GENERAL

Linear instability and adiabaticity of a dark state during conversion of two species of fermionic atoms to stable molecules

Linear instability and adiabaticity of a dark state during conversion of two species of fermionic atoms to stable molecules*

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In the present paper, we investigate the linear instability and adiabaticity of a dark state during conversion of two species of fermionic atoms to stable molecules through the stimulated Raman adiabatic passage aided by Feshbach resonance. We analytically obtain the regions for the appearance of linear instability. Moreover, taking ^{40}K and ^{6}Li atom–molecule conversion systems as examples, we give the unstable regions numerically. We also attempt to obtain the adiabatic criterion for this nonlinear system with classical adiabatic dynamics and study the adibaticity of the dark state with the adiabatic condition.

Keywords: linear instability, adiabaticity, dark state, atom–molecule conversion

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1. Introduction

Creation of ultracold molecules from their atomic counterparts[1−8] by means of photoassociation[9−11] (PA) or magnetic Feshbach resonance[12,13] (FR) offers a whole new dimension in the study of the ultracold atomic physics. In particular, it is no longer a pure Bose system as the molecular Bose–Einstein condensation (BEC) can be a system of Bose–Fermi[14] or Fermi–Fermi mixtures,[15,16] which makes the theoretical structure rich and colorful. Since the molecules created through PA and FR usually may suffer from fast decay due to the vibrational excitation, the stimulated Raman adiabatic passage (STIRAP) in PA[17−20] or aided by FR[21−23] has been regarded as an effective approach to create ground-state molecules. The success of the STIRAP technique requires the existence of the coherent population trapping state or dark state[24,25] which is the superposition of the free atomic and ground molecular states. Therefore, the adiabatic evolution on this state can suppress the spontaneous emission losses in the excited electronic level and hence create ultracold stable molecules with high conversion efficiency.

However, different from the traditional STIRAP in an Λ-atomic system, the atom–molecule STIRAP contains nonlinearities that stem from the mean-field treatment of the interparticle interactions and the conversion process of atoms to molecules. The existence of these nonlinearities make it very difficult to analyze the adiabaticity of the atom–molecule conversion systems because of the absence of the superposition principle. The nonlinear interparticle collisions could also bring forth linear instability,[26−28] which is driven by the emergence of the complex intrinsic frequencies of the system in certain regions of the parameter space. These complex frequencies will cause an exponential growth in the eigenmode of the system and hence make the condensate become unstable against infinitesimal perturbations. In the STIRAP, the linear instability could make the quantum evolu-

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tion deviate from the dark state rapidly even in adiabatic limit.[21] Therefore, it is important to avoid such instability for the success of the STIRAP.

In this paper, we investigate the linear instability and adiabaticity of a dark state during conversion of two species of fermionic atoms to stable molecules through the STIRAP aided by FR. We analytically obtain the regions for the appearance of linear instability and find that, different from the pure bosonic system, such instability is not only caused by the bosonic interparticle interactions but also by the fermionic Pauli blocking terms. We also attempt to obtain the adiabatic criterion for this nonlinear system through the classical adiabatic dynamics. Taking ^{40}K and ^{6}Li atom–molecule conversion systems as examples, we give the unstable regions and study the adibaticity with the adiabatic condition numerically.

Our paper is organized as followed. In Section 2, we model the systems and derive the CPT state solution. In Section 3, through casting the nonlinear Schrödinger equation into an effective classical Hamiltonian, we investigate the linear instability by analyzing the eigenvalues of the Hamiltonian–Jacobi matrix obtained via linearizing the nonlinear equations of motion at the fixed point that corresponds to the CPT state. In Section 4, we study the adiabaticity of the CPT state quantitatively based on the adiabatic condition obtained through the classic Hamiltonian dynamics. In Section 5, our conclusion is presented.

