Adiabatic geometric phase for a Bose-Einstein condensate coupled to a cavity

Sheng-Chang Li,[1,2] Li-Bin Fu,[1,3] and Jie Liu[1,3,*]

[1]*National Key Laboratory of Science and Technology on Computation Physics, Institute of Applied Physics and Computational Mathematics, Beijing 100088, China*

[2]*Graduate School, China Academy of Engineering Physics, Beijing 100088, China*

[3]*Center for Applied Physics and Technology, Peking University, Beijing 100084, China*

(Received 10 August 2011; published 14 November 2011)

We investigate the geometric phase in a model of a Bose-Einstein condensate coupled to an optical cavity in which both the condensate and the cavity are described with coherent states. When the argument of the atom-cavity coupling term varies in time slowly from zero to 2π, we calculate the geometric phase accumulated by the ground state and obtain its analytic expression in explicit form. We find that the adiabatic geometric phase jumps from zero to nontrivial π at a critical value that corresponds to the normal-superradiant phase-transition point. The magneticlike flux interpretation of the geometric phase is also discussed.

DOI: 10.1103/PhysRevA.84.053610 PACS number(s): 67.85.Hj, 03.65.Vf, 05.30.Rt

I. INTRODUCTION

The Berry phase [1], which reveals the gauge structure associated with a phase shift in quantum adiabatic processes, has received renewed interest recently due to its important applications in condensed-matter physics [2,3] and in the implementation of quantum computing gates [4–8]. Most of the previous works, however, focused mainly on single quantum systems, e.g., single spin systems with parameters such as magnetic fields varying in time slowly [9]. Recently, geometric phase studies have been extended to composite quantum systems, and some interesting results have been obtained. For instance, in the composite quantum system of a single spin interacting with a quantized field, the vacuum-field-induced Berry phase was found [10].

In contrast to the above simple composite system, a Bose-Einstein condensate (BEC) in an optical cavity is a complicated composite system of greater interest. In such a system, the interaction between bosonic particles is important and can be adjusted precisely through the Feshbach resonance technique [11]. In the absence of interaction, the system reduces to the Dicke model [12], in which the critical property [13] and scaling behavior [14] of the geometric phase have been analyzed; in the presence of particle interaction, however, the system becomes more complicated and calls for further investigations. By applying the coherent-state description to both the BEC and the cavity field, the system is cast into a nonlinear Schrödinger equation where the nonlinearity arises both from the atom-atom interaction and from the atom-cavity coupling. On the other hand, studies on the geometric properties of nonlinear systems have just begun [15–18], and the adiabatic geometric phase in the nonlinear coupled BEC-cavity system is of interest both in theory and in practice because of the experimental progress in loading ultracold atoms in tiny volume of a high-finesse microcavity [19–21].

In the present paper, we extend the geometric phase issue to the nonlinear system of a BEC coupled to an optical cavity. We first derive an effective Hamiltonian for the system in the bad-cavity limit, and then explore the ground-state properties in the coherent-state description. The ground state exhibits a normal-superradiant quantum phase transition (QPT) when the atom-cavity coupling strength exceeds a critical value. We then calculate the adiabatic geometric phase and obtain its expression in explicit form. In particular, we find that the geometric phase has a jump at the critical point. Our above results for the geometric phase including the nonlinear corrections are evidently different from those in the Dicke model [13,14]. Finally, we interpret the geometric nature of the quantum phase with the help of the virtual magnetic fields.

The paper is organized as follows. In Sec. II, we deduce an effective Hamiltonian from the second-quantized Hamiltonian. In Sec. III, we study the properties of the ground state and the phase transition in the coherent-state description. In Sec. IV, we calculate the ground-state geometric phase and the corresponding magneticlike field. Section V presents our discussion and conclusion.

II. MODEL

We consider a system of a two-level atomic BEC (e.g., a BEC with two different hyperfine levels $|F=2,m_f=1\rangle$ and $|F=1,m_f=-1\rangle$ of ^{87}Rb [22]) interacting with a single quantized cavity mode [23,24]. In the two-mode approximation (i.e., two stationary spatial wave functions have been assumed for the atoms), each atomic mode is associated with an annihilation operator $\hat{c}_j$ $(j=1,2)$. For convenience, we use the Schwinger angular momentum operators: $\hat{J}_x=(\hat{c}_1^\dagger\hat{c}_2+\hat{c}_2^\dagger\hat{c}_1)/2$, $\hat{J}_y=(\hat{c}_1^\dagger\hat{c}_2-\hat{c}_2^\dagger\hat{c}_1)/2i$, and $\hat{J}_z=(\hat{c}_1^\dagger\hat{c}_1-\hat{c}_2^\dagger\hat{c}_2)/2$. In terms of these operators, the Hamiltonian of the system takes the form ($\hbar=1$ throughout the paper)

