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Double Barrier Resonant Tunneling in Spin-Orbit Coupled Bose–Einstein Condensates *

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We study the double barrier tunneling properties of Dirac particles in spin-orbit coupled Bose–Einstein Condensates. The analytic expression of the transmission coefficient of Dirac particles penetrating into a double barrier is obtained. An interesting resonance tunneling phenomenon is discovered in the Klein block region which has been ignored before.

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Resonant tunneling is one of the most important issues in quantum physics theoretically and experimentally. It not only allows us to realize and to reconsider some physical phenomena but also has a variety of novel applications in many fields. Therefore, the research on resonant tunneling arouses great interest from scientists all over the world and a lot of impressive achievements[1−11] have been made in recent years. Furthermore, the double barrier (DB) structure provides us with an ideal platform for the research of the quantum dynamics. It is widely used in the research of x-ray transmission and Fabry–Perot (FP) resonance.[12] The study of how particles penetrate into a double barrier provides vital guidance to the development of laser resonant cavity as well as the research of the process of photons travelling through fiber Bragg gratings (FBGs). Therefore, it is very important to study the scattering properties of Dirac particles penetrating into a double barrier.

However, it is an extremely difficult task to achieve the direct experimental verification of the dynamic properties of Dirac particles due to the limitations of experimental conditions in the past. To break through the limitations and manacles of previous experiments, scientists decided to use ultra-cold atoms as an alternative to conduct their research on the dynamic nature of Dirac particles. Very recently, researchers have designed a scheme to realize quasi Dirac particles of spin-orbit coupled (SOC) Bose–Einstein condensates (BECs) by using the interaction between artificial gauge potential and ultra-cold atoms.[13] To our delight, the SOC BECs of ^{87}Rb have been realized for the first time by the NIST group with the scheme of BEC in a non-Abelian gauge field.[13−15] These achievements make the experimental verification of the dynamic nature of Dirac particles possible, and then some researchers follow up[15] and related works have been reported by different groups on the relativistic phenomenon, such as Klein tunneling and Zitterbewegung effects.[16,17] Although there are many works on tunneling phenomenon in BECs,[18−22] so far only a few papers which have been released are concerning the resonance effect of SOC BECs. In this Letter, we devote ourselves to the study of the double barrier resonant tunneling (DBRT) phenomena with SOC BECs.

First, the situation of quasi-Dirac particles passing through a double barrier is taken into our consideration. We can obtain the SOC BECs by using the Λ-level scheme of combining a non-Abelian gauge field and ultra-cold atoms.[13] By simple derivation, we obtain the 1+1 dimensional Dirac equation[17] $i\frac{\partial}{\partial t}\Psi = H_0\Psi$, in which we adopt the natural units $\hbar = c = 1$, while γ is the rest mass of the Dirac particle. Let $V(x)$ be the double barrier potential with its width of barriers d and its amplitude V. The Dirac Hamiltonian is[17]

$$H_0 = -\sigma_y p + V(x) + \sigma_z \gamma. \tag{1}$$

Then, we have the wave function of in different regions expressions in the forms for $x < 0$,

$$\psi = T_1 \begin{pmatrix} ik \\ E-\gamma \end{pmatrix} e^{ikx} + R_1 \begin{pmatrix} -ik \\ E-\gamma \end{pmatrix} e^{-ikx} \tag{2}$$

for $0 < x < d$, and

$$\psi = T_2 \begin{pmatrix} ik_{\rm B} \\ V-E-\gamma \end{pmatrix} e^{ik_{\rm B}x} + R_2 \begin{pmatrix} -ik_{\rm B} \\ V-E-\gamma \end{pmatrix} e^{-ik_{\rm B}x} \tag{3}$$