2. Theoretical model and the CPT state

Consider a mixture of two species of fermionic atoms initially prepared in a trapped state $|a,b\rangle$ which are coupled to bosonic molecules via the STIRAP aided by the FR as shown in Fig. 1. For convenience, let $|a\rangle$ and $|b\rangle$ stand for the ground states of the two kinds of fermionic atoms in the open channel and $|m\rangle$ and $|g\rangle$ for the quasi-bound and ground molecular states in the closed channel, respectively. The quasi-bound molecule state $|m\rangle$ is coupled with $|a\rangle$ and $|b\rangle$ through Feshbach resonance with coupling strength λ' and detuning δ, while the quasi-bound state $|m\rangle$ and the target state $|g\rangle$ are coupled by a laser field with Rabi frequency Ω' and detuning Δ. The energy density[22,23] corresponding to the Hamiltonian describing the above system under the Hartree approximation is given by

$$E = \hbar\Big[\frac{1}{2}\sum_{i\neq j}\chi'_{ij}\left|\psi_i\right|^2\left|\psi_j\right|^2 + \delta\psi_m^*\psi_m + \Delta\psi_g^*\psi_g + \frac{\lambda'}{2}\left(\psi_m^*\psi_a\psi_b + \text{h.c.}\right) - \frac{\Omega'}{2}\left(\psi_g^*\psi_m + \text{h.c.}\right) + \frac{1}{2}\sum_{i=\{m,g\}}\chi'_{ii}\left|\psi_i\right|^4 + \frac{3}{5}\sum_{i=\{a,b\}}A'_i\left|\psi_i\right|^{10/3}\Big], \quad (1)$$

where ψ_i represents the complex probability amplitude of the i-th component, the coefficients $\chi'_{ii} = 4\pi\hbar a_i/m_i$ and $\chi'_{ij} = \chi'_{ji} = 2\pi\hbar a_{ij}/m_{ij}$ (a_i and a_{ij} are the s-wave scattering lengths, m_i is the mass of species i, and m_{ij} is the reduced mass between states i and j) characterize the bosonic intrastate and interstate interaction strengths, respectively, and the term proportional to A'_i represents the effective self-interaction related to fermions and is called the Pauli blocking term with $A'_i = \hbar^2(6\pi^2)^{2/3}/2m_i$.

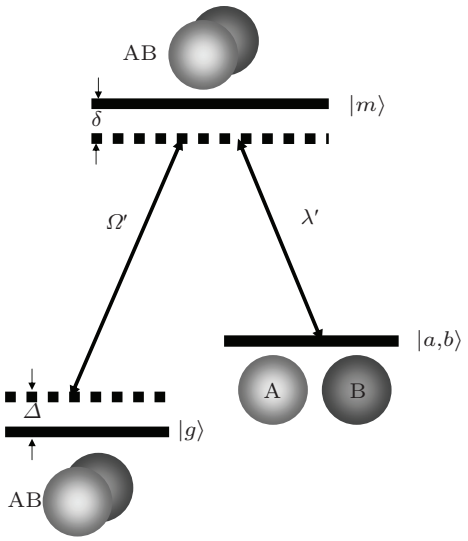

Fig. 1. The energy diagram of three-level atom–molecule system involving free quasi-bound–bound transitions. Conversion of atoms in $|a,b\rangle$ is accomplished by FR, while the coupling between the quasi-bound molecular state $|m\rangle$ and the ground molecular state $|g\rangle$ is provided by laser light. δ and Δ are the one-photon and two-photon detunings, respectively.

In order to investigate the dynamics conveniently and guarantee the conservation of the total particle numbers for different species, as in Refs. [17], [22], and [23], we introduce the mean-field Lagrange density with two Lagrange multipliers $\hbar\mu_a n_a$ and $\hbar\mu_b n_b$,

$$L = \hbar\sum_i\left[\frac{\mathrm{i}\hbar}{2}\left(\psi_i^*\frac{\partial\psi_i}{\partial t} - \psi_i\frac{\partial\psi_i^*}{\partial t}\right)\right] - E - \hbar\mu_a n_a - \hbar\mu_b n_b, \quad (2)$$

where $n_a = |\psi_a|^2 + |\psi_m|^2 + |\psi_g|^2$ and $n_b = |\psi_b|^2 + |\psi_m|^2 + |\psi_g|^2$, and $\hbar\mu_a$, $\hbar\mu_b$ are identified as the chemical potentials of the corresponding atoms.

