Deflection of an Atomic Beam in a Large Period Quantized Standing Light Wave*

WANG Xiaoguang, SUN Changpu
Institute of Theoretical Physics, Northeastern Normal University, Changchun 130024

(Received 17 March 1994)

By using the Wei-Norman's method, the deflection of an atomic beam with a large period quantized standing light wave is analyzed for the case with two-level atom and single-mode cavity field. It is concluded that after passing through the standing light wave in cavity, the atomic beam is split into infinite beams symmetrically, and each momentum shift for a split component is proportional to the interaction time. It is also shown that the output momentum distribution of atoms is sensitive to the statistical properties of the cavity field.

PACS: 03.65.-w, 32.80.-t, 42.50.Vk

With the development of experimental techniques in quantum optics, the study of mechanical effects of light on neutral atoms has made a great progress in last years. Especially, the interference of two atomic beams employing atomic matter-wave interferometers has been observed in several delicate experiments.[1,2] There are several methods to construct atomic interferometers, one of which uses standing light wave as splitter.[3] The standing light wave is also applied to adiabatically cool atoms and deflect atomic beams.[4] The deflection of atomic beams has been extensively studied both experimently and theoretically.[5-8] In fact, ones have experimentlly investigated the relevant imaging and focusing of atomic beams in a large period standing light wave.[9] Our motivation is to analyze the deflection of an neutral atomic beam in a large period quantized standing light wave by making use of a purely analytical method other than a numerical caculations.

Let us consider a two-level atom interacting with a quantized resonant standing light wave perpendicular to the direction of the moving atomic beam. In this case the longitudinal momentum of the atom is unchanged, and so our analysis only concerns the transverse atomic momentum. With a rotating wave approximation, the Hamiltonian of atom-field system is written as[5,8]

$$\hat{H}=\frac{\hat{p}^2}{2m}+\frac{1}{2}\hbar\omega\sigma_z+\hbar\omega a_+a+\frac{\hbar}{2}g(\sigma_+a+\sigma_-a^+)\sin(k\hat{x})\ , \tag{1}$$

where p is the momentum along x direction, $\sigma_\pm=\sigma_x\pm i\sigma_y$, and $\sigma_x,\sigma_y,\sigma_z$ the Pauli matrices; a and a^+ the annihilation and creation operaters of the cavity field mode. In the interaction picture, the Hamiltonian becomes

$$\hat{H}_I=\frac{\hat{p}^2}{2m}+\frac{1}{2}\hbar g(\sigma_+a+\sigma_-a^+)\sin(k\hat{x})\ . \tag{2}$$

* Partially supported by the National Natural Science Foundation of China, and the Fok Yin-Tung Education Foundation.

By taking into account the large period case that the wave number k is very small, the above Hamiltonian can be linearized by keeping the first-order term of the Maclaurin expansion of $\sin(k\hat{x})$,[5]

$$\hat{H}_e = \frac{\hat{p}^2}{2m} + \frac{1}{2}\hbar gk(\sigma_+ a + \sigma_- a^+)\hat{x} \ . \tag{3}$$

Let us solve the eigenvalue problem of the operator $\hat{A} = \sigma_+ a + \sigma_- a^+$. In a subspace spanned by $|n\rangle \otimes |\uparrow\rangle$ and $|n+1\rangle \otimes |\downarrow\rangle$, where $|n\rangle$ is the Fock state, and $|\downarrow\rangle, |\uparrow\rangle$ are the ground state and excited state, the representation of the operator $\hat{A}$ is

$$\hat{A} = \frac{1}{2}\hbar gk\sqrt{n+1}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{4}$$

Obviously, the eigenvalues and corresponding eigenstates are

$$\lambda_{\pm n} = \pm\frac{1}{2}\hbar gk\sqrt{n+1} \ ,$$

$$|\lambda_{\pm n}\rangle = \frac{1}{\sqrt{2}}(|n\rangle \otimes |\uparrow\rangle \pm |n+1\rangle \otimes |\downarrow\rangle) \ . \tag{5}$$

The above eigenstates are the dressed states of the system. In addition to these eigenstates, $|0, \downarrow\rangle$ is also the eigenstate of $\hat{A}$ with zero eigenvalue. Now we assume the initial state

$$\begin{aligned} |\psi(0)\rangle &= |p_0\rangle \otimes \sum_0^{\infty} C_n |n\rangle \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \\ &= \frac{1}{\sqrt{2}}C_0 |p_0\rangle \otimes |0\rangle \otimes |\downarrow\rangle + \sum_{-\infty}^{\infty} B_n |p_0\rangle \otimes |\lambda_n\rangle \ , \end{aligned} \tag{6}$$

where p_0 is the initial momentum of mass center, and $B_{\pm n} = (C_n \pm C_{n+1})/2$ is related to the field photon distribution. Then, the wave function at time t can be obtained from Eq. (3) formally,

$$\begin{aligned} |\psi(t)\rangle = e^{-i\hat{H}_e t/\hbar}|\psi(0)\rangle &= \frac{1}{\sqrt{2}}C_0 e^{-itp_0^2/2m\hbar} |p_0\rangle \otimes |0\rangle \otimes |\downarrow \\ &+ \sum_{-\infty}^{\infty} B_n e^{-it/\hbar(\frac{\hat{p}^2}{2m} + \lambda_n\hat{x})} |p_0\rangle \otimes |\lambda_n\rangle \ . \end{aligned} \tag{7}$$

