Eur. Phys. J. D **65**, 557–561 (2011)
DOI: 10.1140/epjd/e2011-20517-1

THE EUROPEAN PHYSICAL JOURNAL D

Regular Article

Adiabatic geometric phase in the nonlinear coherent coupler

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Received 9 June 2011 / Received in final form 24 October 2011
Published online 1st December 2011 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2011

Abstract. We introduce the concept of geometric phase to the nonlinear coherent coupler. With considering the adiabatic change of the distance-dependent phase mismatch, we calculate the adiabatic geometric phase related to the supermode of the coupler analytically. We find that the phase depends on the input light intensity explicitly. In particular, in the low and high intensity limits, the phase equals half of the area on the Poincare sphere enclosed by the evolution loop of the system. At the critical intensity where different supermodes merge, the phase diverges, which can be considered as the signal of a continuous phase transition.

1 Introduction

A close examination of the quantum adiabatic theorem [1–3] leads to the discovery of the adiabatic geometric phase, or the Berry phase [4]. Recently, this phase and its extensions [5–9] have received renewed interest due to their important applications in quantum computation and quantum information [10–13]. According to the adiabatic theorem, when a parameter of the system changes adiabatically, the system, which is initially in an energy eigenstate, will remain in this eigenstate, and thus will evolve with the parameter simultaneously. When the parameter returns to its initial value, the system will acquire a adiabatic geometric phase as well as the dynamical phase. Motivated partly by the studies of Bose-Einstein condensation [14–19], both the adiabatic theorem and the adiabatic geometric phase have been extended to nonlinear systems [20–23]. In particular, the effect of nonlinearity on the adiabatic process has been investigated in the coupled waveguide system [24]. Some nonlinear waveguide systems, described by the nonlinear Schrödinger equation, can be directly used to observe the interplay between adiabatic evolution and nonlinearity. In addition, they can serve as direct analogies to various other quantum processes [24]. Therefore, these nonlinear waveguide systems provide ideal models to study the nonlinear adiabatic evolution.

One of the most simple nonlinear waveguide systems is known as the nonlinear coherent coupler, which consists of two parallel optical waveguides with Kerr nonlinearity [25]. When the two waveguides in the coupler are different, the coupler is called asymmetric. In this paper, we consider the phase mismatch in the asymmetric coupler changing adiabatically with distance, and calculate the adiabatic geometric phase associated with the supermode of the coupler analytically. We find that the phase is dependent of the input light intensity, and show the characteristics of the phase at the critical light intensity where different supermodes merge as well as in the low and high intensity limits. Because the nonlinear coherent coupler perform a number of useful functions in optical communications, including power division, power coupling, and switching [26], we can expect that the geometric phase presented here may have many prospective applications in practice.

The plan of this paper is as follows. In Section 2, we review the model describing the nonlinear coherent coupler, and investigate the supermode of the coupler briefly. In Section 3, we calculate the adiabatic geometric phase related to the supermode analytically, and discuss its characteristics. Section 4 is the conclusion.

2 Coupler and its supermodes

The model describing the propagation of the laser field inside the coupler can be derived from the standard coupled mode theory [27]. To be clear and self-contained, we first introduce this model briefly. We begin by considering a linearly polarized laser field propagating inside the coupler along the $+z$-direction. According to the coupled mode theory, the electric field can be expressed as

$$\mathbf{E}(x,y,z,t) = \frac{1}{2}\sum_l A_l(z)\mathbf{E}_l(x,y)e^{i(\beta_l z-\omega t)} + c.c., \quad (1)$$

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where ω is the frequency, $\mathbf{E}_l (l = 1, 2)$ is the only confined mode of waveguide l, A_l and $\beta_l > 0$ are the corresponding amplitude and propagation constant respectively. The mode function $\mathbf{E}_l$ satisfies the orthonormalization relation $\frac{\beta_l}{2\omega\mu_0} \int \mathbf{E}_l^* \cdot \mathbf{E}_{l'} dxdy = \delta_{ll'}$ and the wave equation $\left(\nabla_\perp^2 + \frac{\omega^2}{c^2}[1 + \chi_l(x, y)]\right) \mathbf{E}_l = \beta_l^2 \mathbf{E}_l$, where $\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, and the susceptibility distribution

$$\chi_l(x, y) = \begin{cases} \chi_l, & \text{waveguide } l, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The electric field $\mathbf{E}$ itself satisfies