$$\hat{H}=\omega\hat{a}^\dagger\hat{a}+\omega_a\hat{J}_z+\frac{\eta}{N}\hat{J}_z^2+\frac{\chi}{\sqrt{N}}(\hat{a}^\dagger+\hat{a})\hat{J}_x, \qquad (1)$$

where $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator for the cavity field with frequency ω, and N represents the total atom number. The parameters ω_a, η, and χ, respectively, denote the energy difference between two atomic modes, the effective atomic interaction strength, and the atom-cavity coupling strength (these parameters are related to the integrals of the equilibrium spatial wave functions [25]). It is noted that in

*liu_jie@iapcm.ac.cn

the absence of atom-atom interaction (i.e., $\eta = 0$), model (1) reduces to the Tavis-Cummings model [26], also known as the Dicke model [12].

In order to study the adiabatic geometric phase, we introduce a unitary transformation $\hat{U}(\phi) = \exp(-i\phi\hat{J}_z)$ to change Hamiltonian (1) to a time-dependent form,

$$\hat{H}(\phi) = \omega\hat{a}^\dagger\hat{a} + \omega_a\hat{J}_z + \frac{\eta}{N}\hat{J}_z^2 + \frac{\chi}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})[\cos(\phi)\hat{J}_x - \sin(\phi)\hat{J}_y], \quad (2)$$

where the argument $\phi = \phi(t)$ is a slowly varying function of time. In practical experiment, the ϕ-dependent coupling term can be achieved by using a Λ-type three-level configuration in which an external microwave field is applied to drive the transition between two internal atomic states [27]. Here, we are interested in the bad-cavity limit [28], where the problem can be simplified considerably and solved analytically. In this limit, we can treat the leakage of cavity photons by introducing a decay rate κ with typical values $\sim$1 MHz, while the time scale for the atomic transition dynamics is much longer (e.g., the measured population oscillation frequency is below 10 Hz for ^{87}Rb [29]). The separation of time scales allows us to assume that the cavity field always follows the atomic dynamics adiabatically. Under this condition, we can eliminate the cavity mode in model (1) and obtain a Hamiltonian for the atomic degrees of freedom alone. From $i\frac{d\hat{a}}{dt} = [\hat{a},\hat{H}] - i\kappa\hat{a} = 0$, we have

$$\hat{a} = -\frac{\chi(\omega + i\kappa)}{(\omega^2 + \kappa^2)\sqrt{N}}[\cos(\phi)\hat{J}_x - \sin(\phi)\hat{J}_y]. \quad (3)$$

Combining Eqs. (2) and (3), we derive an effective Hamiltonian $\hat{H}'$ as follows:

$$\hat{H}' = \omega_a\hat{J}_z + \frac{\eta}{N}\hat{J}_z^2 - \frac{\Omega}{N}[\cos(\phi)\hat{J}_x - \sin(\phi)\hat{J}_y]^2, \quad (4)$$

where $\Omega = \frac{\chi^2\omega}{(\omega^2+\kappa^2)}$ describes the effective coupling between the BEC atoms and the cavity field.

III. COHERENT-STATE DESCRIPTION AND GROUND STATE

In the following study, we will adopt the coherent-state treatment by replacing the operator $\hat{c}_j$ with the corresponding complex number $\sqrt{N}c_j$ ($j = 1,2$). Then the evolution of the system is governed by a nonlinear Schrödinger equation,

$$i\frac{d|\psi\rangle}{dt} = H|\psi\rangle, \quad (5)$$

with the Hamiltonian

$$H = \begin{pmatrix} \frac{\omega_a}{2} + \frac{\eta}{2}(|c_1|^2 - |c_2|^2) & -\frac{\Omega}{2}(c_1^* c_2 e^{i\phi} + c_2^* c_1) \\ -\frac{\Omega}{2}(c_1^* c_2 + c_2^* c_1 e^{-i\phi}) & -\frac{\omega_a}{2} - \frac{\eta}{2}(|c_1|^2 - |c_2|^2) \end{pmatrix}, \quad (6)$$

