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We study the optical bistability for a Bose–Einstein condensate of atoms in a driven optical cavity with a Kerr medium. We find that both the threshold point of optical bistability transition and the width of optical bistability hysteresis can be controlled by appropriately adjusting the Kerr interaction between the photons. In particular, we show that the optical bistability will disappear when the Kerr interaction exceeds a critical value.

Keywords: Bose–Einstein condensate, optical bistability, Kerr interaction

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1. Introduction

The Bose–Einstein condensate (BEC) in a driven ultrahigh-finesse optical cavity is a nice example of a nonlinear system. The nonlinearity arises from the dispersive atom–light interaction when the cavity resonance is far detuned from the atomic resonance. This atom–photon interaction imprints a position-dependent phase shift on the cavity field. In turn, this shift can affect the mechanical motion of atoms. This nonlocal interaction is quite different from the usual local atom–atom interaction. Many interesting results, such as self-organization of atoms,[1] quantum phase transitions in the Dicke model[2] and the Bose–Hubbard model,[3] adiabatic geometric phase,[4] cavity-enhanced super-radiant Rayleigh scattering,[5] the Josephson effect,[6,7] and optical bistability,[8] have been reported.

Optomechanics is another important field of research, where the center-of-mass motion of a mechanical oscillator is manipulated by the radiation pressure force of a single-mode Fabry–Pérot resonator.[9] Optomechanics is a paradigmatic system to explore the correlation between light and mesoscopic objects, which may possibly be applied to quantum information processing. And it can be simply described with only a few modes of the cavity field and one mode for the motion of the mirror.[10,11] Recently, optomechanics and ultracold atoms in optical resonators are unified, the collective motion of an ensemble of atoms can be considered to play the role of the movable mirror.[12]

Optical bistability was extensively studied in the 1980s due mostly to the prospect of its use as an optical switch in all-optical computers.[13] However, there are limited applications because of the lack of controllability. Recently, more intriguing phenomena, the controlled threshold points of optical bistability transition and width of optical bistability hysteresis curve, have been studied theoretically and observed experimentally.[14,15] The ultracold atoms in optical resonators also show strong matter-wave bistability[16] and optical bistability.[12,17]

In this paper, we consider a BEC in a pumped optical cavity, and the cavity is filled with an additional Kerr medium, which gives rise to a strong nonlinear interaction between photons. A single photon in the cavity can block the injection of the second photon due to the photon blockade effect induced by the Kerr interaction. This could be used to realize a single-photon turnstile device in quantum computation.[18] We show here that the bistability of photons in the optical cavity can be controlled by the Kerr interaction. With a weak Kerr interaction, the intracavity photon

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number shows clear bistability. The threshold point of optical bistability transition shifts upward and the width of optical bistability hysteresis decreases with the increase of the Kerr interaction. Above a critical Kerr interaction, the bistable behavior disappears. Therefore, we can realize a controllable optical switch conditioned on the Kerr interaction.

2. Model of system

We consider a BEC of N two-level atoms each with mass m inside a high-Q optical cavity of length L. The BEC is tightly confined in the transverse direction, so the transverse size of the condensate is smaller than the waist of the cavity field. Thus, we only consider the dynamics along the cavity x axis. In the large detuning limit and the rotating frame with the pump frequency, the Hamiltonian of the whole system is ($\hbar = 1$)[12]

$$H = H_{\rm a} + H_{\rm c} + H_{\rm d}, \tag{1}$$

where

$$\begin{aligned} H_{\rm a} = & \int {\rm d}x\ \psi^\dagger(x)\bigg[-\frac{1}{2m}\frac{{\rm d}^2}{{\rm d}^2x} + V_{\rm ext} \\ & + U_0\cos^2(kx)a^\dagger a\bigg]\psi(x) \end{aligned} \tag{2}$$

is the corresponding Hamiltonian for the condensate atoms including the atom–cavity interaction, and

$$\begin{aligned} H_{\rm c} &= \varDelta_{\rm c}a^\dagger a + {\rm i}\eta(a^\dagger - a) + \frac{1}{2}\chi a^{\dagger 2}a^2, \\ H_{\rm d} &= -{\rm i}\kappa a^\dagger a \end{aligned} \tag{3}$$