$$\left(\nabla^2 + \frac{\omega^2}{c^2}[1 + \sum_l \chi_l(x, y)]\right) \mathbf{E} = \mu_0 \frac{\partial^2}{\partial t^2} \mathbf{P}_{NL}, \tag{3}$$

where $\mathbf{P}_{NL}$ is the nonlinear polarization. Because the laser field is linearly polarized, there is only one component of the third-order nonlinear susceptibility, denoted by $\chi_l^{(3)}$, responsible for the Kerr nonlinearity, and the nonlinear polarization

$$\mathbf{P}_{NL} = \frac{3}{2} \sum_l \chi_l^{(3)}(x, y) |\mathbf{E}|^2 \mathbf{E} + c.c., \tag{4}$$

where the nonlinear susceptibility distribution

$$\chi_l^{(3)}(x, y) = \begin{cases} \chi_l^{(3)}, & \text{waveguide } l, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

We now take the scalar product of equation (3) with $\mathbf{E}_l^*(x, y)$, and integrate over the entire $x - y$ plane. Moreover, using the slowly varying amplitude approximation $|\beta_l \frac{d}{dz} A_l| \gg |\frac{d^2}{dz^2} A_l|$, and assuming that $\int \chi_l^{(3)}(x, y) |\mathbf{E}_l|^4 dxdy$ is much larger than the other integrals related to the nonlinearity, we finally obtain the coupled nonlinear Schrödinger equations

$$i\frac{d}{dz}A_1 + \alpha_1 A_1 + k_{12} e^{i(\beta_2 - \beta_1)z} A_2 + \gamma_1 |A_1|^2 A_1 = 0, \tag{6}$$

$$i\frac{d}{dz}A_2 + \alpha_2 A_2 + k_{21} e^{-i(\beta_2 - \beta_1)z} A_1 + \gamma_2 |A_2|^2 A_2 = 0, \tag{7}$$

where

$$\alpha_1 = \frac{\omega\epsilon_0}{4} \int \chi_2(x, y) |\mathbf{E}_1|^2 dxdy, \tag{8}$$

$$\alpha_2 = \frac{\omega\epsilon_0}{4} \int \chi_1(x, y) |\mathbf{E}_2|^2 dxdy, \tag{9}$$

$$k_{12} = \frac{\omega\epsilon_0}{4} \int \chi_1(x, y) \mathbf{E}_1^* \cdot \mathbf{E}_2 dxdy, \tag{10}$$

$$k_{21} = \frac{\omega\epsilon_0}{4} \int \chi_2(x, y) \mathbf{E}_2^* \cdot \mathbf{E}_1 dxdy, \tag{11}$$

$$\gamma_1 = \frac{3\omega}{4} \int \chi_1^{(3)}(x, y) |\mathbf{E}_1|^4 dxdy, \tag{12}$$

$$\gamma_2 = \frac{3\omega}{4} \int \chi_2^{(3)}(x, y) |\mathbf{E}_2|^4 dxdy. \tag{13}$$

Before proceeding further, we present a few comments on equations (6), (7), and the parameters given in equations (8)–(13). First, we note that the phase mismatch factors in the coupling terms of equations (6) and (7) exist only when $\beta_1 \neq \beta_2$. Second, the parameters α_1 and α_2 serve only to modify β_1 and β_2. Third, from the conservation of the total light intensity $I = |A_1|^2 + |A_2|^2$ in the z-direction, we find $k_{12} = k_{21}^*$. For simplicity, we assume that $k_{12} = k_{21} = k > 0$. Fourth, if $\chi_l^{(3)} > 0$, and thus $\gamma_l > 0$, we say that the nonlinear mechanism of waveguide l is self-focusing; if $\chi_l^{(3)} < 0$, and thus $\gamma_l < 0$, we say that it is defocusing [28]. In the self-focusing case, the refractive index increases locally with the power, while in the defocusing cases it decreases.