where $|\psi\rangle = (c_1,c_2)^T$ represents the state vector satisfying the normalized condition $\langle\psi|\psi\rangle = 1$. Taking advantage of this condition, we write the variables c_1 and c_2 as $c_1 = \sqrt{p}e^{i(\lambda+q)}$ and $c_2 = \sqrt{1-p}e^{i\lambda}$, where p, λ, and q, respectively, denote the population probability of the atoms in the upper mode, the overall phase, and the relative phase between two atomic modes. Then the state vector can be expressed as $|\psi\rangle = e^{i\lambda}(\sqrt{p}e^{iq}, \sqrt{1-p})^T$. Inserting this expression into Eq. (5) and its complex conjugate, we have

$$\frac{d\lambda}{dt} = \frac{\omega_a}{2} + \frac{\eta}{2}(2p-1) + \Omega p\cos^2(\phi - q), \quad (7)$$

$$\frac{dp}{dt} = \Omega(1-p)p\sin[2(q-\phi)], \quad (8)$$

$$\frac{dq}{dt} = -\omega_a + \eta(1-2p) + \Omega(1-2p)\cos^2(q-\phi). \quad (9)$$

For fixed interatomic interaction η, Eqs. (8) and (9) are used to establish a connection between the projective Hilbert space spanned by the vector $\mathbf{S} = (p,q)$ and the parameter space spanned by the vector $\mathbf{R} = (\omega_a,\Omega,\phi)$. Note that the variables p and q form a pair of canonical variables, and the corresponding classical Hamiltonian is given by

$$\mathcal{H} = -\frac{\omega_a}{2}(1-2p) + \frac{\eta}{4}(1-2p)^2 - \Omega(1-p)p\cos^2(\phi - q). \quad (10)$$

Here we denote the fixed points in the projective Hilbert space by $(\bar{p},\bar{q})$, which can be determined by the equilibrium equations $\frac{\partial\mathcal{H}}{\partial p}|_{(\bar{p},\bar{q})} = 0$ and $\frac{\partial\mathcal{H}}{\partial q}|_{(\bar{p},\bar{q})} = 0$. Obviously, the classical system (10) has two phase-independent fixed points (to be precise, they are two lines in the projective Hilbert space), namely $\bar{p} = 0$ and 1. The rest of the fixed points are

$$(\bar{p},\bar{q}) = \begin{cases} \left(\frac{\eta-\omega_a}{2\eta},\ \phi + \frac{\pi}{2} \text{ or } \phi + \frac{3\pi}{2}\right), \\ \left(\frac{\eta+\Omega-\omega_a}{2(\eta+\Omega)},\ \phi \text{ or } \phi + \pi\right). \end{cases} \quad (11)$$

The number of fixed points depends on the parameters η and Ω (see Fig. 1). Note that the fixed points characterize the stationary points of the classical Hamiltonian (10), and thus they correspond to the eigenstates of the nonlinear system (5) [30]. Indeed, we can express the eigenvalue or chemical potential corresponding to the eigenstate

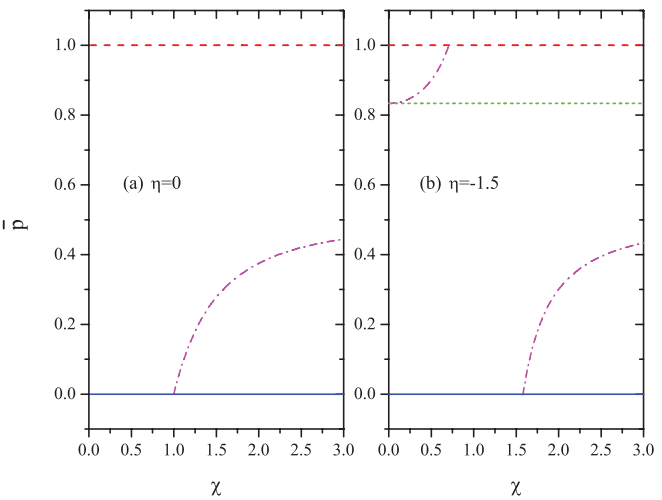

FIG. 1. (Color online) Fixed points of the classical Hamiltonian $\mathcal{H}$ with $\omega_a = 1$ and $\omega/(\omega^2 + \kappa^2) = 1$. (a) $\eta = 0$ and (b) $\eta = -1.5$.