Substituting the above mean-field Lagrangian density into the Euler–Lagrange equation

$$\frac{\partial L}{\partial \psi_i^*} - \partial_\nu \left(\frac{\partial L}{\partial (\partial_\nu \psi_i^*)} \right) = 0,$$

one can obtain a set of equations for the complex probability amplitudes (with $\hbar = 1$),

$$\begin{aligned}
\mathrm{i}\dot{\phi_a} &= \left(\sum_{i \neq a} \chi_{ai} |\phi_i|^2 + A_a |\phi_a|^{4/3} - \mu_a \right) \phi_a \\
&\quad + \frac{\lambda}{2} \phi_b^* \phi_m, \\
\mathrm{i}\dot{\phi_b} &= \left(\sum_{i \neq b} \chi_{bi} |\phi_i|^2 + A_b |\phi_a|^{4/3} - \mu_b \right) \phi_b \\
&\quad + \frac{\lambda}{2} \phi_a^* \phi_m, \\
\mathrm{i}\dot{\phi_m} &= \left(\sum_{i} \chi_{mi} |\phi_i|^2 - \mu_a - \mu_b - \mathrm{i}\gamma + \delta \right) \phi_m \\
&\quad + \frac{\lambda}{2} \phi_a \phi_b - \frac{\Omega}{2} \phi_g, \\
\mathrm{i}\dot{\phi_g} &= \left(\sum_{i} \chi_{gi} |\phi_g|^2 - \mu_a - \mu_b \right) \phi_g \\
&\quad + \Delta \phi_g - \frac{\Omega}{2} \phi_m,
\end{aligned} \tag{3}$$

where $\phi_i = \psi_i/\sqrt{n}$, $\chi_{ij} = n\chi'_{ij}$, $\lambda_i = \lambda'_i\sqrt{n}$, $\Omega_i = \Omega'_i\sqrt{n}$, and $A_i = A'_i n^{2/3}$ are the renormalized quantities with the density of the total particle number $n = n_a + n_b$, and the term proportional to γ is introduced phenomenologically to simulate the loss of intermediate molecules.

We now consider the stationary state solutions of Eq. (3). We know that the existence of stationary solutions of Eq. (3) requires that $\dot{x} = 0$, $x = \phi_a, \phi_b, \phi_m, \phi_g$. However, it is difficult to find the exact solutions of the above dynamical equations when $\dot{x} = 0$. In analogy to the closely related models,[22,23] we assume that the system supports a CPT state with $\phi_m = 0$ and let $n_a = n_b$, then one can easily derive the following CPT solutions:

$$\begin{aligned}
&|\phi_a^0|^2 = |{\phi_b}^0|^2 = (-\Omega^2 + \sqrt{\Omega^4 + 2\Omega^2\lambda^2})/(2\lambda), \\
&|\phi_g^0|^2 = 1/2 - |\phi_a^0|^2,
\end{aligned} \tag{4}$$

with the following chemical potentials and two-photon resonance condition:

$$\begin{aligned}
\mu_a &= \left(\chi_{ab} |\phi_b^0|^2 + \chi_{ag} |\phi_g^0|^2 + A_a |\phi_a^0|^{4/3} \right), \\
\mu_b &= \left(\chi_{ab} |\phi_a^0|^2 + \chi_{bg} |\phi_g^0|^2 + A_b |\phi_g^0|^{4/3} \right), \\
\Delta &= (\chi_{ag} + \chi_{bg} - \chi_{gg}) |\phi_g^0|^2 + A_a |\phi_a^0|^{4/3} + A_b |\phi_b^0|^{4/3} \\
&\quad + (\chi_{ab} - \chi_{ag}) |\phi_a^0|^2 + (\chi_{ab} - \chi_{bg}) |\phi_b^0|^2.
\end{aligned} \tag{5}$$

From Eqs. (4) and (5), we can conclude that, by dynamically maintaining the two-photon resonance condition, population can be concentrated in atomic states and bound molecular states under the respective limit $\lambda/\Omega \to 0$ and $\lambda/\Omega \to \infty$, which facilitates the adiabatic coherent population transfer between fermionic atoms and bosonic molecules.