Because the concept of adiabatic geometric phase originates from quantum mechanics, we need to map the above model into a nonlinear quantum model. Specifically, introducing the nonlinear Hamiltonian

$$H(|A_1|^2, |A_2|^2) = -\begin{pmatrix} \alpha_1 + \gamma_1 |A_1|^2 & k e^{i(\beta_2 - \beta_1)z} \\ k e^{-i(\beta_2 - \beta_1)z} & \alpha_2 + \gamma_2 |A_2|^2 \end{pmatrix}, \tag{14}$$

we can express the coupled nonlinear Schrödinger equations (6) and (7) as

$$i\frac{d}{dz}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = H(|A_1|^2, |A_2|^2) \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \tag{15}$$

Here we note that the evolution of $(A_1, A_2)^T$ in the $+z$-direction corresponds to the time evolution of a nonlinear two-level system in quantum mechanics. The eigenequation of $H(|\bar{A}_1|^2, |\bar{A}_2|^2)$ reads

$$\mu \begin{pmatrix} \bar{A}_1 \\ \bar{A}_2 \end{pmatrix} = H(|\bar{A}_1|^2, |\bar{A}_2|^2) \begin{pmatrix} \bar{A}_1 \\ \bar{A}_2 \end{pmatrix}, \tag{16}$$

where μ is the eigenvalue, and $(\bar{A}_1, \bar{A}_2)^T$ is the eigenstate, or the supermode of the coupler [26].

For convenience and without loss of generality, we further write

$$\begin{aligned} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} &= \sqrt{I} \begin{pmatrix} \cos(\theta/2) e^{-i\phi_1} \\ \sin(\theta/2) e^{-i\phi_2} \end{pmatrix} \\ &= \sqrt{I} \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) e^{-i\phi} \end{pmatrix} e^{-i\phi_1}, \end{aligned} \tag{17}$$

where $0 \leq \theta \leq \pi$, $\phi = \phi_2 - \phi_1$, and ϕ_1 has been split off as the overall phase. From equation (17), we know that the state of the system, except for an overall phase, can be denoted by (θ, ϕ). Because θ and ϕ span a unit sphere, called the Poincare sphere [26,27,29,30], the evolution of the system without the overall phase corresponds to the movement of the system on the Poincare sphere. Introducing $\beta = \beta_2 - \beta_1$, $\alpha = \alpha_2 - \alpha_1$, and substituting equation (17) into equation (15), we obtain

$$\frac{d\theta}{dz} = -\,2k\sin(\phi-\beta z), \tag{18}$$

$$\frac{d\phi}{dz} = -\,\alpha - 2k\cot\theta\cos(\phi-\beta z) - I\gamma_2 + I(\gamma_1+\gamma_2)\cos^2(\theta/2), \tag{19}$$

$$\frac{d\phi_1}{dz} = -\,\alpha_1 - k\tan(\theta/2)\cos(\phi-\beta z) - I\gamma_1\cos^2(\theta/2). \tag{20}$$

Similar to equation (17), we write the supermode $(\bar{A}_1, \bar{A}_2)^T = (\cos(\bar{\theta}/2), \sin(\bar{\theta}/2)e^{-i\bar{\phi}})^T$, so that we can denote the supermode by $(\bar{\theta}, \bar{\phi})$. From equation (16), we have

$$\bar{\phi} = \beta z \quad\text{and}\quad \bar{\phi} = \beta z + \pi, \tag{21}$$

$$\pm 2k\cot\bar{\theta} = -\,\alpha - I\gamma_2 + I(\gamma_1+\gamma_2)\cos^2(\bar{\theta}/2), \tag{22}$$

$$\mu = -\,\alpha_1 \mp k\tan(\bar{\theta}/2) - I\gamma_1\cos^2(\bar{\theta}/2). \tag{23}$$

Here and below, the upper sign is for $\bar{\phi} = \beta z$, and the lower sign for $\bar{\phi} = \beta z + \pi$.

Introducing $t = \tan(\bar{\theta}/2)$, we can rewrite equation (22) as

$$t^4 \mp \frac{\alpha + I\gamma_2}{k}t^3 \mp \frac{\alpha - I\gamma_1}{k}t - 1 = 0. \tag{24}$$

Because $0 \le \bar{\theta} \le \pi$, only $t \ge 0$ corresponds to the supermode of the coupler. Here we note that, if t_0 is a solution to equation (24) for the upper sign, $-t_0$ will be a solution to it for the lower sign. Therefore, we can find all supermodes by solving equation (24) for either sign, and changing the signs of the minus solutions. Actually, without having to solve equation (24), we can analytically prove that, only when

$$[2\alpha + I(\gamma_2-\gamma_1)]^{2/3} + (4k)^{2/3} = [I(\gamma_1+\gamma_2)]^{2/3}. \tag{25}$$

Equation (24) has three solutions for either sign. If the left-hand side of equation (25) is greater (less) than the right-hand side, equation (24) has two (four) solutions for either sign. This indicates that equation (25) determines the parameter region where the number of the supermode changes.