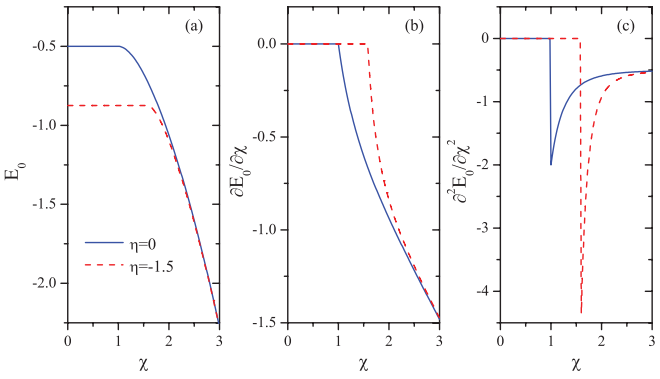

FIG. 2. (Color online) The energy of the ground state and its first and second derivatives with respect to χ. The parameters are $\omega_a = 1$ and $\omega/(\omega^2+\kappa^2) = 1$.

$|\bar{\psi}\rangle = (\sqrt{\bar{p}}e^{i\bar{q}}, \sqrt{1-\bar{p}})^T$ as

$$\mu = \langle\bar{\psi}|H|\bar{\psi}\rangle = \frac{\omega_a}{2}(2\bar{p}-1) + \frac{\eta}{2}(1-2\bar{p})^2 - 2\Omega(1-\bar{p})\bar{p}\cos^2(\phi-\bar{q}). \tag{12}$$

Substituting the concrete expression for the fixed points into Eq. (12), we obtain the corresponding eigenvalues in explicit form,

$$\mu = \begin{cases} \frac{\eta-\omega_a}{2} & \text{for} \quad \bar{p}=0, \\ \frac{\eta+\omega_a}{2} & \text{for} \quad \bar{p}=1, \\ 0 & \text{for} \quad \bar{p}=\frac{\eta-\omega_a}{2\eta}, \\ -\frac{\Omega}{2} & \text{for} \quad \bar{p}=\frac{\eta+\Omega-\omega_a}{2(\eta+\Omega)}. \end{cases} \tag{13}$$

For our nonlinear system (5), the chemical potential is not equal to the energy, and the relation between them is determined by $\mu = \mathcal{H}(\bar{p},\bar{q}) + \frac{\eta}{4}(1-2\bar{p})^2 - \Omega(1-\bar{p})\bar{p}\cos^2(\phi-\bar{q})$. The ground-state energy E_0 and its first and second derivatives with respect to χ have been demonstrated in Fig. 2. We see that the second derivative of the ground-state energy [see Fig. 2(c)] possesses a discontinuity at a critical point that corresponds to the point of intersection of the curves for μ. The critical point is $\Omega_c = \omega_a - \eta$ or $\chi_c = \sqrt{(\kappa^2+\omega^2)(\omega_a-\eta)/\omega}$ with $\omega_a \geqslant \eta$. The divergence of the derivative $\partial^2 E_0/\partial\chi^2$ implies that the system exhibits a second-order phase transition at the critical point in the thermodynamic limit, where the ground state changes from a normal phase (i.e., $\bar{p}=0$) to a superradiant phase where the atom occupation in the upper mode acquires macroscopic values (i.e., $\bar{p}>0$).

It is easy to find that, in the absence of cavity decay and atomic interactions (i.e., $\kappa=\eta=0$), the critical point reduces to $\chi_c = \sqrt{\omega\omega_a}$, which is the critical point found in the Dicke model [31].

In order to see the scaling behavior of the ground state in the vicinity of the critical point, we rewrite the ground-state solution in the superradiant phase as follows:

$$\bar{p} = \frac{(\chi^2-\chi_c^2)\omega}{2[\chi^2\omega+\eta(\kappa^2+\omega^2)]}. \tag{14}$$

We find that, above the critical value (i.e., $\chi > \chi_c$), the atomic population in the upper mode vanishes as $\chi-\chi_c$ approaches the critical point, and the corresponding critical exponent is thus 1. When $\eta=0$, the scaling behavior of the ground state is the same as that in the Dicke model [32].