The operator $\hat{U}(t) = e^{-it/\hbar(\frac{\hat{p}^2}{2m} + \lambda_n\hat{x})}$ can be factorized by using Wei-Norman's method.[10,11] In fact, the operators $\hat{p}^2, \hat{p}, \hat{x}$, and 1 form a closed four-dimension Lie algebra. Let us substitute the Wei-Norman's Ansatz solution

$$\hat{U}(t) = e^{\alpha(t)\hat{p}^2} e^{\beta(t)\hat{p}} e^{\nu(t)\hat{x}} e^{\chi(t)} \tag{8}$$

into the effective equation

$$i\hbar\frac{\partial\hat{U}(t)}{\partial t} = (\frac{\hat{p}^2}{2m} + \lambda_n\hat{x})\hat{U}(t) \tag{9}$$

to determine $\alpha(t), \beta(t), \nu(t)$, and $\chi(t)$.

With the help of Baker-Hausdoff formula, the following equations are achieved:

$$\dot{\alpha}(t) = -2mi\hbar\ , \qquad \dot{\beta}(t) = 2i\dot{\nu}(t)\alpha(t)\ ,$$
$$\dot{\nu}(t) = -i\lambda_n/\hbar\ , \qquad \dot{\chi}(t) = i\dot{\nu}(t)\beta(t)\ . \tag{10}$$

Under the initial conditions $\alpha(0) = \beta(0) = \nu(0) = \chi(0) = 0$, the solution of the above equations is

$$\alpha(t) = \frac{-it}{2m\hbar}\ , \qquad \beta(t) = \frac{-i\lambda_n t^2}{2m\hbar}\ ,$$
$$\nu(t) = \frac{-i\lambda_n t}{\hbar}\ , \qquad \chi(t) = \frac{-i\lambda_n^2 t^3}{6m\hbar}\ . \tag{11}$$

Finally, the factorization of $\hat{U}(t)$ is obtained as

$$\hat{U}(t) = e^{\frac{-it\hat{p}^2}{2m\hbar}} e^{\frac{-i\lambda_n t^2 \hat{p}}{2m\hbar}} e^{\frac{-i\lambda_n t\hat{x}}{\hbar}} e^{\frac{-i\lambda_n^2 t^3}{6m\hbar}}\ . \tag{12}$$

By substituting Eq. (12) into Eq. (7), the wave function at time t is written in a explicit form:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} C_0 |\, p_0\rangle \otimes |0\rangle \otimes |\downarrow\rangle + \sum_{-\infty}^{\infty} B_n e^{\frac{-i\hbar\lambda_n^2 t^3}{6m\hbar}} |\, p_n\rangle \otimes |\lambda_n\rangle\ , \tag{13}$$

where $p_0 = 0$ and $p_n = -\lambda_n t$. It should be noticed that we have assumed the initial momentum of the mass center is zero in obtaining the above simple result.

It is observed from Eq. (13) that after the atomic beam passed through the cavity, the atomic wave function is truly spatially seperated and the beam is split into infinite beams symmetrically. Physically, this symmetrically split results from the discrete transverse momentum, which is proportional to the interaction time t and the eigenvalues $\lambda_{\pm}n = \pm(\hbar gk\sqrt{n+1})/2$ for different dressed states. Because the longitudinal momentum is unchanged in this process and the eigenvalue is related to the photon number, the longer the interaction time and the larger the photon number, the larger the deflection angle. The probability of finding the momentum $\lambda_{\pm}n$ is $|B_{\pm}|^2$ which depends on both the initial field photon distribution and the initial probability of finding atom in the ground and excited states by Eq. (6). Therefore, it is concluded that the output momentum distribution of atoms is sensitive to the statistical properties of the field and conversely the statistical properties of electromagnetic field can be directly probed by using the output momentum distribution.

REFERENCES

[1] M. Kasevich and S. Chu, Phys. Rev. Lett. 67 (1991) 181.
[2] F. Riehle, Th. Kister, A. Witte, J. Helmcke and Ch. J. Borde, Phys. Rev. Lett. 67 (1991) 177.
[3] P. J. Martin, B. G. Oldaker, A. H. Miklich and D. E. Prichard, Phys. Rev. Lett. 60 (1988) 515.
[4] T. Zaugg, M. Wilkens, P. Meystry and G. Lenz, Opt. Commun. 97 (1993) 189.
[5] A. P. Kazantsev, G. A. Gyabenko, G. I. Surdutovich and V. P. Yakovlev, Phys. Rep. 129 (1985) 75.
[6] M. Wilkens, E. Schumacher and P. Mestry, Phys. Rev. A 44 (1991) 3130.
[7] S. Glasgow, P. Mestry, M. Wilkens and E. M. Wright, Phys. Rev. A 43 (1991) 2455.
[8] P. Mestry, E. Schmucher and S. Stenholm, Opt. Commun. 73 (1989) 443.
[9] T. Sleator, T. Pfau, V. Balykin and J. Mlynek, Appl. Phys. B 54 (1992) 375.
[10] J. Wei and E. Norman, J. Math. Phys. 4 A (1963) 575.
[11] C. P. Sun and Q. Xiao, Commun. Theor. Phys. 16 (1991) 359.