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Quantum Cyclotron Orbits of a Neutral Atom Trapped in a Triple Well with a Synthetic Gauge Field *

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The strong effective magnetic fields with flux to the order of one flux quantum per plaquette has been realized for ultracold atoms, and the quantum cyclotron orbit of a single atom in a single plaquette exposed to the magnetic field was directly revealed recently [Phys. Rev. Lett. 107 (2011) 255301]. We study the quantum cyclotron orbits of a bosonic atom in a triple well with a synthetic gauge field, and find that the dynamics of the atom in real space is similar to a classical dynamic billiard. It is interesting that the billiard-like motion is a signature of the quantum evolution of the three-level system, and its behaviors are determined by the ratio of the two energy gaps of the three energy levels.

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Based on the Berry phase effect and its non-Abelian generation,[1,2] the creation of synthetic gauge fields in neutral atoms by controlling atom-light interaction has attracted great interest in recent theoretical studies[3−16] and has been realized by many experiments.[17−20] The neutral atoms in a generated effective magnetic field behave like electrons in an electromagnetic field.[18−20] Two-dimensional electron gases with magnetic fields have led to the discovery of many quantum phenomena, such as the quantum Hall effect. Ultracold atoms provide clean and well controlled experimental systems not only for studying such systems, but also for exploring new physical regimes, which are not attainable in traditional condensed matter systems. Most recently, the strong effective magnetic fields with flux to the order of one flux quantum per plaquette has been realized for ultracold atoms in an optical lattice, and the quantum cyclotron orbit of a single atom in a single plaquette with four sites exposed to the magnetic field is directly revealed.[21]

The system of two-dimensional three coupled potential wells (triple wells), which can be realized as an isolated single plaqette with three sites in a triangle optical lattice, is the simplest system used to investigate the quantum dynamics in the presence of a magnetic field. In this Letter, we study the quantum cyclotron orbit of a single bosonic atom in such a triple well. It is found that the dynamics of the single atom in real space is similar to a classical dynamic billiard. It is interesting that the billiard-like motion is a signature of the quantum evolution of a three-level system, and its behaviors are determined by the ratio of the energy gaps of the three energy levels.

A system of three coupled potential wells constitutes the simplest concept for investigating tunneling dynamics in the presence of a magnetic field. We consider a single triple well potential occupied by bosonic atoms in the presence of a uniform artificial magnetic field in the transverse direction. If the the vibrational level space in each well is much larger than all other relevant energy scales, the system can be described by a three-site version of the Bose-Hubbard Hamiltonian

$$H = -J\sum_{i,j}^{3} e^{i\epsilon_{ij}\phi/3} a_i^\dagger a_j + U_0 \sum_{i}^{3} n_i(n_i-1), \qquad (1)$$

where $a_i^\dagger$, a_i, and n_i are the boson creation, annihilation and number operators, respectively; $\epsilon_{ij} = 1$ for $ij = 12, 23, 31$, and $\epsilon_{ij} = -1$ for $ij = 21, 32, 13$. The first term gives us the nearest neighbor hopping where the hopping rate $J = -\int d\boldsymbol{r} w_i(\boldsymbol{r})[-\hbar^2\nabla^2/2m + V_{trap}(\boldsymbol{r})]w_j(\boldsymbol{r})$ is for two neighbors, $\phi = \int \boldsymbol{A}(\boldsymbol{r})\cdot d\boldsymbol{r}$ is flux and $\boldsymbol{A}(\boldsymbol{r})$ is the vector potential corresponding to the artificial gauge field. The second term describes the interaction between atoms on the same lattice site and $U_0 = g\int |w_i|^4 dr$ characterizes the on-site interactions. Here $w_i(\boldsymbol{r})$ are the Wannier functions for the lowest band. The state of the system can be described with the Fock states

$$|n, m, N-n-m\rangle = \frac{a_1^{\dagger n} a_2^{\dagger m} a_3^{\dagger N-n-m}}{\sqrt{n!m!(N-n-m))}}|0\rangle.$$

For the one-atom case, the system can be diago-

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nalized readily. The eigenstates are

$$|E_l\rangle = \frac{1}{\sqrt{3}}(|1,0,0\rangle + e^{i\frac{2(l-1)\pi}{3}}|0,1,0\rangle + e^{i\frac{4(l-1)\pi}{3}}|0,0,1\rangle) \quad (2)$$

with their corresponding eigenvalues $E_l = -2\cos(\frac{\phi}{3} + \frac{2(l-1)\pi}{3})$ $(l = 1, 2, 3)$ respectively. It is interesting that the eigenvectors are independent of flux ϕ, but the energy spectrum can be well controlled by the flux ϕ, see Fig. 1. Here the period of the flux ϕ is 6π.