3. Linear instability of the CPT state

Now we turn to study the linear stability of the CPT state. The existence of the CPT state cannot guarantee that it can always be followed adiabatically. This is because the bosonic interparticle nonlinear collisions or the fermionic blocking terms may bring forth the linear instability to the dynamics of the system and hence destroy the adiabatic evolution from the CPT state, as has been shown in Refs. [27] and [29]. Therefore, it is important to avoid the occurrence of the dynamical linear instability of the CPT state for the successful achievement of the STIRAP. For this purpose, we investigate the stability of the CPT state with the linear stability analysis[30,31] of the classical Hamiltonian.

By making use of the canonical transformation, we recast the grand canonical Hamiltonian $K = E - \hbar\mu_a n_a - \hbar\mu_b n_b$ corresponding Eq. (3) into the form of a classical one by substituting old 'variables' $-\phi_i$ (complex numbers $\phi_a = x_a + \mathrm{i}y_a$, $\phi_b = x_b + \mathrm{i}y_b$, $\phi_m = x_m + \mathrm{i}y_m$, $\phi_g = x_g + \mathrm{i}y_g$) into new ones x_i and y_i. Here x_i are canonical momenta, while y_i are the coordinates. They are governed by the following differential equations:

$$\dot{x_i} = \frac{\partial K}{\partial y_i}, \qquad \dot{y_i} = -\frac{\partial K}{\partial x_i}.$$

By setting $\dot{x_i} = \dot{y_i} = 0$, it can be proved that the CPT state corresponds to the fixed point: $x_a = |\phi_a^0|$, $y_a = 0$, $x_b = |\phi_b^0|$, $y_b = 0$, $x_m = 0$, $y_m = 0$, $x_g = |\phi_g^0|$, $y_g = 0$ with the chemical potentials and the two-photon resonance conditions in Eqs. (5).

The stability of the fixed points depends on the Hamiltonian–Jaccobi matrix. Once the eigenvalues of the Hamiltonian–Jaccobi matrix are pure imaginary,

the system is dynamical stable. Otherwise, it is dynamical unstable and the adiabaticity will therefore break down. Let $x_a = z_1$, $y_a = z_2$, $x_b = z_3$, $y_b = z_4$, $x_d = z_5$, $y_d = z_6$, $x_g = z_7$, $y_g = z_8$, then elements of the Hamiltonian–Jaccobi matrix can be written elegantly as

$$J_{ij} = \frac{(-1)^i}{2} \frac{\partial^2 K}{\partial z_i \partial z_{j\pm 1}} \Big|_{\rm CPT},$$

where i, j are respectively indexes of rows and columns, and the plus (subtraction) sign is for odd (even) j. Substituting the CPT state into the matrix elements, we find the Hamiltonian–Jacobi matrix around this fixed point (CPT state) for the atom–molecule conversion system is

$$J = \begin{pmatrix} 0 & \rho & 0 & \alpha & 0 & -\mu & 0 & \beta \\ 0 & 0 & 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & \alpha & 0 & \varsigma & 0 & -\nu & 0 & \gamma \\ 0 & 0 & 0 & 0 & \nu & 0 & 0 & 0 \\ 0 & -\mu & 0 & -\nu & 0 & -\xi & 0 & \zeta \\ \mu & 0 & \nu & 0 & \xi & 0 & -\zeta & 0 \\ 0 & \beta & 0 & \gamma & 0 & \zeta & 0 & \varrho \\ 0 & 0 & 0 & 0 & -\zeta & 0 & 0 & 0 \end{pmatrix}, \quad (6)$$