IV. GEOMETRIC PHASE AND VIRTUAL MAGNETIC FIELD

In this section, we will calculate the geometric phase for the ground state of the system. To this end, for simplicity, we first treat the parameters ω_a and Ω as constants, and then a closed loop C in the parameter space can be formed by changing the parameter ϕ with time from 0 to 2π. We assume that the Hamiltonian $H(\phi)$ travels along the cyclic path adiabatically. The dimensionless adiabatic parameter can be defined as $\epsilon \sim |\frac{d\phi}{dt}| \sim \frac{1}{T}$ (where T is the period of the cyclic evolution), which is small enough here, i.e., $\epsilon \ll 1$. Initially, the system is prepared in an eigenstate of $H(\phi)$, then undergoes a cyclic adiabatic evolution along the path C, and finally, the eigenstate will acquire a geometric phase during the adiabatic process [33] besides the usual dynamical phase (relative to the instantaneous energy of the system). In order to obtain the pure geometric part, we will employ the procedure introduced in Refs. [16,17] to separate the geometric phase from the total phase. Since the adiabatic parameter ϵ is small but finite, the system will fluctuate around the eigenstate during the adiabatic evolution. This allows us to expand the total phase λ in a perturbation series in the adiabatic parameter, that is,

$$\frac{d\lambda}{dt} = \lambda_0(\epsilon^0) + \lambda_1(\epsilon^1) + O(\epsilon^2). \tag{15}$$

In the adiabatic limit (i.e., $\epsilon \to 0$), a trip along the cyclic path in the parameter space will take an infinitely long time (i.e., $T \to \infty$). The time integral of the zero-order term in Eq. (15) gives the dynamic phase, while the time integral of the first-order term gives the geometric phase, and the contribution from the higher-order terms will vanish.

In the subsequent study, we will restrict our discussion to the geometric phase for the ground state. We note that, in the normal phase (i.e., $\chi < \chi_c$ or $\Omega < \Omega_c$), the ground state (i.e., $\bar{p}=0$) is independent of the system parameters. From Eq. (7), we have

$$\frac{d\lambda}{dt} = \frac{1}{2}(\omega_a - \eta) = -\mu \tag{16}$$

and then

$$\lambda = \int_0^T \lambda_0 dt = -\int_0^T \mu\, dt. \tag{17}$$

This implies that the geometric phase vanishes.

In the superradiant phase (i.e., $\chi > \chi_c$ or $\Omega > \Omega_c$), the ground state depends on the system parameters, and we can expand the variables as $p = \bar{p}(\mathbf{R}) + \delta p(\mathbf{R})$ and $q = \bar{q}(\mathbf{R}) + \delta q(\mathbf{R})$, where $(\bar{p},\bar{q})$ is the instantaneous ground state corresponding to the global minimum energy. δp and δq denote the fluctuations of the ground state induced by the slow variation of the system, which depend on the adiabatic parameter and are of order of ϵ. Substituting the above expressions back into Eq. (7) and using Eq. (12), we have

$$\lambda_0 = -\mu(\mathbf{R}), \tag{18}$$

$$\lambda_1 = (\eta+\Omega)\delta p. \tag{19}$$

Moreover, from Eqs. (9) and (11), we have

$$\frac{d\bar{q}}{dt} = -2(\eta + \Omega)\delta p + O(\epsilon^2). \tag{20}$$

To deduce Eqs. (19) and (20), the equilibrium condition $\frac{\partial \mathcal{H}}{\partial p}|_{(\bar{p},\bar{q})} = 0$ and the relation $\frac{d}{dt}\delta q \sim O(\epsilon^2)$ have been used. Substituting Eq. (20) back into Eq. (19) and considering $\bar{q} = \phi$ (or $\phi + \pi$) for the ground state, we obtain

$$\lambda_1 = -\frac{1}{2}\frac{d\phi}{dt}. \tag{21}$$

Integrating this equation over the evolution period of the system, we get the ground-state geometric phase

$$\lambda_G = -\int_0^T \lambda_1 dt = \frac{1}{2}\int_0^{2\pi} d\phi = \pi. \tag{22}$$

This result is very interesting because it cannot be obtained directly by using Berry's formula. Indeed, for our nonlinear system, the total geometric phase includes two parts: the first part is the traditional Berry phase $\lambda_{\rm BP}$, which is a circuit integral of the Berry connection, i.e., $\lambda_{\rm BP} = -i\int_0^{2\pi}\langle\bar{\psi}(\mathbf{R})|\nabla_{\mathbf{R}}|\bar{\psi}(\mathbf{R})\rangle = \pi\frac{\eta+\Omega-\omega_a}{\eta+\Omega}$; the second part is a nonlinear correction term, which is given by $\lambda_{\rm NL} = \lambda_G - \lambda_{\rm BP} = \pi\frac{\omega_a}{\eta+\Omega}$. Our above theoretical predictions have been verified by numerically integrating Eqs. (7)–(9) along the circle path in the parameter space with ω_a and Ω fixed, and ϕ varying with a very small rate. Plotted in Fig. 3 are our numerical results showing a good agreement with the theoretical predictions. The inset shows the convergence of the adiabatic geometric phase with the time duration T.