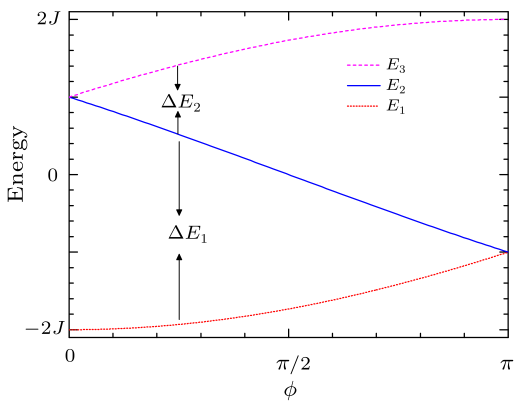

Fig. 1. Energy spectrum of the system. The energy gaps ΔE_1 and ΔE_2 can be well modulated by the flux ϕ.

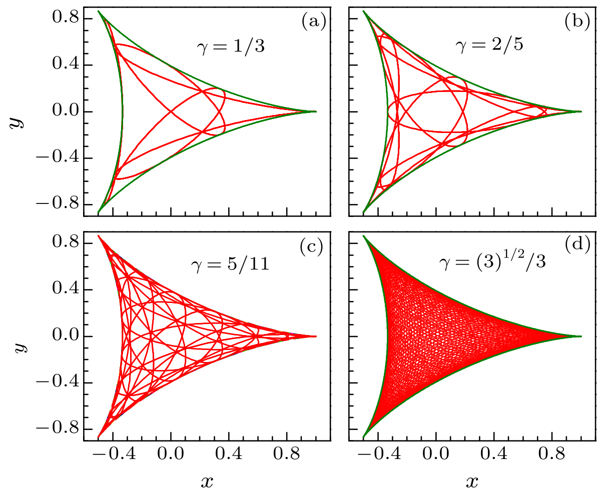

Fig. 2. The quantum cyclotron orbits in real space for different ratios of energy gaps γ, 1/3 (a), 2/5 (b), 5/11 (c), and $\sqrt{3}/3$ (d).

Let us define $\Delta E_1 = E_2 - E_1$, $\Delta E_2 = E_3 - E_2$ and $\gamma = \Delta E_{\min}/\Delta E_{\max}$, where $\Delta E_{\min(\max)}$ is the minimum (maximum) of ΔE_1 and ΔE_2, respectively. We can see that the ratio of energy gaps γ is modulated by the flux ϕ from 0 to 1 continuously.

To investigate the quantum cyclotron motions we put the atom in one site at the beginning, i.e., the system evolves with an initial state $\psi(0) = |1,0,0\rangle$.[21] In order to exhibit the cyclotron motions in real space we note the locations of the three sites as $S_1 = (1, 0)$, $S_2 = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$, and $S_3 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, respectively. Then at time t, the average atom position is $\langle X\rangle = n_1 - \frac{1}{2}(n_2 + n_3)$ and $\langle Y\rangle = \frac{\sqrt{3}}{2}(n_2 - n_3)$, where n_i is the probability of occupations on ith site. From the superposition principle, we can easily obtain $n_1 = \frac{1}{3} + \frac{2}{9}\{\cos[\omega_{12}t] + \cos[\omega_{13}t] + \cos[\omega_{23}t]\}$, $n_2 + n_3 = \frac{2}{3} - \frac{2}{9}[\cos(\omega_{12}t) + \cos(\omega_{23}t) + \cos(\omega_{13}t)]$, $n_2 - n_3 = \frac{2\sqrt{3}}{9}[-\sin(\omega_{12}t) - \sin(\omega_{23}t) + \sin(\omega_{13}t)]$, in which $\omega_{ij} = E_i - E_j$.

For generality, by rescaling time as $\tau = \Delta E_{\max}t$ and denoting $(x(t), y(t)) = (\langle X\rangle, \langle Y\rangle)$, we obtain

$$x(t) = \frac{1}{3}\{\cos(\tau) + \cos(\gamma\tau) + \cos[(1+\gamma)\tau]\}, \quad (3)$$

$$y(t) = \frac{1}{3}\{-\sin(\tau) - \sin(\gamma\tau) + \sin[(1+\gamma)\tau]\}. \quad (4)$$

In the following, we will show that the quantum cyclotron orbits described by the above equations are similar to a classical dynamic billiard.

The above two equations are trigonometric functions, so the cyclotron orbits must have a boundary, at which $R = x^2 + y^2$ reaches its local extreme value. We can obtain the formula of R, i.e. $R = \frac{1}{3} + \frac{2}{9}[\cos(\zeta - \eta) + \cos(\zeta + 2\eta) + \cos(2\zeta + \eta)]$, where we have substituted τ with ζ and $\gamma\tau$ with η respectively. It is easily known that R reaches its local extreme value under the condition $\zeta - \eta = 2n\pi$. Then, we can come to the conclusion that the motions are bounded by the curve

$$x(\tau) = \frac{1}{3}[\cos(2\zeta) + 2\cos\zeta],$$
$$y(\tau) = \frac{1}{3}[\sin(2\zeta) - 2\sin\zeta]. \quad (5)$$

It is just the quantum cyclotron orbit for $\gamma = 1$.

