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CLASSICAL ANALOGUES OF QUANTUM BERRY'S PHASE[1]

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With the classical generalized harmonic oscillator as an example, we show that the analogues of the quantum Berry's phase can appear in the classical mechanic systems with adiabatically-changing parameters, which have the Schrödinger-type evolution equations. The corresponding guage structures are naturally induced in these systems.

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The Berry's phase[1] has quickly developed a reputation as an observable topological phase. It appears in many areas of quantum physics[2] and has been experimentally verified[3]. For classical cases the parallel analyses[4−6] have been given. More recently, a kind of classical analogues of the quantum Berry's phase was derived by considering locally inertial coordinated frames in classical mechanics[7]. In fact, so long as the evolution equation of a classical system is Schrödinger-type and has adiabatically changing parameters, the Berry's phase-like nature probably appears in evolution states of this system. For example, the propogation of polarized photons with high intensity in a helical fibre can be described by a classical Schrödinger-type equation, and the corresponding phase is similar to the Berry's phase[8].

In this paper we use the classical generalized harmonic oscillator to illustrate the above idea. The Hamiltonian of our problem is

$$H(t) = \frac{1}{2}[X(t)q^2 + 2Y(t)p\cdot q + Z(t)p^2] \quad , \tag{1}$$

where the slowly changing parameters $X(t)$, $Y(t)$ and $Z(t)$ form a closed curve C : $\{R(t)|R(T) = R(0)\}$ in the parameter space $\{R = (X,Y,Z)\}$ and satisfy $X.Z > Y$. The Hamilton's canonical equations $p{=}-\partial H/\partial q$ and $q{=}\partial H/\partial p$ can be expressed in matrix form as

$$\frac{d}{dt}\begin{bmatrix} p(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} -Y(t) & -X(t) \\ Z(t) & Y(t) \end{bmatrix} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix} \quad , \tag{2}$$

where $[p(t), q(t)]^T$ is called an evolution state. Because (2) is just a Schrödinger-type equation, the high-order adiabatic approximation method suggested by the present author[9−11] can be used to deal with it. Let

$$\phi(t) = \sum_{k=1}^{2} C_k(t) \exp\left[-i\int_0^t \lambda_k[R(t')]dt'\right]\phi_k[R(t)] \tag{3}$$

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be a solution of (2), where

$$\phi_1[R(t)] = \frac{1}{\sqrt{N(t)}}\begin{bmatrix} -X(t) \\ Y(t)+i\omega(t)\end{bmatrix} \quad , \quad \phi_2[R(t)] = \frac{1}{\sqrt{N(t)}}\begin{bmatrix} -X(t) \\ Y(t)-i\omega(t)\end{bmatrix} \tag{4}$$

are two eigenvectors of the matrix in (2) respectively corresponding to eigenvalues $\lambda_1(t)=\omega(t)$ and $\lambda_2(t)=-\omega(t)$. Here, $N(t)=X(t)^2+X(t)Z(t)$, $\omega(t)=[X(t)Z(t)-Y(t)^2]^{\frac{1}{2}}$. Substituting (3) into (2), we obtain

$$\begin{aligned} \dot{C}_k+\phi_k^+[R(t)]\cdot\dot{\phi}_k[R(t)]C_k(t) &= -\exp\left\{i\int_0^t(\lambda_k[R(t')]-\lambda_l[R(t')])dt'\right\} \quad \cdot \\ \phi_k^+[R(t)]\frac{\partial}{\partial t}&\left[\dot{C}_l(t)\phi_l[R(t)]\right] \quad , \quad l\neq k \ , \ l,k=1,2 \ . \end{aligned} \tag{5}$$