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for $d < x < d + L$,

$$\psi = T_3 \begin{pmatrix} ik \\ E-\gamma \end{pmatrix} e^{ikx} + R_3 \begin{pmatrix} -ik \\ E-\gamma \end{pmatrix} e^{-ikx} \quad (4)$$

for $d + L < x < 2d + L$,

$$\psi = T_4 \begin{pmatrix} ik_{\rm B} \\ V-E-\gamma \end{pmatrix} e^{ik_{\rm B}x} + R_4 \begin{pmatrix} -ik_{\rm B} \\ V-E-\gamma \end{pmatrix} e^{-ik_{\rm B}x} \quad (5)$$

for $x > 2d + L$,

$$\psi = T_5 \begin{pmatrix} ik \\ E-\gamma \end{pmatrix} e^{ikx}, \quad (6)$$

where d is the width of barriers, L is the separation of two barriers, k and $k_{\rm B}$ are the wave vectors outside and inside the barrier, respectively; $k = \sqrt{E^2 - \gamma^2}$ and $k_{\rm B} = -\sqrt{(V-E)^2 - \gamma^2}$. It is well known that in the regions $0 < x < d$ and $d + L < x < 2d + L$, the scattering properties of a negative energy state is similar to the properties of electric charge conducted by a process known as the hole conduction. When $V - \gamma > \gamma$, the continuum bands of the particle overlap the negative energy continuum bands, and a tunneling region comes into existence where the so-called Klein tunneling[5,18] occurs.

In order to study the tunneling properties of a double barrier, we make use of the continuity of the spinor solution (2)–(6) at $x = 0$, $x = d$, $x = d + L$ and $x = 2d + L$. For simplicity, we consider the width and amplitude of two barriers to be d and V respectively. The transmission coefficient T can be obtained explicitly as

$$T_d = |T_5|^2, \quad (7)$$

$$\begin{aligned} T_5 = {}& 16e^{-2id(k-k_{\rm B})}k^2k_{\rm B}^2SS_{\rm B}/\{-e^{4idk_{\rm B}}(k_{\rm B}S - kS_{\rm B})^4 \\ & - 2e^{2idk_{\rm B}}(-1 + e^{2ikL})(k_{\rm B}^2S^2 - k^2S_{\rm B}^2)^2 \\ & + e^{2ikL}(k_{\rm B}^2S^2 - k^2S_{\rm B}^2)^2 - (k_{\rm B}S + kS_{\rm B})^4 \\ & + e^{4idk_{\rm B}+2ikL}(k_{\rm B}S - kS_{\rm B})^2(k_{\rm B}S + kS_{\rm B})^2]\}, \end{aligned} \quad (8)$$

where $S = E - \gamma z$, $S_{\rm B} = V - E - \gamma z$.

It is not difficult to verify that the expression will reduce to the single barrier transmission coefficient when the distance $L = 0$.[5]

For photons, we present the basic equations and expressions of transmission coefficient T as follows:[5,18]

$$E\psi = i\sigma_z \frac{\partial \psi}{\partial x} + \sigma_y k_{\rm B} V(x)\psi,$$
$$T = |t|^2 = 1/\cosh^2(2k_{\rm B}V(x)L);$$

for non-relativistic electrons we have

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi = 0,$$
$$T = |t|^2 = 1/\cosh^2(\chi L);$$

for relativistic Dirac particles travelling through a double barrier, we have

$$E\psi = (-\sigma_y p + V(x) + \sigma_z\gamma)\psi,$$
$$T = T_d;$$

where the transmission coefficient is derived from directly solving continuity conditions so as to seize the resonance solutions that have been lost by using the scattering matrix approach.

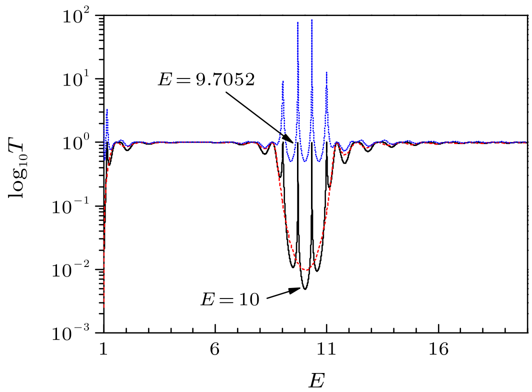

Fig. 1. (Color online) Transmission coefficient line versus energy for Dirac particle penetrating into a double barrier for $V = 10$, and $\gamma = 1$.