3 Adiabatic geometric phase

We consider an asymmetric coupler so that the phase mismatch βz is nonvanishing, and the supermode changes with z according to equation (21). Mathematically, the supermode corresponds to the fixed point on the Poincare sphere, therefore the change of the supermode corresponds to the movement of the fixed point on the sphere. Moreover, we assume that β is small enough so that, according to the nonlinear version of the adiabatic evolution condition [20,21], the system, which is in a fixed point at $z = 0$, can remain in this fixed point, and thus also moves on the Poincare sphere as z increases. When z reaches $2\pi/\beta$ at last, the system will return its initial position on the Poincare sphere, and the overall phase ϕ_1 will acquire an increasing amount, which consists of an adiabatic geometric phase Φ as well as the dynamical phase. To obtain the expression for Φ, we need to use the method introduced in reference [22,23] to separate the Φ-related term from equation (20). This is equivalent to calculate Φ as the difference between the overall phase and the dynamical phase [31]. To proceed, we first note that, because β is small but finite, the system will fluctuate around the supermode during the evolution process, i.e., $\phi = \bar{\phi} + \delta\phi$ and $\theta = \bar{\theta} + \delta\theta$, where $\delta\phi \sim \delta\theta \sim O(\beta)$. Then, using equations (18)–(23) and ignoring the terms $\sim O(\beta^2)$, we have

$$\frac{d\bar{\theta}}{dz} = \mp\, 2k\delta\phi, \tag{26}$$

$$\frac{d\bar{\phi}}{dz} = \frac{1}{2}\left[\frac{\pm 4k}{\sin^2\bar{\theta}} - I(\gamma_1+\gamma_2)\sin\bar{\theta}\right]\delta\theta, \tag{27}$$

$$\frac{d\phi_1}{dz} = \mu - \frac{1}{2}\left[\frac{\pm k}{\cos^2(\bar{\theta}/2)} - I\gamma_1\sin\bar{\theta}\right]\delta\theta. \tag{28}$$

Combining equations (27) and (28), we have

$$\frac{d\phi_1}{dz} = \mu - \frac{\beta}{2}\left[1 - \frac{\cos\bar{\theta} \mp (I/4k)(\gamma_2-\gamma_1)\sin^3\bar{\theta}}{1 \mp (I/4k)(\gamma_1+\gamma_2)\sin^3\bar{\theta}}\right]. \tag{29}$$

Integrating the zero-order term μ over z from 0 to $2\pi/\beta$, we obtain the dynamical phase. Likewise, integrating the Φ-related first-order term, we obtain the adiabatic geometric phase

$$\Phi = -\pi\left[1 - \frac{\cos\bar{\theta} \mp (I/4k)(\gamma_2-\gamma_1)\sin^3\bar{\theta}}{1 \mp (I/4k)(\gamma_1+\gamma_2)\sin^3\bar{\theta}}\right]. \tag{30}$$

From equation (30), we know that the geometric phase Φ is dependent of the total light intensity I. In the low intensity limit $I \to 0$, the nonlinear effect is negligible. Then, equation (30) reduces to $\Phi = -\pi(1-\cos\bar{\theta})$, which denotes half of the area on the Poincare sphere enclosed by the evolution loop of the system except for a sign due to the definition of the overall phase. Noting that this result is in accordance with the result obtained in linear systems [4–9], we conclude that the geometric phase Φ is a nonlinear extension of its linear counterpart. In the low intensity limit, from equation (22), we have $\cos\bar{\theta} \to \mp\frac{\alpha}{\sqrt{\alpha^2+4k^2}}$, and thus $\Phi \to -\pi(1 \pm \frac{\alpha}{\sqrt{\alpha^2+4k^2}})$.

In the high intensity limit $I \to \infty$, the nonlinear effect is dominant. Then, using equation (22) to obtain $\cos\bar{\theta}$, we find that equation (30) still reduces to $\Phi = -\pi(1-\cos\bar{\theta})$. That is to say, the geometric meaning of Φ is still the same as in the low intensity limit.

To illustrate the geometric phase Φ and the supermode in the high power intensity limit, we give them in Tables 1 and 2. From these two tables, we find that the supermodes in Table 1 and in the last two lines of Table 2 confine the laser field in one of the two waveguides. Then $\Phi = 0$ or -2π trivially. The supermodes in the first two lines of Table 2 have an intensity distribution determined by the

Table 1. All supermodes and Φ's in the high intensity limit $I \to \infty$ with $\gamma_1\gamma_2 < 0$.