It is very interesting that the quantum cyclotron orbit will be limited in a region in the x–y plane with the above boundary (see Fig. 2), and the atom behaves likes a billiard ball. The billiard-like behaviors depend on the ratio of the gaps. If γ is a rational number, the motion is similar to the regular motion of a dynamic billiard (see, for example, Figs. 2(a)–2(c)). However, if γ is an irrational number, the motion will be ergodic in the bounded region (see Fig. 2(d)). The reason is quite apparent because if γ is rational the evolution is qusi-period while γ is irrational.

The quantum cyclotron orbits are a kind of 'strange' billiard system, the motion of the ball is not in a straight line and reflections from the boundary are not specular reflections. In fact, these orbits are similar to a ball with electric charge moving in a magnetic field, the curved trajectories exhibit the effect of the Lorentz force, and the quantum cyclotron orbits reveal the effective magnetic fields.

For the quantum cyclotron motions, the boundary is not a real boundary as it is for a billiard. The boundary of the quantum cyclotron motions is just the turning point at which the (average) velocity of atom changes its direction largely and is similar to a backward scattering process. In Figs. 3(a)–3(d), we plot the average velocities $(dx/dt, dy/dt)$ of the cyclotron motions. We can clearly see the turning points. Since γ is a rational number, the total number of turning points is a finite number. We find that if $\gamma = p/q$, the total number of turning points is equal to $2q + p$, where p and q are two coprime integers.

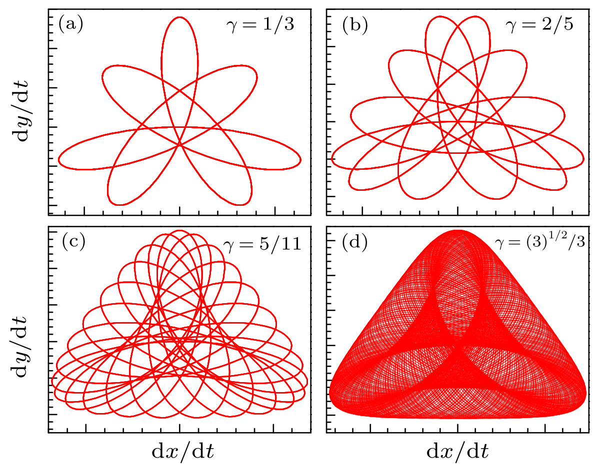

Fig. 3. The average velocities of quantum cyclotron orbits in real space for different ratios of energy gaps γ, 1/3 (a), 2/5 (b), 5/11 (c), and $\sqrt{3}/3$ (d).

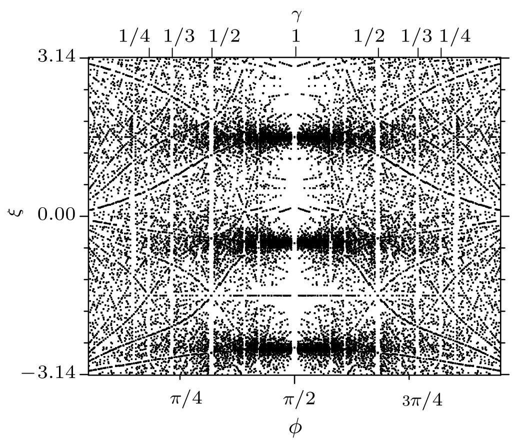

Fig. 4. The angles of turning points $\xi = \arctan((dy/dt)/(dx/dt))$ for different flux (ratio of energy gap γ).

The above property can be well exhibited by employing the angles of the turning points defined as $\xi = \arctan((dy/dt)/(dx/dt))$ (mod in $-\pi$ and π). In Fig. 4 we plot the angle ξ for different γ with it being rational. When γ is rational and equal to p/q with p and q relatively prime, there will be $2q + p$ turning points, and then we will obtain $2q + p$ angles ξ, for example, for $\gamma = 1/2$, there will be five points in ξ–γ plane. When γ is irrational, the turning points is a cantor set, and so is the angle corresponding to them, and we have not plotted them in Fig. 4.

In conclusion, the quantum cyclotron orbits of a bosonic atom in a triple well with a synthetic gauge field has been studied in detail. The dynamics of the atom in real space has a boundary and is similar to a classical dynamic billiard. It is interesting that the billiard-like motions are determined by the ratio of the two energy gaps of the three energy levels, if the ratio is a rational number, the motion is similar to the regular motion of a dynamic billiard. However, if the ratio is an irrational number, the motion will be ergodic in the bounded region. The relation between rational numbers and the turning points of quantum cyclotron orbits has also been exhibited.

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