where

$$\begin{aligned} &\alpha = -2\chi_{ab}|\phi_a^0||\phi_b^0|, \quad \beta = -2\chi_{ag}|\phi_a^0||\phi_g^0|, \\ &\gamma = -2\chi_{bg}|\phi_b^0||\phi_g^0|, \quad \mu = (\lambda/2)|\phi_b^0|, \\ &\nu = (\lambda/2)|\phi_a^0|, \quad \zeta = \Omega/2, \\ &\rho = -(4/3)A_a|\phi_a^0|^{4/3}, \quad \varrho = -2\chi_{gg}|\phi_g^0|^2, \\ &\varsigma = -(4/3)A_b|\phi_b^0|^{4/3}, \\ &\xi = (\chi_{am} - \chi_{ab})\,|\phi_a^0|^2 + (\chi_{bm} - \chi_{ab})\,|\phi_b^0|^2 \\ &\quad + (\chi_{mg} - \chi_{ag} - \chi_{bg})\,|\phi_g^0|^2 + \delta \\ &\quad - A_a|\phi_a^0|^{4/3} - A_b|\phi_b^0|^{4/3}. \end{aligned}$$

We can see there are lots of zero matrix elements in the Hamiltonian–Jacobi matrix J of Eq. (6). A straightforward calculation of the eigenvalues (other than the zero-mode frequency) of Eq. (6) with an analytic expression can be obtained,

$$\begin{aligned} &\omega_{1,2\pm} = \pm\frac{\rm i}{2}\sqrt{b \pm \sqrt{b^2 - c}}, \\ &b = 2\xi^2 + \Omega^2 + 2\lambda^2|\phi_a^0|^2, \\ &c = \Omega^4 - 8\xi^4 - 8\Omega^2\xi^2 + 4\left(\lambda^4 - 12\chi_{ab}\lambda^2\xi\right)|\phi_a^0|^4 \\ &\quad + 4\lambda|\phi_a^0|^2\left(\lambda\left(\Omega^2 - 4\xi^2\right) + 12(\chi_{ag} + \chi_{bg})\xi\Omega|\phi_g|\right) \\ &\quad - 16(A_a + A_b)\xi\lambda^2|\phi_a^0|^{10/3} - 24\chi_{gg}\xi\Omega^2|\phi_g^0|^2. \quad (7) \end{aligned}$$

When $\omega_\pm$ becomes real or complex, the corresponding CPT state is dynamically unstable. We can see from Eq. (7) that $b > 0$. Hence the unstable regime is given by either $c < 0$ or $c > b^2$. In the absence of nonlinear collisions and the Pauli-blocking terms, i.e., $\chi_{ij} = A_i = 0$, we find $c < b^2$. Hence the eigenvalues are always pure imaginary. Once either of the interparticle interactions and the Pauli blocking terms is included, they may induce $c < 0$ or $c > b^2$, and hence complex frequencies will emerge and bring the instability in population dynamics. Therefore, the instability here is not only caused by nonlinear collisions but also by the Pauli-blocking term, which is different from the bosonic system where the dynamical instability is only caused by the interparticle interactions.

Now we consider the instability of two concrete systems, respectively, for ^{40}K and ^{6}Li atoms with concrete magnetic and laser fields. As in Ref. [23], for the ^{40}K atom–molecule conversion system, the magnetic field is chosen as 201.7 Gs (1 Gs = 10^{-4} T) and the particle density n is about 10^{20} m^{-3}, then we can obtain $\lambda' = 16.6 \times 10^{-39}$ J, $A_a = 0.16\lambda$, $\chi_{ab} = 0.24\lambda$, $\delta = -4.4\lambda$. For ^{6}Li atom–molecule conversion system with the same particle density n, the magnetic field is about 543.6 Gs, then one obtains $\lambda' = 3.29 \times 10^{-37}$ J, $A_a = 0.055\lambda$, $\chi_{ab} = 0.0027\lambda$, $\delta = -0.01125\lambda$. So far, there are no good estimates on molecular scattering lengths. So we take the collisional coefficients involving the molecular levels to zero.

