1}{Z'_S} \sum_n e^{-\beta E_n} \xi(n) |n\rangle\langle n|, \tag{6}$$

where $Z'_S = \sum_n \exp(-\beta E_n) \xi(n)$ and

$$\xi(n) = \text{Tr}_A [\langle n | \exp(-\beta(H_A + V_{\text{eff}})) | n \rangle] \tag{7}$$

is a formal factor depending on the system state $|n\rangle$ and vanishes trivially when there is no coupling.

Here, we consider a manipulation process. (i) Initially, no probing apparatus is coupled to the system, which is in equilibrium with the heat bath with temperature T . (ii) The apparatus then begins to probe the system at time $t=0$. As the evolution described in Figure 1, the total system $S+A$ reaches the state with the same temperature T . We observe that the thermodynamic effect of the measurement implied by eq. (6) allows two interpretations. These two interpretations are illustrated in Figure 2 as those in [10–12].

The first interpretation is as follows. The change from the close thermal state of the system $\rho_{Sc} = \exp(-\beta H_S)/Z_S$ to the modified thermal state ρ_S is understood as a quantum isometric process, represented by the solid line between points A and C in Figure 2. In this process, the temperature is constant, but the energy level spacings alternate with the interaction with the apparatus. Accordingly, we define $\xi(n) = \exp(-\beta \Delta E_n)$; ρ_S can then be written in a form with clear physical meaning:

$$\rho_S = \frac{1}{Z'_S} \sum_n e^{-\beta(E_n + \Delta E_n)} |n\rangle\langle n|. \tag{8}$$

In the present case, it is natural to regard the inner energy change $\Delta U = \text{Tr}_S [H_S (\rho_S - \rho_{Sc})]$ as a witness of the thermodynamic role of measurement.

In the second interpretation, the change from ρ_{Sc} to ρ_S is understood as a quantum isothermal process, represented

by the dotted line between points A and B in Figure 2. In this process, the energy level spacings are fixed. To justify the term ‘‘isothermal process’’ here, at least to some extent, we define the effective temperature [6,7] via $\beta(n) = \ln(P_n/P_{n+1})/\Delta_n$ or

$$\beta(n) = \beta + \frac{1}{\Delta_n} \ln \frac{\xi(n)}{\xi(n+1)}, \tag{9}$$

where $\Delta_n = E_{n+1} - E_n$ is the n th energy-level spacing. We note that this generalized temperature cannot generally be regarded as an effective temperature because it depends on the energy levels of the system. However, if $\beta(n+1) - \beta(n) = 0$ or

$$\left[\frac{\xi(n+1)}{\xi(n+2)} \right]^{\Delta_n} = \left[\frac{\xi(n)}{\xi(n+1)} \right]^{\Delta_{n+1}}, \tag{10}$$

then $\beta(n)$ is a well-defined thermodynamic parameter independent of n , which we denote by β_{eff} . In this case, the above-defined generalized temperature allows the physical interpretation of effective temperature, and we have $\rho_S = \sum_n e^{-\beta_{\text{eff}} E_n} |n\rangle\langle n| / Z'_S$, and the inner energy change

$$\Delta U = \sum_n E_n [e^{-\beta_{\text{eff}} E_n} - e^{-\beta E_n}] \tag{11}$$

reflects the thermalization effect.

3 Quantum role of measurement and renormalization

As an explicit illustration, we model an apparatus with weak coupling to a probed system as a collection of harmonic oscillators. According to [13–15] (where arguments are made for bath modeling but apply to our setup of the apparatus), the coupling of the system to the apparatus is linear

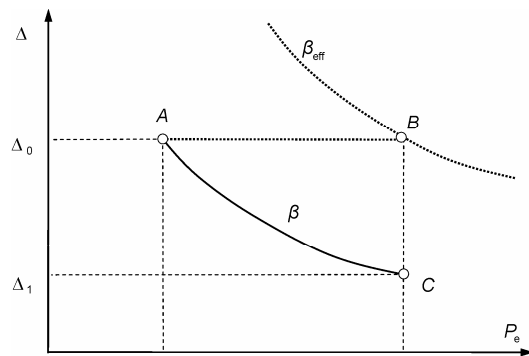


Figure 2 Two interpretations. (i) Isometric process ($A \rightarrow C$, the solid line): the inverse temperature β is fixed in the process. (ii) Isothermal process ($A \rightarrow B$, the dotted line): the level spacing Δ is fixed in the process.

with respect to the coordinates of the bath harmonic oscillators. Let b_j^+ (b_j) be the creation (annihilation) operator of the j th mode of the bath with eigenfrequency ω_j , and let $\lambda_n g_j$ be the coefficient of the coupling of the system state $|n\rangle$ to the j th mode. The total Hamiltonian is then obtained as

$$H = \sum_n E_n |n\rangle\langle n| + \sum_k \omega_k b_k^+ b_k + \sum_{nk} \lambda_n |n\rangle\langle n| (g_k b_k^+ + h.c.). \quad (12)$$

In this case, the above-defined generalized inverse temperature is

$$\beta(n) = \beta \left(1 - \frac{|\lambda_{n+1}|^2 - |\lambda_n|^2}{E_{n+1} - E_n} \varepsilon \right), \quad (13)$$

where

$$\varepsilon = \sum_k \frac{|g_k|^2}{\omega_k} = \int \rho(\omega) \frac{|g_k|^2}{\omega_k} dk$$

represents the self-energy of the apparatus, which results in a Lamb shift of the coupled system. As pointed out above, to define reasonably an effective temperature for the system, the generalized inverse temperature $\beta(n)$ should be independent of the energy level. A simple example satisfying this condition is the system of a harmonic oscillator with energy levels $E_n = (n+1/2)\omega$ and coupling strengths $\lambda_n = \sqrt{n}$. In this example, the well-defined effective inverse temperature is

$$\beta_{\text{eff}} = \beta \left(1 - \sum_k \frac{|g_k|^2}{\omega \omega_k} \right), \quad (14)$$

and the corresponding effective temperature $T_{\text{eff}} = (k_B \beta_{\text{eff}})^{-1}$ of the system is higher than that of the environment $T = (k_B \beta)^{-1}$.