are the Hamiltonians for the cavity field and its damping. In the above formulas, $\psi^\dagger$ is the creation operator of the BEC, and $a^\dagger$ is the creation operator of the cavity photon with frequency $\omega_{\rm c}$, wave vector $k = 2\pi/\lambda$, and mode function $\cos(kx)$. The maximum light shift experienced by an atom in the cavity mode is given by $U_0 = g_0^2/\varDelta_{\rm a}$ with the single atom–photon coupling constant being g_0. The pump laser with strength η and frequency $\omega_{\rm p}$ is detuned from the empty cavity resonance frequency $\omega_{\rm c}$, namely, $\varDelta_{\rm c} = \omega_{\rm c} - \omega_{\rm p}$. The κ denotes the cavity decay. Giant optical Kerr nonlinearities are obtained by placing an $\xi^{(3)}$ medium inside the cavity,[19] with $\chi = 3\omega_{\rm c}^2{\rm Re}[\xi^{(3)}]/2\epsilon_0V_{\rm c}$, where ϵ_0 is the dielectric constant of the medium, $V_{\rm c}$ is the volume of the cavity, and $\xi^{(3)}$ is the third-order nonlinear susceptibility. In the case of weak atom–atom interactions and a shallow external trapping potential $V_{\rm ext}$, we can neglect the atom–atom interactions and set $V_{\rm ext} = 0$.[12]

The photon recoil associated with the absorption and the stimulated emission of light by the BEC results in the generation of symmetric momentum side modes at $\pm 2lk$, where l is an integer. We consider here the relatively simple situation in which the optical field is weak enough so that only $l = 1$ and 2 side modes are significantly populated. To account for this effect, we then expand the field operator in Eq. (2) as[20]

$$\psi(x) \simeq [c_0 + \sqrt{2}\cos(2kx)c_1 + \sqrt{2}\cos(4kx)c_2]/\sqrt{L}, \tag{4}$$

where c_0, c_1, and c_2 are the bosonic annihilation operators for atoms in the zero-momentum state and for the side-mode components with $l = 1$ and 2, respectively. With the help of the above expansion, the Hamiltonian (2) reduces to

$$\begin{aligned} H_{\rm a} = & 4\omega_{\rm r}(c_1^\dagger c_1 + 4c_2^\dagger c_2) + \frac{1}{4}U_0a^\dagger a(2N + \sqrt{2}c_0^\dagger c_1 \\ & + \sqrt{2}c_1^\dagger c_0 + c_1^\dagger c_2 + c_2^\dagger c_1), \end{aligned} \tag{5}$$

where $\omega_{\rm r} = k^2/2m$ is the atomic recoil energy, and $N = \sum_{i=0}^{2} c_i^\dagger c_i$ denotes the total number of atoms. From now on, $\omega_{\rm r}$ is used to rescale all the parameters in the Hamiltonian.

The Heisenberg equations of motion for the BEC atoms are

$$\begin{aligned} {\rm i}\frac{\rm d}{{\rm d}t}c_0 &= \frac{1}{4}U_0a^\dagger a\sqrt{2}c_1, \\ {\rm i}\frac{\rm d}{{\rm d}t}c_1 &= 4c_1 + \frac{1}{4}U_0a^\dagger a(\sqrt{2}c_0 + c_2), \\ {\rm i}\frac{\rm d}{{\rm d}t}c_2 &= 16c_2 + \frac{1}{4}U_0a^\dagger ac_1, \end{aligned} \tag{6}$$

and that for the optical field is

$${\rm i}\frac{\rm d}{{\rm d}t}a = \varDelta_{\rm c}a + {\rm i}\eta + \chi a^\dagger a^2 - {\rm i}\kappa a + \frac{U_0N}{2} + \boldsymbol{c}^\dagger\boldsymbol{H}_2\boldsymbol{c}, \tag{7}$$

where $\boldsymbol{c} = (c_0, c_1, c_2)^{\rm T}$, and $\boldsymbol{H}_2 = \frac{1}{4}U_0\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

In the experiment of Brennecke *et al.*,[12] $\kappa \sim$ MHz and $\omega_{\rm r} \sim$ kHz, i.e., the cavity decay is almost three orders of magnitude faster than the motion of the condensate. So it is reasonable to assume that the cavity field follows the condensate adiabatically, then the cavity field is solved as

$$a = \frac{\eta}{\kappa + {\rm i}(\varDelta_{\rm c}' + \boldsymbol{c}^\dagger\boldsymbol{H}_2\boldsymbol{c} + \chi a^\dagger a)}, \tag{8}$$

with $\varDelta_{\rm c}' = \varDelta_{\rm c} + NU_0/2$.