Figures 2(a)–2(d) show the instability diagrams and examples of the occurrence of instability in the population dynamics for ^{40}K and ^{6}Li atom–molecule conversion systems, respectively. In Figs. 2(a) and 2(b), the black (white) areas correspond to the unstable (stable) regions. From these two figures, we see that, for the above two systems, there are two unstable regions corresponding to the two cases $c > b^2$ or $c < 0$, respectively. Region I corresponds to the unstable region obtained by setting $c > b^2$, whose width shrinks as Ω increases; Region II is the unstable region obtained by setting $c < 0$, whose width becomes fat with increasing δ. For ^{40}K (^{6}Li) atom–molecule conversion system, when $\delta = 3$ ($\delta = 7$), the dynamics of the system is unstable once $0 < \Omega < 0.84$ ($0 < \Omega < 0.37$). The examples of the occurrence of instability with the parameters (labeled by $*$) in the unstable regions for ^{40}K and ^{6}Li atom–molecule conversion systems are shown in Figs. 2(c) and 2(d). For ^{40}K (^{6}Li) atom–molecule conversion system, when $\Omega = 0.445$, $\delta = 3$ ($\Omega = 0.3$, $\delta = 7$), the linear insta-

bility occurs at $t = 180$ ($t = 1000$), as is shown in Fig. 2(c) [2(d)]. Hence it is crucial for adiabatic evolution to avoid these unstable regimes when designing the route of adiabatic passage. Moreover, by comparing Fig. 2(a) with Fig. 2(b), we find that the range of δ which induces the instability in ^{6}Li atom–molecule conversion system is smaller than that of ^{40}K atom–molecule conversion system. Therefore, in the present atom–molecule conversion STIRAP scheme in which Ω decreases with time, it is more easy to introduce instability in ^{40}K atom–molecule conversion system than that in ^{6}Li atom–molecule conversion system.