It is useful to evaluate the difference between our results and the previous work in the Dicke model [14]. When the particle interaction is neglected (i.e., $\eta = 0$), our model reduces to the Dicke model. In this case, our result for the usual Berry phase $\lambda_{\rm BP} = \pi(1 - \omega_a/\Omega)$ is consistent with that in Ref. [14] in the thermodynamic limit. However, a novel correction term

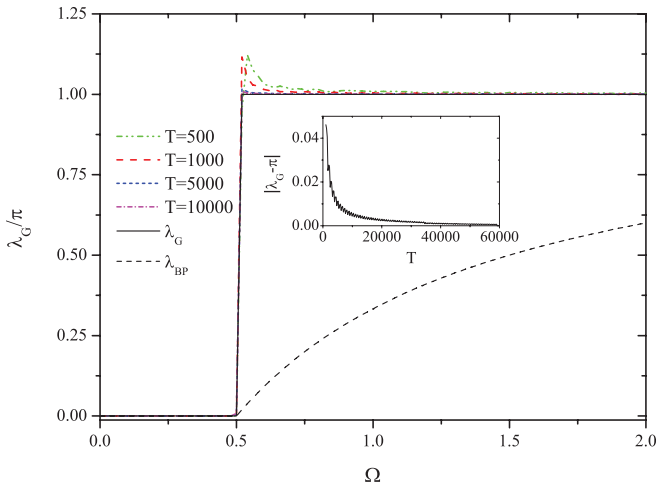

FIG. 3. (Color online) Numerical results and comparison with theoretical prediction for the ground-state geometric phase. The inset shows the convergence of the geometric phase with the evolution period with $\Omega = 0.6$. The parameters are $\omega_a = 1$ and $\eta = 0.5$.

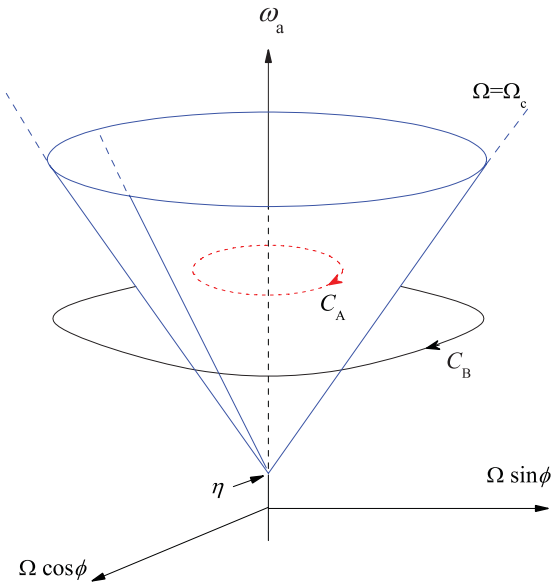

FIG. 4. (Color online) A qualitative illustration of the paths followed by the parameter. The circular conical surface corresponds to the critical region $\Omega = \Omega_c$. If the parameter follows the path C_A, the ground state will acquire a trivial zero Berry phase; while it follows the path C_B (encircling the conical surface), a nontrivial geometric phase (i.e., $\lambda_G = \pi$) is acquired.

$\lambda_{\rm NL} = \pi\omega_a/\Omega$ is still found in our calculation due to the nonlinear atom-cavity coupling.

Our above findings indicate that, in the linear system, the high-order correction to the adiabatic approximate solution can often be neglected. However, in the nonlinear system, since the Hamiltonian is a function of the instantaneous states, and the fluctuations induced by the slow change of the system can be fed back to the Hamiltonian, the high-order correction could be accumulated during the adiabatic evolution with an infinite time duration and contributes a finite value to the geometric phase in the adiabatic limit.

It is worth emphasizing that the different geometric phase behaviors of the normal and the superradiant phases are due to the level-crossing nature of the transition between them. From Fig. 4, we see that the geometric phase would vanish as long as the loop C_B is pushed down into the $\Omega < \omega_a - \eta$ (normal phase) region. In practice, we are computing the geometric phase of two different states, which happen to be the ground state in different regions of the phase diagram. Thus the above "counter-topological" feature of the geometric phase for the ground state is associated with the level-crossing structure.