As we all know, the tunneling rate is very high when $\gamma < E < V - \gamma$ due to the Klein tunneling for Dirac particles, but there exists a region from $V - \gamma$ to $V + \gamma$ where particles cannot pass through any one barrier. We called it the Klein block.

Figure 1 shows the corresponding relationship between the behavior of the transmission coefficient T and the incident particle energy E. The red line corresponds to $d = 1.5$ and $L = 0$, which indicates that the double barrier reduces to a single $d = 3$ square barrier case. It is clear that when $E < V - \gamma$, Dirac particles can tunnel across the barrier because of the effect of the Klein tunneling phenomenon, which will not be observed in the non-relativistic system. However as the energy gradually enhances, there appears to be a region of block of $V - \gamma < E < V + \gamma$ as shown in Fig. 1. When the incident energy exists in this region, particles cannot pass through the barrier, because of the effect of the Klein block. When $E > V + \gamma$, it is well known that the particle can pass through the potential barriers. The black line of Figure 1 corresponds to $\sqrt{T_d}$(as shown in Eq. (7)) with $d = 3$ and $L = 6$. It means that we add another barrier with width $d = 3$ behind the first one and have created a double barrier structure. As shown in Fig. 1, the phenomenon in both the Klein tunneling region and the classical transmission region is the same as for the single barrier case. However, there exists an interesting resonance phenomenon with energy in the region of the Klein block which has never been found before. In certain circumstances, Dirac particles can still penetrate into the barriers or even total transmission can occur while the phenomenon cannot be calculated by the traditional scattering matrix approach. The blue line corresponds to the ratio of the red line to the black one. It shows that in the Klein tunneling and classical transmission region the transmission coefficient in the single barrier case is similar to the square root of the double barrier case. This means that in this region, we can obtain the same results by the scattering matrix method. It just resembles particles penetrating into two isolated single barriers. However, when energy exists in the Klein block region, the single barrier and the square root of the double barrier case are quite different from each other. This means that the scattering matrix method cannot obtain the resonant solutions in this region.

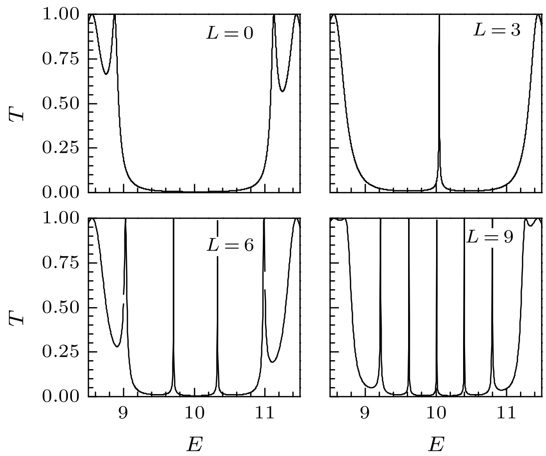

Fig. 2. Transmission coefficient T versus energy E with $L = 0$, $L = 3$, $L = 6$, $L = 9$, for $V = 10$, $\gamma = 1$ and $d = 3$.