$\bar{\phi}(\gamma_1 > 0, \gamma_2 < 0)$	$\bar{\phi}(\gamma_1 < 0, \gamma_2 > 0)$	$\cos\bar{\theta} \to$	$\Phi \to$
βz	$\beta z + \pi$	1	0
$\beta z + \pi$	βz	-1	-2π

Table 2. Same as Table 1 but with $\gamma_1\gamma_2 > 0$.

$\bar{\phi}(\gamma_1, \gamma_2 > 0)$	$\bar{\phi}(\gamma_1, \gamma_2 < 0)$	$\cos\bar{\theta} \to$	$\Phi \to$
βz	βz	$\frac{\gamma_2-\gamma_1}{\gamma_1+\gamma_2}$	$-\frac{2\gamma_1}{\gamma_1+\gamma_2}\pi$
$\beta z + \pi$	$\beta z + \pi$	$\frac{\gamma_2-\gamma_1}{\gamma_1+\gamma_2}$	$-\frac{2\gamma_1}{\gamma_1+\gamma_2}\pi$
βz	$\beta z + \pi$	1	0
βz	$\beta z + \pi$	-1	-2π

ratio between γ_1 and γ_2. Then the values of Φ are also determined by this ratio.

In contrast to the geometric phases in linear systems, a remarkable feature of Φ lies in its divergence when

$$\sin^3\bar{\theta} = \pm\frac{4k}{I(\gamma_1+\gamma_2)}. \qquad (31)$$

From equations (22) and (31), we obtain

$$\cos^3\bar{\theta} = \frac{2\alpha + I(\gamma_2-\gamma_1)}{I(\gamma_1+\gamma_2)}. \qquad (32)$$

Combining equations (31) and (32) yields equation (25) exactly. This indicates that, only when equation (25) holds, the geometric phase Φ may diverge. Noting that equation (25) determines the parameter region where the number of the supermode changes, we know that the divergence of Φ is caused by the merging of the supermodes. Actually, besides the divergence condition, from equations (31) and (32), we can determine the merged supermode completely. This indicates that the geometric phase Φ characterizes the supermode precisely.

To illustrate the divergence of Φ when the supermodes merge, we take $\alpha = 0$ and $\gamma_1 = \gamma_2$, and give all supermodes and Φ's in Tables 3 and 4. Here we note that the supermodes and Φ's in the last two lines of these two tables exist only when $|I\gamma_1| > 2k$, and the geometric phase Φ's in the second lines diverge when $|I\gamma_1| = 2k$. At the critical intensity $I = 2k/|\gamma_1|$, the supermodes in the last three lines of these two tables have the same $\bar{\phi}$ and $\cos\bar{\theta}$, and thus merge together. On the other hand, at the same critical intensity, the geometric phase Φ's in the last three lines diverge.

In the perspective of the spontaneous symmetric breaking [32], the supermodes in the first two lines of Tables 3 and 4 are symmetric supermodes, and the supermodes in the last two lines are broken supermodes. As the total intensity I increases adiabatically, the system, which is initially in a symmetric supermode, can eventually settle in a broken supermode. This process is actually a continuous phase transition occurs when $|I\gamma_1| = 2k$. The divergence of Φ can be considered as the signal of the phase transition. Recently, the relation between geometric phases and phase transitions is proposed [33–39].

Table 3. All supermodes and Φ's when $\alpha = 0$ and $\gamma_1 = \gamma_2 > 0$.

$\bar{\phi}$	$\cos\bar{\theta}$	Φ
$\beta z + \pi$	0	$-\pi$
βz	0	$-\pi$ (unless $I\gamma_1 = 2k$)
βz	$[1-(\frac{2k}{I\gamma_1})^2]^{\frac{1}{2}}$	$-\pi + \pi[1-(\frac{2k}{I\gamma_1})^2]^{-\frac{1}{2}}$
βz	$-[1-(\frac{2k}{I\gamma_1})^2]^{\frac{1}{2}}$	$-\pi - \pi[1-(\frac{2k}{I\gamma_1})^2]^{-\frac{1}{2}}$

Table 4. Same as Table 3 but with $\gamma_1 = \gamma_2 < 0$.