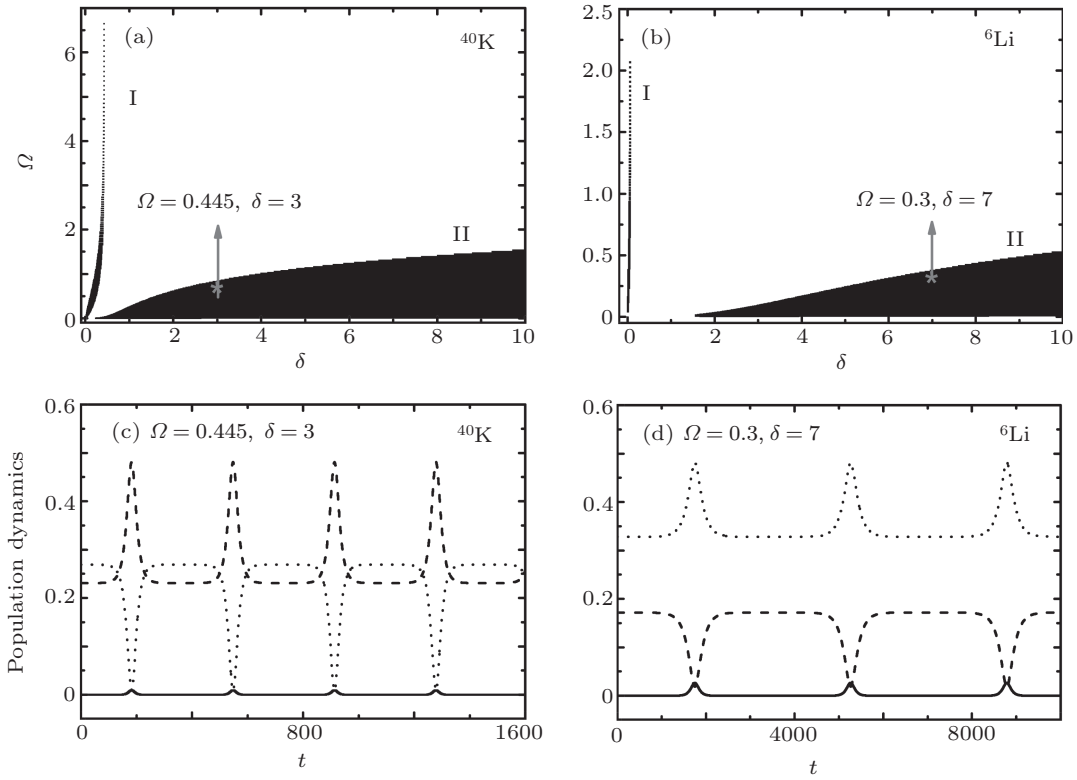

Fig. 2. Panels (a) and (b) are the instability diagrams for ^{40}K and ^{6}Li atom–molecule conversion systems, where the black areas correspond to the unstable regions. Panels (c) and (d) are examples of instability in the population dynamics with the parameters labeled by $*$ in panels (a) and (c). The left figures are for ^{40}K atom–molecule conversion system, where the parameters are $A_a = A_b = 0.16$, $\chi_{ab} = 0.24$, $\chi_{am} = \chi_{ag} = \chi_{bm} = \chi_{bg} = \chi_{mg} = 0$. The right figures are for ^{6}Li atom–molecule conversion system, where the parameters are $A_a = A_b = 0.055$, $\chi_{ab} = 0.0027$, $\chi_{am} = \chi_{ag} = \chi_{bm} = \chi_{bg} = \chi_{mg} = 0$. Time is in units of λ^{-1}. All other parameters are in units of λ.

4. Adiabaticity of the CPT state

In the stable region, the existence of the CPT state facilitates the adiabatic coherent population transfer between the fermionic atoms and the bosonic molecules. However, owing to the invalidation of the superposition principle in the nonlinear atom–molecule conversion system, it is not justified to apply the adiabatic condition of quantum mechanics to study the adiabatic evolution of the CPT state. To overcome this difficulty, in this section, we attempt to obtain the adiabatic criterion for this nonlinear system with classical adiabatic dynamics.[32] Therefore, the adiabatic parameter of this nonlinear system can be defined as the ratio between the change rate of the external parameters and the fundamental frequencies of periodic orbits around the fixed point which corresponds to the CPT state.[32,33]

For a classical system, the adiabatic evolution[33] requires that the external parameter R changes slowly in comparison with the intrinsic frequency of the system, i.e., the adiabatic parameter satisfies

$$\varepsilon = \frac{2\pi}{\omega} \cdot \alpha \ll 1, \tag{8}$$

where

$$\alpha = \max\Big\{\Big|\frac{\dot{R}}{R}\Big|, (0, T)\Big\},$$

ω is the intrinsic frequency of the system, and $\varepsilon \to 0$ corresponds to the adiabatic limit.

For this atom–molecule conversion process accomplished by the STIRAP aided by FR, $R = \Omega$

and $\alpha = |\dot{\Omega}/\Omega|$. Moreover, in this multiple-degree-of-freedom system, ω is the smallest magnitude of the frequencies of the system, i.e.,

$$\omega = \min\{|\omega_{1+}|, |\omega_{1-}|, |\omega_{2+}|, |\omega_{2-}|\}.$$

In the following discussions, we will use the adiabatic parameter ε to study the adiabaticity of the system.

In our calculations, we adopt the time-dependent Rabi frequency,

$$\Omega(t) = \Omega_0 \left[1 - \tanh\left(\frac{t - t_0}{\tau}\right)\right], \tag{9}$$

where the parameters Ω_0, t_0, and τ are determined by the applied laser field that couples the two molecular states. In the numerical calculation, at the initial time ($t = 0$), only the atomic states are populated, i.e., $\phi_a = \phi_b = 1/2$, $\phi_{m,g} = 0$, and we solve the nonlinear Schrödinger equation using the fourth-fifth order Runge–Kutta adaptive-step algorithm.

Figures 3(a) and 3(b) show the adiabatic parameter ε and the population dynamics for ^{40}K and ^{6}Li atom–molecule conversion system, respectively. As can be seen in Fig. 3(a) [Fig. 3(b)], for ^{40}K [^{6}Li] atom–molecule conversion system, the adiabatic condition is approximately satisfied in the first stage of the evolution, i.e., $\varepsilon \ll 1$, which implies that the system can adiabatically evolve along the CPT state. However, for ^{40}K [^{6}Li] atom–molecule conversion system, at the time $t = 827$ ($t = 799$), the adiabatic condition begins to be dissatisfied, i.e., $\varepsilon > 1$, which denotes that the system deviates from the CPT state from that time on and cannot maintain adiabaticity completely during the entire evolution. That is why, at the later stage of evolution, the real evolution state departs from the CPT solution slightly. Hence not all particles but 88% of ^{40}K [93% of ^{6}Li] atoms in the initial states can be converted to the stable molecules.