By using the same analyses as that in refs.9–11, under the adiabatic condition

$$\begin{aligned} &|\phi_i\cdot\dot{\phi}_j|/|\lambda_1-\lambda_2| << 1 \quad (i\neq j \ ; \ i,j=1,2) \ , \ \text{or} \\ &|w\cdot\dot{u}/N\omega| << 1 \quad , \quad |w^2\dot{u}/N^2\omega| << 1 \quad , \quad w,u=X,Y,Z \end{aligned} \tag{6}$$

an adiabatic approximation solution of (5) is

$$C_k^{ad}(t) = C_k(0)\exp[i\Gamma_k(t)] \quad , \quad k=1,2 \tag{7}$$

where

$$\Gamma_k(t) = i\int_0^t \phi_k^+[R(t')]\dot{\phi}_k[R(t')]dt' = i\int_C^{R(t)}\phi_k^+[R]d\phi_k[R] \tag{8}$$

are defined as the classical Berry's phases for the classical generalized harmonic oscillator, which is the classical analogues of the quantum Berry's phase. Because $\Gamma_k(t)$'s are path -dependent, we cannot choose the phases of the states $\phi_k[R(t)]$ so that $\Gamma'_k(t)$'s are zero for the states $\phi'_k[R(t)] = e^{i\beta(t)}\phi'_k[R]$ that satisfy $\phi'_k[R(T)]=\phi'_k[R(0)]$.

Under local U(1) gauge transformation

$$\phi_k[R(t)] \longrightarrow \phi'_k[R(t)] = e^{i\beta[R]}\phi_i[R(t)] \quad , \tag{9}$$

the phase 1-form

$$A_k[R] \equiv i\phi_k^+[R]d\phi_k[R] \tag{10}$$

transforms as a guage potential:

$$A_k[R] \longrightarrow A'_k[R] = A_k[R]-d\beta[R] \quad , \tag{11}$$

and the phase 2-form $F_k[R]=dA_k[R]$ is invariant. Then, the classical Berry's phase

$$\Gamma_k(c) = \Gamma_k(T) = \oint_c A_k[R] = \iint_{s:\{\partial s=c\}} F_k[R] \tag{12}$$

is also invariant. Thus, the guage structure is induced in classical mechanic system. This case is just similar to that the guage structure appears in simple quantum systems in virtue of the quantum Berry's phase[12]. It is worth while pointing out that the classical Berry's phase is different from the Hannay angles and is observable in principle.

Finally, we discuss a manifestation of this phase in practice. If the oscillator is initially in a point (0, A) of the phase space $\{(p,q)\}$, then

$$C_1(0) = -C_2(0) = [A/2i\omega(0)][X(0)Z(0) + X^2(0)]^{\frac{1}{2}} \tag{13}$$

that gives the adiabatic trajectory in the phase space

$$\begin{aligned} q(t) = \left[\frac{X(0)Z(0)+X^2(0)}{X(t)Z(t)+X^2(t)}\right]^{\frac{1}{2}} &\left\{\frac{A}{\omega(0)}\cdot\omega(t)\cos\left[\Gamma_1(t) - \int_0^t \omega(t')dt'\right]\right. \\ &\left.+\frac{A}{\omega(0)}Y(t)\sin\left[\Gamma_1(t) - \int_0^t \omega(t')dt'\right]\right\} \end{aligned}$$

$$P(t) = -\frac{A}{\omega(0)}\left[\frac{X(0)Z(0)+X^2(0)}{X(t)Z(t)+X^2(t)}\right]^{\frac{1}{2}}\cdot X(t)\cdot\sin\left[\Gamma_1(t) - \int_0^t \omega(t')dt'\right] \quad . \tag{14}$$

At some time T, if

$$\int_0^T \omega(t')dt' = 2n\pi \quad , \quad (n = 1,2,3\cdots) \tag{15}$$

one easily see that the oscillation momentum is

$$P(t) = \frac{A}{\omega(0)}X(0)\sin[\Gamma_1(c)]$$

which obviously depends on the classical Berry's phase $\Gamma_1(c)$. This effect is observable at least in principle.

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