By applying the mean-field approximation $\boldsymbol{c} \sim \sqrt{N}\boldsymbol{z}$, $a \sim \alpha$, the equation of motion for the condensate atoms is found to be

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\tilde{t}}\boldsymbol{z} = (\boldsymbol{H}_1 + n_{\mathrm{ph}}\boldsymbol{H}_2)\boldsymbol{z}, \tag{9}$$

and the photon number is

$$n_{\mathrm{ph}} = \frac{\eta^2}{\kappa^2 + (\varDelta_{\mathrm{c}}' + N\boldsymbol{z}^{\dagger}\boldsymbol{H}_2\boldsymbol{z} + \chi n_{\mathrm{ph}})^2}, \tag{10}$$

where $\tilde{t} = \omega_{\mathrm{r}} t$ and $\boldsymbol{H}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 16 \end{pmatrix}$.

Formally, the equations of motion are very similar to their counterparts in Ref. [21], except for the additional Kerr term in the steady photon number in Eq. (10). Actually, they are very different, as the photon number is partly determined by the Kerr term in Eq. (10). With the interaction between the photon cavity and the atoms taking into consideration, the Kerr interaction of photon will play a non-trivial role in the properties of the system.

3. Results and discussion

With a sufficiently slowly ramped up pumping field, it is expected that the condensate and the optical cavity will follow their self-sustained steady states, i.e., the two stationary sides are consistent with and dependent on each other. Therefore, we should firstly investigate the steady states of the system. We write the steady state corresponding to a solution of Eq. (9) as

$$\boldsymbol{z}_{\mathrm{s}}(\tilde{t}) = \boldsymbol{z}_{\mathrm{s}} \exp(-\mathrm{i}E_{\mathrm{s}}\tilde{t}), \tag{11}$$

then equation (9) reduces to

$$(\boldsymbol{H}_1 + n_{\mathrm{ph}}\boldsymbol{H}_2)\boldsymbol{z}_{\mathrm{s}} = E_{\mathrm{s}}\boldsymbol{z}_{\mathrm{s}}, \tag{12}$$

which is a nonlinear eigenvalue problem, because the Hamiltonian $\boldsymbol{H}_1 + n_{\mathrm{ph}}\boldsymbol{H}_2$ depends on the eigenstate $\boldsymbol{z}_{\mathrm{s}}$. We solve it by making use of the following strategy.[21] At the first step, we take an arbitrary trial photon number n_{tr}, solve the ground state of the Hamiltonian, and then substitute it into Eq. (10) to obtain an output photon number n_{out}. If $n_{\mathrm{out}} = n_{\mathrm{tr}}$, then the solution is self-consistent, and a steady state is obtained. Note the nonlinearity of the system, the possibility of multiple steady-state solutions for a given set of parameters is expected.[22–24]