To better understand our above geometric phases, we calculate the corresponding virtual magnetic field in the parameter space. For convenience, we represent the three parameters ω_a, Ω, and ϕ as cylindrical coordinates of a derived three-parameter space spanned by the vector $\mathbf{R} = (\omega_a, \Omega, \phi) = (z, \rho, \phi)$. We first define the vector potentials $\mathbf{A}_{\rm BP}$ and $\mathbf{A}_{\rm NL}$ by the relations

$$\lambda_{\rm BP} = \oint \mathbf{A}_{BP}\cdot d\mathbf{R}, \quad \lambda_{\rm NL} = \oint \mathbf{A}_{\rm NL}\cdot d\mathbf{R}, \tag{23}$$

and then we have

$$\mathbf{A}_{\mathrm{BP}} = \frac{\eta + \Omega - \omega_a}{2\Omega(\eta + \Omega)}\mathbf{e}_\phi, \tag{24}$$

$$\mathbf{A}_{\mathrm{NL}} = \frac{\omega_a}{2\Omega(\eta + \Omega)}\mathbf{e}_\phi. \tag{25}$$

The corresponding virtual magnetic fields $\mathbf{B}_{\mathrm{BP}}$ and $\mathbf{B}_{\mathrm{NL}}$ can be defined by $\mathbf{B}_{\mathrm{BP}} = \nabla \times \mathbf{A}_{\mathrm{BP}}$ and $\mathbf{B}_{\mathrm{NL}} = \nabla \times \mathbf{A}_{\mathrm{NL}}$. A straightforward calculation gives

$$\mathbf{B}_{\mathrm{BP}} = \frac{1}{2\Omega(\eta + \Omega)}\mathbf{e}_\rho + \left[\frac{\omega_a}{2\Omega(\eta + \Omega)^2} + \pi\delta(\Omega)\right]\mathbf{e}_z, \tag{26}$$

$$\mathbf{B}_{\mathrm{NL}} = -\frac{1}{2\Omega(\eta + \Omega)}\mathbf{e}_\rho - \frac{\omega_a}{2\Omega(\eta + \Omega)^2}\mathbf{e}_z, \tag{27}$$

and the total magneticlike field reads

$$\mathbf{B}_T = \mathbf{B}_{\mathrm{BP}} + \mathbf{B}_{\mathrm{NL}} = \pi\delta(\Omega)\mathbf{e}_z. \tag{28}$$

This result implies that, in the superradiant phase, the virtual magnetic field can be viewed as one field line along the positive z axis and the field only distributes in the critical region, which is denoted by a paraboloid [i.e., $\chi_c = \sqrt{(\kappa^2 + \omega^2)(\omega_a - \eta)/\omega}$] or a circular conical surface (i.e., $\Omega_c = \omega_a - \eta$) in the parameter space (see Fig. 4). Therefore, the flux of the virtual magnetic field through the surface enclosed by the closed path (i.e., the loop C_B in Fig. 4) is π, which gives the physical interpretation of our total geometric phase. In the normal phase, there does not exist the virtual magnetic field because the geometric phase is zero there. It is worth mentioning that, in the superradiant phase, as long as the loop encircles the critical region, the total geometric phase is independent both of the shape and of the size of the loop in the parameter space.

V. CONCLUSION AND DISCUSSION

In this work, we have extended the nonlinear geometric phase issue to the system of a BEC in an optical cavity. First, we have derived the adiabatic geometric phase for the ground state analytically. We find that it equals zero when the BEC-cavity coupling strength is less than the critical value, and it becomes π when the coupling strength exceeds this value. The discontinuous behavior of the ground-state geometric phase is found to be a signal of the normal-superradiant phase transition in the system. Second, we have calculated the virtual magnetic field corresponding to the above geometric phase in explicit form. An effective δ-function-type magneticlike field in the superradiant phase is shown while no virtual magnetic field is found in the normal phase. We believe that our present work will stimulate interest in carrying out further investigations on the geometric issue of the model at finite temperature to account for coupling to the environment, dissipation, decoherence, etc., possibly adopting a master-equation approach [34,35].

ACKNOWLEDGMENTS

This work is supported by the National Fundamental Research Program of China (Contract No. 2011CB921503) and the National Natural Science Foundation of China (Contracts No. 10725521, No. 91021021, No. 11075020, and No. 11078001).