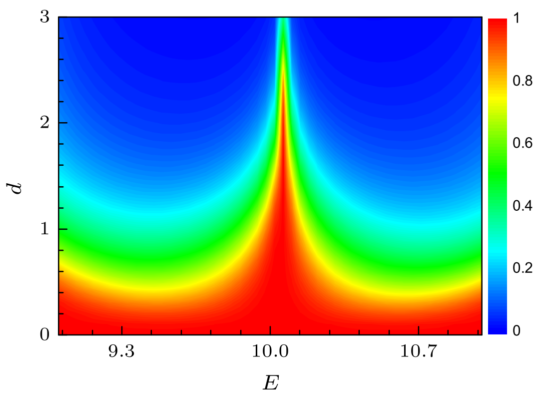

Fig. 3. (Color online) The phase diagram of T with $L = 3$ versus d and E. The other parameters we take here are the same as those of Fig. 1.

Figure 2 shows the behavior of the transmission coefficient T versus the energy for a double barrier with equal barrier width $d = 3$ and separation $L = 0, 3, 6, 9$ respectively. Obviously, when the other parameters are constant, the larger the L is, the greater the resonance peaks will appear in Klein block region. Furthermore, we suppose $L = 3$ with a view to facilitate the research on the behavior of the transmission coefficient versus the barrier width d. Keeping the other parameters constant, Fig. 3 illustrates the relationship between the resonance width and the barrier width d. It is obvious that the width of the resonance peak decreases with the increasing d.

To better understand this kind of FP-like resonance, we take two typical values $E = 9.7052$ and $E = 10$ respectively (see Fig. 1), to depict the characteristics of their corresponding plane waves in Fig. 4. It is obvious that there is no particle with energy in most of the area of the Klein block region able to penetrate into a barrier potential. In barriers, Dirac particles present mono-exponential declination as shown in the lower panel of Fig. 4. However, when we set a special value of energy to $E = 9.7052$ (which is also in the Klein block region) while the other parameters remain constant, the plane wave solution will no longer exhibit the one-way declination in barriers but show exponential increase in the first barrier and then decrease in the second one. Finally, the system shows a nearly total transmission state just as shown in the upper panel of Fig. 4. This suggests that double barrier resonant tunneling happens in the Klein block region in certain circumstances. Dirac particles can pass through or even totally transmit a double barrier when the energy is close to the resonance value.

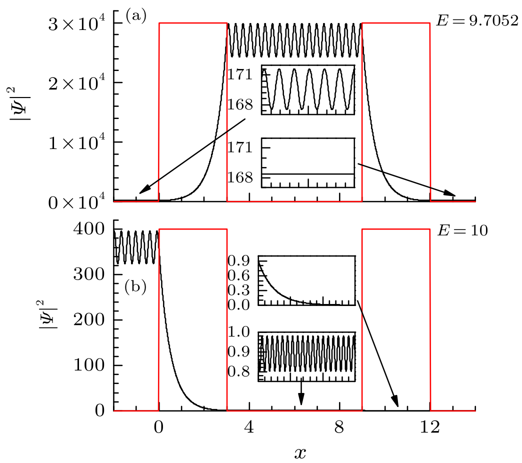

Fig. 4. (Color online) The plane waves $|\Psi|^2$ with energy $E = 9.7052$ (upper) and $E = 10$ (lower). The insets are the magnified images of particular areas. The other parameters we take here are the same as those of Fig. 1.

We give a brief explanation for this phenomenon. Although the plane wave shows exponential decrease when passing through the first barrier, there will always be part of the wave packet that succeeded in penetrating the barrier into the Klein block region. It will finally reach a plateau by the effort of constant accumulation as shown in Fig. 4. Then if we emit one more particle into the already saturated resonant cavity, one particle will be knocked out of the cavity and then the system shows a representation of total transmission. The process elaborated above is to some extent resembling things that happen in a reservoir.

In summary, we have studied the scattering process of Dirac particles and found that an interesting resonance occurs in the Klein block region. We present the expression of the transmission coefficient of DBRT. Moreover, we consider that the resonant mechanism represents the repeated captivity of particles due to the high reflectivity between two barriers. Furthermore, we study the behavior of the transmission versus the separation and the width of the barrier. The plane wave solutions of the double barrier scattering are also depicted with both resonance and common parameters.

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