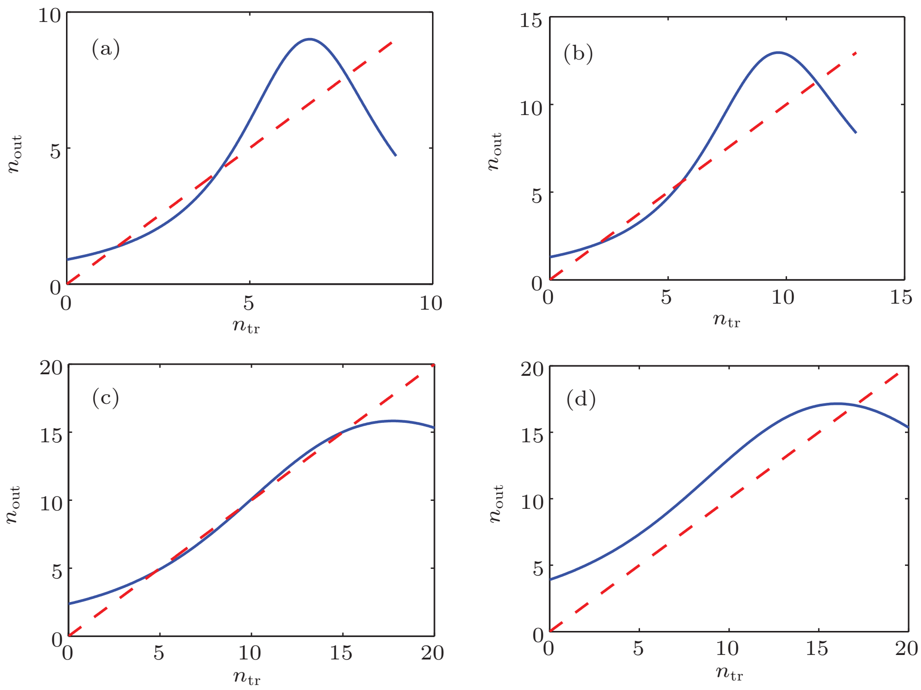

Fig. 1. (color online) Output photon number n_{out} as a function of the trial photon number n_{tr}. The intersection points of the solid curves with the diagonal dotted lines correspond to steady states. In panels (a)–(d), Kerr parameter χ and η are (0, 1.20×10^3), (50, 1.44×10^3), (100, 1.95×10^3), and (110, 2.5×10^3), respectively. The other parameters are $N = 4.8 \times 10^4$, $U_0 = 0.25$, and $(\kappa, \varDelta_{\mathrm{c}}) = (0.4, 1.2) \times 10^4$.[21]

Figure 1 shows the output photon number n_{out} as a function of the trial photon number n_{tr} when the pump strength η is varied and the other parameters are fixed. For each curve, its intersection with the dotted line of $n_{\mathrm{out}} = n_{\mathrm{tr}}$ corresponds to a steady state. It is clear that when the Kerr interaction is weak (Figs. 1(a)–1(c)), there are three solutions. When the Kerr parameter χ is larger than the critical value ($\chi_{\mathrm{c}} = 110$ in Fig. 1(d)), only one intersection point exists for each η.

This result becomes more evident in Fig. 2, where the photon number at the steady state n_{st} is plotted as a function of η. When the Kerr interaction is weak (Figs. 2(a)–2(c)), each η corresponds to three steady states in the middle interval, and the middle solution is the unstable one. With a large enough Kerr interaction (Fig. 2(d)), one η only corresponds to a single n_{st} in the whole interval.

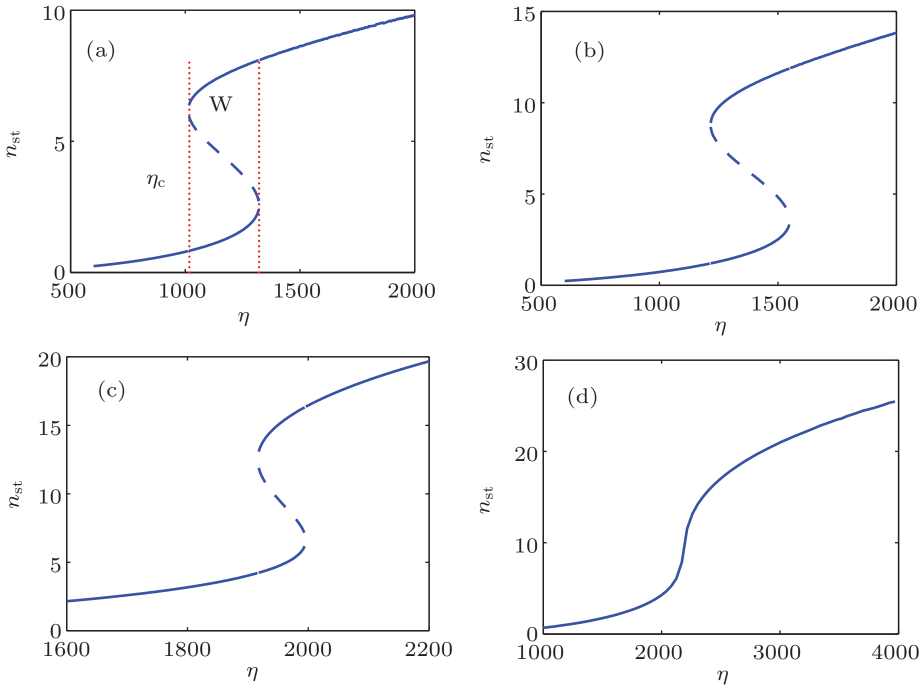

Fig. 2. (color online) Photon number at steady state n_{st} as a function of pump strength η. In the hysteresis regime, the middle dashed line denotes the unstable solution. The parameters in panels (a)–(d) are the same to those in the corresponding panels of Fig. 1.