$\bar{\phi}$	$\cos\bar{\theta}$	Φ
βz	0	$-\pi$
$\beta z + \pi$	0	$-\pi$ (unless $I\gamma_1 = -2k$)
$\beta z + \pi$	$[1-(\frac{2k}{I\gamma_1})^2]^{\frac{1}{2}}$	$-\pi + \pi[1-(\frac{2k}{I\gamma_1})^2]^{-\frac{1}{2}}$
$\beta z + \pi$	$-[1-(\frac{2k}{I\gamma_1})^2]^{\frac{1}{2}}$	$-\pi - \pi[1-(\frac{2k}{I\gamma_1})^2]^{-\frac{1}{2}}$

The geometric phase Φ in the continuous phase transition provides a paradigm for this relation, and the general correspondence between the divergence of Φ and the merging of the supermode can be regarded as an extension of this relation.

To illustrate the geometric phase Φ in other cases, we need to calculate the supermode and Φ numerically. As an example, in Figure 1, we show the changes of $\cos\bar{\theta}$ and Φ as γ_2 tends to $\gamma_1 = 1$ with $\alpha = 0$ and $k = 1$. From this figure, we can confirm the following features of $\cos\bar{\theta}$ and Φ. First, in the low intensity limit $I \to 0$, we have $\cos\bar{\theta} \to 0$ and $\Phi \to -\pi$ for both $\bar{\phi} = \beta z$ and $\bar{\phi} = \beta z + \pi$ as given before. Second, when the intensity I is large enough, $\cos\bar{\theta}$ and Φ tend to the values given in Table 2. Third, when two supermodes merge at the critical intensity obtained from equation (25), the geometric phase Φ diverges. Fourth, when γ_2 tends to γ_1, $\cos\bar{\theta}$ and Φ tend to the values given in Table 3.

4 Conclusion

In conclusion, we have calculated the adiabatic geometric phase associated with the supermode of the nonlinear coherent coupler analytically. We found that, in the linear and strong nonlinear limits, the phase is equal to half of the area on the Poincare sphere enclosed by the evolution loop of the system. At the critical intensity where two or three supermodes merge, the geometric phase diverges, which can be regarded as the signal of a continuous phase transition. Because the Poincare sphere representation of the supermode has a qubit structure [29,30], the extension of our findings to the quantum domain may lead to useful results for quantum information processes. We expect that the adiabatic geometric phase presented here will be confirmed experimentally in the near future.

This work is supported by the National Fundamental Research Program of China (Contact Nos. 2007CB814800 and 2011CB921503), the National Natural Science Foundation of China (Contact Nos. 10905027, 10725521, 91021021, 11075020, and 11078001).

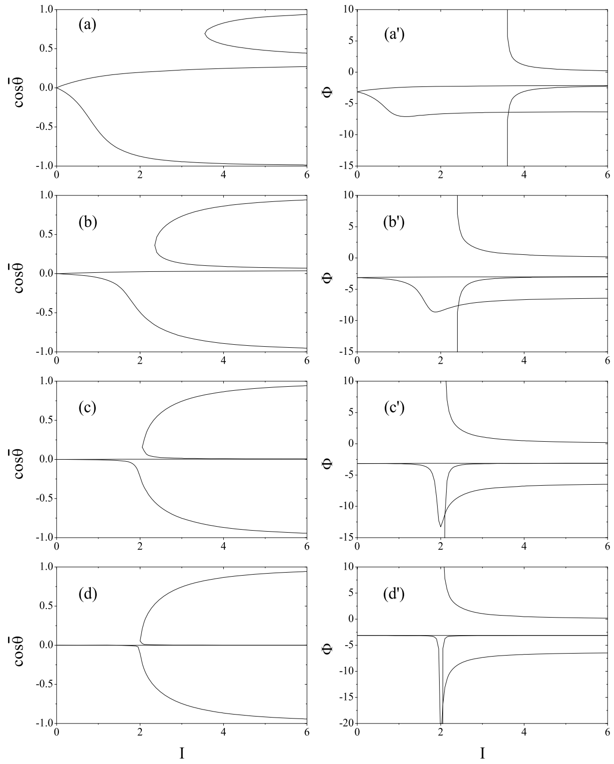

Fig. 1. The changes of $\cos\bar{\theta}$ and Φ as γ_2 tends to $\gamma_1 = 1$ with $\alpha = 0$ and $k = 1$. $(\mathrm{a}, \mathrm{a}')$ $\gamma_2 = 2$, $(\mathrm{b}, \mathrm{b}')$ $\gamma_2 = 1.1$, $(\mathrm{c}, \mathrm{c}')$ $\gamma_2 = 1.01$, $(\mathrm{d}, \mathrm{d}')$ $\gamma_2 = 1.001$.

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