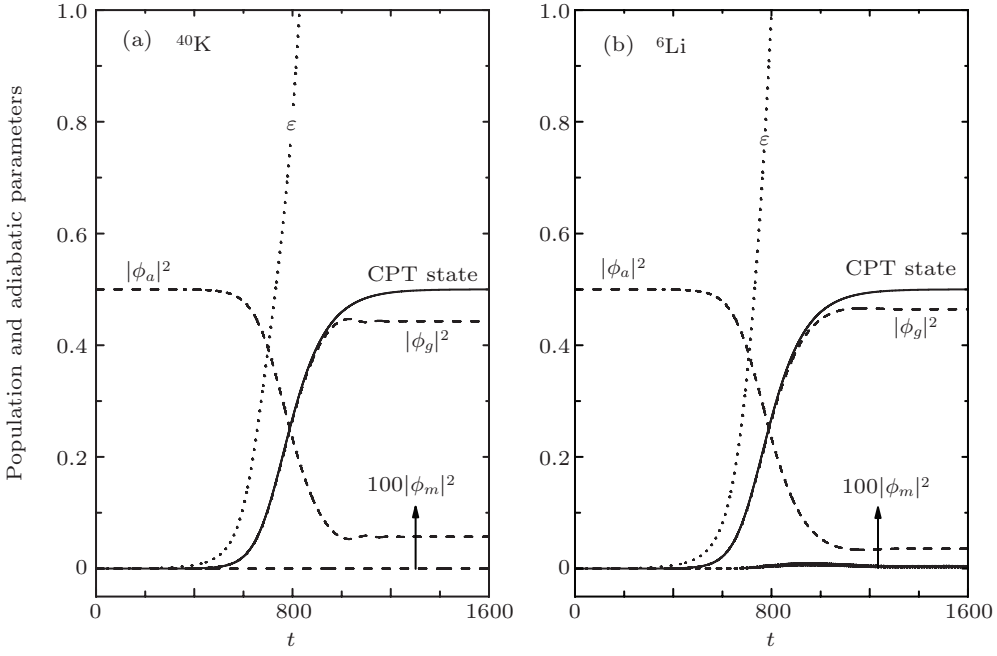

Fig. 3. Adiabatic parameters and populations as functions of time for ^{40}K and ^{6}Li atom–molecule conversion systems, respectively. Panel (a) is for the ^{40}K atom–molecule conversion system, where $\gamma = 0$, $\Omega_0 = 200$, $\delta = -4.4$, $\tau_0 = 600$, $\tau = 200$. Panel (b) is for the ^{6}Li atom–molecule conversion system, in which $\gamma = 0$, $\Omega_0 = 200$, $\delta = -3$, $\tau_0 = 600$, $\tau = 200$. All other parameters are the same as those in Fig. 2.

5. Conclusion

In conclusion, we investigated the linear instability and adiabaticity of a dark state during conversion of two species of fermionic atoms to stable molecules through the stimulated Raman adiabatic passage aided by Feshbach resonance. We analytically obtained the regions for the appearance of linear instability which is caused not only by the bosonic interparticles but also by the Pauli blocking terms. Taking ^{40}K and ^{6}Li atom–molecule conversion systems as examples, we gave the unstable regions numerically. Moreover, we also attempted to obtain the adiabatic criterion for this nonlinear system with classical adiabatic dynamics. We showed that the adiabatic condition of the atom–molecule dark state for the ^{40}K [or ^{6}Li] atom–molecule conversion system is only satisfied

in the first stage of the evolution. Therefore, the small discrepancy from the CPT solution at the latter stage of the evolution can be attributed to the fact that the system cannot maintain adiabaticity completely.

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