The relationships among the threshold points of optical bistability transition η_{c}, the width of optical bistability hysteresis W, and the Kerr parameter χ are also interesting. They are shown in Fig. 3. We find numerically that η_{c} increases very quickly (Fig. 3(a)) and W undergoes a decrease (Fig. 3(b)) when the Kerr parameter χ is increased. All these results imply that we can achieve the controllable optical bistability of the cavity field with the Kerr medium.

These results can be understood as follows. On one hand, if χ is small, the main interaction of the system is the dispersive atom–light interaction, which can induce the optical bistability of the cavity field. On the other hand, when χ is very large, if a photon from the driving field has been injected into the cavity, the second injected photon will be blocked. Only after the first photon has left the cavity, the second one can be injected. This photon blockade can drastically reduce the photon number fluctuation, which implies a reduced interaction between the side modes of the BEC and the cavity field (note in Figs. 2(a)–2(d), with increasing Kerr interaction χ, the photon number decreases at a fixed η value). It is reasonable to expect that there is only one steady solution in the limit of sufficiently large χ. In the middle case, with the increase of χ, the smooth changes of the bistability of the optical field are that the threshold points of optical bistability transition shift upward and the width of optical bistability hysteresis decreases. Actually, the photon number Eq. (10) can be reduced to

$$n_{\mathrm{ph}}^3 + n_{\mathrm{ph}}^2 \frac{2\varDelta_{\mathrm{c}}'}{\chi + \beta_\chi} + n_{\mathrm{ph}} \frac{\kappa^2 + \varDelta_{\mathrm{c}}'^2}{(\chi + \beta_\chi)^2} - \frac{\eta^2}{(\chi + \beta_\chi)^2} = 0$$

with the linear approximation (to the lowest order of photon number) $N \boldsymbol{z}^\dagger \boldsymbol{H}_2 \boldsymbol{z} \simeq \beta_\chi n_{\mathrm{ph}} + \mathcal{O}(n_{\mathrm{ph}}^2)$.[17]

Due to the interdependence of the BEC and the photon, the subscript of β is explicitly written to denote the relying on χ (we numerically checked that $\beta_\chi \in [-175, -150]$ in our case). According to the theory of the roots of the polynomial equation, it is relatively simple to theoretically estimate the relationship between η_c and χ as

$$\eta_c = \sqrt{-\frac{2}{27}\frac{x+y}{\beta_\chi+\chi}} \sim \sqrt{\frac{s_1}{|\beta_\chi+\chi|}},$$

with $x = \Delta_c'(\Delta_c'^2 + 9\kappa^2)$, $y = \sqrt{(\Delta_c'^2 - 3\kappa^2)^3}$, and $s_1 \sim 10^8$. Therefore, η_c increases very quickly with a small change of χ. And the width W is given by $W = \eta_c' - \eta_c$ with $\eta_c' = \sqrt{-\frac{2}{27}\frac{x-y}{\beta_\chi+\chi}}$. According to this expression, we numerically find that W is approximately a constant when χ is small compared to β, then decreases with the increase of the Kerr parameter χ. For a sufficiently large χ, the bistability hysteresis disappears.