[1] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).
[2] A. C. M. Carollo and J. K. Pachos, Phys. Rev. Lett. **95**, 157203 (2005).
[3] S.-L. Zhu, Phys. Rev. Lett. **96**, 077206 (2006).
[4] *Geometric Phases in Physics*, edited by A. Shapere and F. Wilczek (World Scientific, Singapore, 1989).
[5] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999).
[6] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Nature (London) **403**, 869 (2000).
[7] G. Falci, R. Fazio, G. M. Palma, J. Siewert, and V. Vedral, Nature (London) **407**, 355 (2000).
[8] A. Joshi and M. Xiao, Phys. Lett. A **359**, 390 (2006).
[9] G. De Chiara and G. M. Palma, Phys. Rev. Lett. **91**, 090404 (2003).
[10] I. Fuentes-Guridi, A. Carollo, S. Bose, and V. Vedral, Phys. Rev. Lett. **89**, 220404 (2002).
[11] S. Inouye *et al.*, Nature (London) **392**, 151 (1998); Ph. Courteille, R. S. Freeland, D. J. Heinzen, F. A. van Abeelen, and B. J. Verhaar, Phys. Rev. Lett. **81**, 69 (1998).
[12] R. H. Dicke, Phys. Rev. **93**, 99 (1954); L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
[13] G. Chen, J. Li, and J.-Q. Liang, Phys. Rev. A **74**, 054101 (2006).
[14] F. Plastina, G. Liberti, and A. Carollo, Europhys. Lett. **76**, 182 (2006).
[15] F. N. Litvinets, A. V. Shapovalov, and A. Yu Trifonov, J. Phys. A **39**, 1191 (2006).
[16] L. B. Fu and J. Liu, Ann. Phys. (NY) **325**, 2425 (2010).
[17] J. Liu and L. B. Fu, Phys. Rev. A **81**, 052112 (2010).
[18] S. C. Li, J. Liu, and L. B. Fu, Phys. Rev. A **83**, 042107 (2011).
[19] F. Brennecke, T. Donner, S. Ritter, T. Bourdel, M. Köhl, and T. Esslinger, Nature (London) **450**, 268 (2007).
[20] S. Slama, S. Bux, G. Krenz, C. Zimmermann, and P. W. Courteille, Phys. Rev. Lett. **98**, 053603 (2007).
[21] Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, Nature (London) **450**, 272 (2007).
[22] M. R. Matthews, D. S. Hall, D. S. Jin, J. R. Ensher, C. E. Wieman, E. A. Cornell, F. Dalfovo, C. Minniti, and S. Stringari, Phys. Rev. Lett. **81**, 243 (1998).
[23] M. G. Moore, O. Zobay, and P. Meystre, Phys. Rev. A **60**, 1491 (1999).
[24] C. K. Law and N. P. Bigelow, Phys. Rev. A **58**, 4791 (1998).
[25] G. Chen, X. Wang, J. Q. Liang, and Z. D. Wang, Phys. Rev. A **78**, 023634 (2008).
[26] M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968).
[27] Y. Li, P. Zhang, and Z. D. Wang, Eur. Phys. J. D **58**, 379 (2010).
[28] J. I. Cirac, Phys. Rev. A **46**, 4354 (1992); J. I. Cirac, M. Lewenstein, and P. Zoller, *ibid.* **51**, 1650 (1995).
[29] M.-S. Chang *et al.*, Nat. Phys. **1**, 111 (2005).

[30] J. Liu, L. Fu, B. Y. Ou, S. G. Chen, D. I. Choi, B. Wu, and Q. Niu, Phys. Rev. A **66**, 023404 (2002).

[31] K. Hepp and E. H. Lieb, Ann. Phys. (NY) **76**, 360 (1973); Y. K. Wang and F. T. Hioe, Phys. Rev. A **7**, 831 (1973); K. Hepp and E. H. Lieb, Ann. Phys. (NY) **8**, 2517 (1973); G. Cromer Duncan, *ibid*. **9**, 418 (1974).

[32] D. Nagy, G. Kónya, G. Szirmai, and P. Domokos, Phys. Rev. Lett. **104**, 130401 (2010).

[33] J. Liu, B. Wu, and Q. Niu, Phys. Rev. Lett. **90**, 170404 (2003); H. Pu, P. Maenner, W. Zhang, and H. Y. Ling, *ibid*. **98**, 050406 (2007).

[34] U. Weiss, *Quantum Dissipative Systems*, 2nd ed. (World Scientific, Singapore, 1999).

[35] F. Pellegrini, C. Negri, F. Pistolesi, N. Manini, G. E. Santoro, and E. Tosatti, Phys. Rev. Lett. **107**, 060401 (2011).