For a two-level system with an excited state $|e\rangle$, ground state $|g\rangle$ and energy-level spacing Δ , the effective inverse temperature is also well defined:

$$\beta_{\text{eff}} = \beta + \frac{1}{\Delta} \ln \frac{\xi_g}{\xi_e}. \quad (15)$$

If the apparatus is a single-mode cavity with frequency ω_b and a two-level system is coupled to it by dipole interaction with coupling strength g , then in the large detuning case $\omega_b \gg \Delta$, the formal factors ξ_e and ξ_g can be explicitly calculated as

$$\begin{aligned} \xi_e &= \sum_n \exp \left\{ -\beta \left[\omega_b n + \frac{|g|^2}{\omega_b - \Delta} (n+1) \right] \right\}, \\ \xi_g &= \sum_n \exp \left\{ -\beta \left[\omega_b n + \frac{|g|^2}{\omega_b - \Delta} n \right] \right\}. \end{aligned} \quad (16)$$

Because $\xi_g > \xi_e$, we conclude that the measurement decreases the temperature of the system by

$$\Delta T = \frac{\frac{1}{\Delta} \ln \frac{\xi_g}{\xi_e}}{\beta + \frac{1}{\Delta} \ln \frac{\xi_g}{\xi_e}} T. \quad (17)$$

Finally, to quantitatively evaluate the thermalized state resulting from the interaction with the environment, we check the fidelity [16] F of such a state to the ordinary canonical state $\rho_{\text{can}} = \exp(-\beta H_S) / \text{Tr}[\exp(-\beta H_S)]$. Generally for the reduced density matrix in eq. (6), the fidelity between the initial state and the final state is

$$F = \frac{\sum_n e^{-\beta E_n} \sqrt{\xi(n)}}{\left(\sum_n e^{-\beta E_n} \right)^{1/2} \left(\sum_n e^{-\beta E_n} \xi(n) \right)^{1/2}}. \quad (18)$$

When the probed system is also the harmonic oscillator discussed above, the fidelity can be analytically obtained as

$$F = \frac{\sqrt{\sinh \frac{\beta \omega}{2} \sinh \frac{\beta(\omega - \lambda)}{2}}}{\sinh \frac{\omega - \lambda/2}{2}}, \quad (19)$$

where $\lambda = \sum_k |g_k|^2 / \omega_k$ characterizes the shift in the energy level of the harmonic oscillator, which reflects the effect because of the coupling to the apparatus. In Figure 3(a), the fidelity is plotted against the level shift. As the coupling between the apparatus and system is turned on, the energy level for the system is effective $E'_n = n(\omega - \lambda)$ on the basis of the first interpretation of the reduced density matrix. The fidelity between the canonical thermal and the reduced density

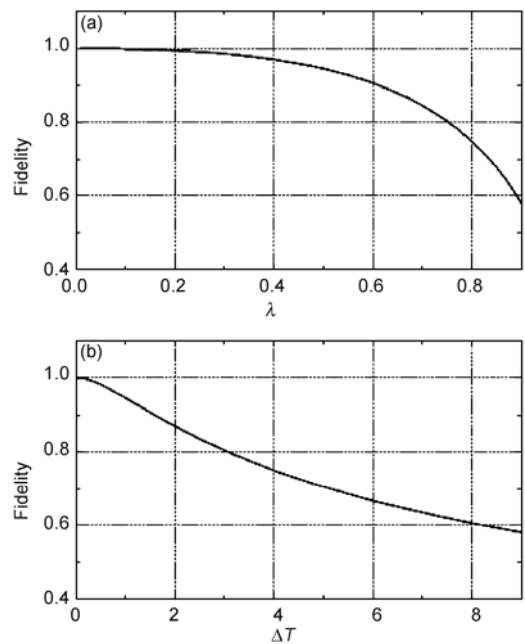


Figure 3 (a) Fidelity F vs. the level shift λ ; (b) Fidelity F vs. the temperature shift ΔT . Here, we take $\omega=1$ and $\beta=1$.

matrix decreases as the coupling strengthens. Therefore, the system gradually deviates from a canonical thermal state, providing evidence for the witness of the apparatus. For the second interpretation, we plot the fidelity F as a function of the temperature shift $\Delta T = 1/\beta_{\text{eff}} - 1/\beta = [\beta(\omega/\lambda - 1)]^{-1}$ in Figure 3(b).

4 Conclusion

In summary, for the weak coupling case, we show the universality of a non-demolition Hamiltonian in connection with the probing process employing the generalized approach of the Frohlich-Nakajima transformation. On the basis of this general non-demolition measurement, we investigate the probing effect on the system when the total measurement is made in a reservoir. It is concluded that the probing of the system can be witnessed through the change in the effective temperature, even though there is no direct energy exchange between the system and detector. To characterize the change in the state of the system, we evaluate the fidelity between the modified canonical thermal state and the original canonical state not being probed.

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