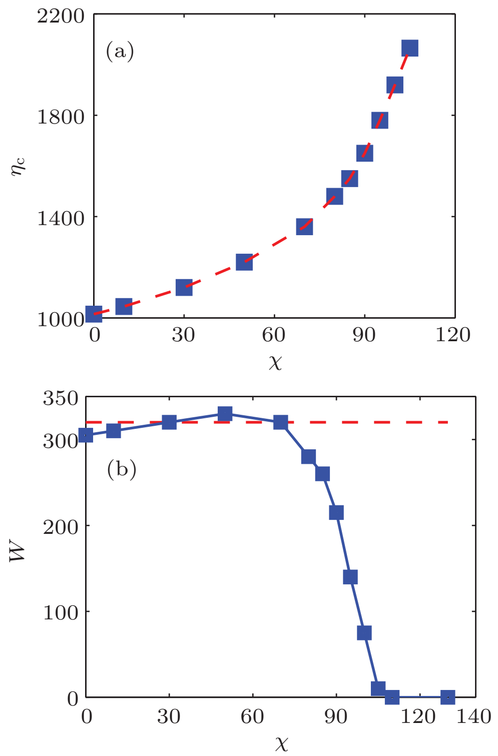

Fig. 3. (color online) Panel (a) shows the variation of the critical strength of the pumping field η_c, with which the bistability just exists, with respect to Kerr parameter χ. Panel (b) shows the variation of the width of the bistability hysteresis W with χ. The blue squares are the numerical results, and the red dash lines are plotted for guiding eyes. The other parameters are the same to those in Fig. 1.

4. Conclusion

In summary, we have studied the effect of the Kerr medium on the behavior of a BEC in a pumped cavity field. We have shown that as the Kerr nonlinearity increases, the threshold point of optical bistability transition shifts upward, and the width of optical bistability hysteresis decreases. Particularly, the bistable behavior can completely disappear when the Kerr interaction exceeds a critical strength. These effects are due to the photon blockade mechanism, which suppresses the photon fluctuation. All the results demonstrate that the Kerr interaction is a new handle to coherently control the dynamics of the BEC in a cavity field and hence could be useful in the realization of tuneable quantum-mechanical devices in the future.

References

[1] Domokos P and Ritsch H 2002 *Phys. Rev. Lett.* **89** 253003
[2] Baumann K, Guerlin C, Brennecke F and Esslinger T 2010 *Nature* **464** 1301
[3] Larson J, Damski B, Morigi G and Lewenstein M 2008 *Phys. Rev. Lett.* **100** 050401
[4] Li S C, Fu L B and Liu J 2011 *Phys. Rev.* A **84** 053610
[5] Slama S, Bux S, Krenz G, Zimmermann C and Courteille P W 2007 *Phys. Rev. Lett.* **98** 053603
[6] Zhang J M, Liu W M and Zhou D L 2008 *Phys. Rev.* A **77** 033620
[7] Wang B W, Tan L, Lu C H and Tan W T 2011 *Chin. Phys.* B **19** 117402
[8] Gupta S, Moore K L, Murch K W and Stamper-Kurn D M 2007 *Phys. Rev. Lett.* **99** 213601
[9] Kippenberg T J and Vahala K J 2009 *Science* **321** 1172
[10] Law C K 1995 *Phys. Rev.* A **51** 2537
[11] Genes C, Mari A, Tombesi P and Vitali D 2008 *Phys. Rev.* A **78** 032316
[12] Brennecke F, Ritter S, Donner T and Esslinger T 2008 *Science* **322** 235
[13] Gibbs H M 1985 *Controlling Light with Light* (Orlando: Academic)
[14] Joshi A and Xiao M 2003 *Phys. Rev. Lett.* **91** 143904
[15] Chang H, Wu H, Xie C D and Wang H 2004 *Phys. Rev. Lett.* **93** 213901
[16] Zhou L, Pu H, Ling H Y and Zhang W 2009 *Phys. Rev. Lett.* **103** 160403
[17] Yang S, Al-Amri M, Evers J and Zubairy M S 2011 *Phys. Rev.* A **83** 053821
[18] Imamoglu A, Schmidt H, Woods G and Deutsch M 1997 *Phys. Rev. Lett.* **79** 1467
[19] Kumar T, Bhattacherjee A B and ManMohan 2010 *Phys. Rev.* A **81** 013835
[20] Zhang K, Chen W, Bhattacharya M and Meystre P 2010 *Phys. Rev.* A **81** 013802
[21] Zhang J M, Cui F C, Zhou D L and Liu W M 2009 *Phys. Rev.* A **79** 033401
[22] Wu B and Niu Q 2000 *Phys. Rev.* A **61** 023402(R)
[23] Pu H, Maenner P, Zhang W and Ling H Y 2007 *Phys. Rev. Lett.* **98** 050406
[24] Wang G F, Fu L B, Zhao H and Liu J 2005 *Acta Phys. Sin.* **54** 5003 (in Chinese)