AIP Journal of Mathematical Physics

Topological tensor current of $\tilde{p}$ -branes in the φ-mapping theory

Yishi Duan, Libin Fu, and Guang Jia

Citation: Journal of Mathematical Physics **41**, 4379 (2000); doi: 10.1063/1.533347
View online: http://dx.doi.org/10.1063/1.533347
View Table of Contents: http://scitation.aip.org/content/aip/journal/jmp/41/7?ver=pdfcov
Published by the AIP Publishing

Topological tensor current of $\tilde{p}$-branes in the ϕ-mapping theory

Yishi Duan
Institute of Theoretical Physics, Lanzhou University, Lanzhou, 730000 Gansu, People's Republic of China

Libin Fu[a)]
LCP, Institute of Applied Physics and Computational Mathematics, P.O. Box 8009 (26), 100088 Beijing, People's Republic of China

Guang Jia
Institute of Theoretical Physics, Lanzhou University, Lanzhou, 730000 Gansu, People's Republic of China

(Received 27 April 1999; accepted for publication 23 February 2000)

We present a new topological tensor current of $\tilde{p}$-branes by making use of the ϕ-mapping theory. It is shown that the current is identically conserved and behaves as $\delta(\vec{\phi})$, and every isolated zero of the vector field $\vec{\phi}(x)$ corresponds to a "magnetic" $\tilde{p}$-brane. Using this topological current, the generalized Nambu action for multi $\tilde{p}$-branes is given, and the field strength F corresponding to this topological tensor current is obtained. It is also shown that the magnetic charges carried by $\tilde{p}$-branes are topologically quantized and labeled by Hopf index and Brouwer degree, the winding number of the ϕ mapping. © *2000 American Institute of Physics.* [S0022-2488(00)01707-2]

I. INTRODUCTION

Extended objects with p spatial dimensional, known as "branes," play an essential role in revealing the nonperturbative structure of the superstring theories and M-theories.[1–4] Antisymmetric tensor gauge fields have been widely studied in the theories of p-branes.[5–8] In the context of the effective $D=10$ or $D=11$ supergravity theory a p-brane is a p-dimensional extended source for a $(p+2)$-form gauge field strength F. It is well-known that the $(p+2)$-form strength F satisfies the field equation

$$\nabla_\mu F^{\mu\mu_1\cdots\mu_{p+1}}=j^{\mu_1\cdots\mu_{p+1}},$$

where $j^{\mu_1\cdots\mu_{p+1}}$ is a $(p+1)$-form tensor current, corresponding to the electric source, and the dual field strength $^*F$ satisfies

$$\nabla_\mu {}^*F^{\mu\mu_1\cdots\mu_{\tilde{p}+1}}=\tilde{j}^{\mu_1\cdots\mu_{\tilde{p}+1}}$$

in which $\tilde{j}^{\mu_1\cdots\mu_{\tilde{p}+1}}$ is a $(\tilde{p}+1)$-form tensor current, corresponding to the magnetic source.[9–11]

The ϕ-mapping theory proposed by Professor Duan[12,13] is important in studying the topological invariant and topological structure of physics systems and has been used to study the topological current of magnetic monopole,[12] topological string theory,[13] topological structure of Gauss–Bonnet–Chern theorem,[14] topological structure of the SU(2) Chern density,[15] and topological structure of the London equation in superconductor.[16] We must point out that the ϕ-mapping theory is also a powerful tool to investigate the topological defects theory,[17–19] and here the vector field $\vec{\phi}$ is looked upon as the order parameters of the defects.

[a)]Author to whom all correspondence should be addressed; electronic mail: lbfu@263.net

In this paper, we present a new topological tensor current of ''magnetic'' $\tilde{p}$-branes by making use of the ϕ-mapping theory. One shows that each isolated zero of the d-dimensional vector field $\vec{\phi}(x)$ corresponds to a $\tilde{p}$-brane ($\tilde{p}=D-d-1$), and this current is proved to be the general current density of multi-$\tilde{p}$-branes. Using this current, the generalized Nambu action for multi-$\tilde{p}$-branes is obtained. This topological tensor current will give rise to the inner structure of the field strength F including the contribution of the ''magnetic'' $\tilde{p}$-branes. Finally, we show that the charges carried by multi-$\tilde{p}$-branes are topologically quantized and labeled by the Hopf index and Brouwer degree, the winding number of the ϕ mapping.

II. THE TOPOLOGICAL TENSOR CURRENT OF $\tilde{p}$-BRANES

Let X be a D-dimensional smooth manifold with metric tensor $g_{\mu\nu}$ and local coordinates $x^{\mu}(\mu,\nu=0,\ldots,D-1)$ with $x^0=t$ as time, and let R^d be an Euclidean space of dimension $d<D$. We consider a smooth map $\phi:X\rightarrow R^d$, which gives a d-dimensional smooth vector field on X,

$$\phi^a=\phi^a(x), \quad a=1,2,\ldots,d. \tag{1}$$

The direction unit field of $\vec{\phi}(x)$ can be expressed as

$$n^a=\frac{\phi^a}{\|\phi\|}, \quad \|\phi\|=\sqrt{\phi^a\phi^a}. \tag{2}$$

In the ϕ-mapping theory, to extend the theory of magnetic monopoles[12] and the topological string theory,[13] we present a new topological tensor current, with the unit ''magnetic'' charge g_m, defined as

$$\tilde{j}^{\mu_1\cdots\mu_{D-d}}=\frac{g_m}{A(S^{d-1})(d-1)!}\left(\frac{1}{\sqrt{g}}\right)\epsilon^{\mu_1\cdots\mu_{D-d}\mu_{D-d+1}\mu_{D-d+2}\cdots\mu_D} \times\epsilon_{a_1a_2\cdots a_d}\partial_{\mu_{(D-d+1)}}n^{a_1}\partial_{\mu_{(D-d-2)}}n^{a_2}\cdots\partial_{\mu_D}n^{a_d}, \tag{3}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and $A(S^{d-1})$ is the area of $(d-1)$-dimensional unit sphere S^{d-1}. Obviously, this ''magnetic'' tensor current is identically conserved,

$$\nabla_{\mu_1}\tilde{j}^{\mu_1\cdots\mu_{D-d}}=0, \quad i=1,\ldots,D-d. \tag{4}$$

From (2) we have

$$\partial_\mu n^a=\frac{1}{\|\phi\|}\partial_\mu\phi^a+\phi^a\partial_\mu\left(\frac{1}{\|\phi\|}\right), \tag{5}$$

$$\frac{\partial}{\partial\phi^a}\left(\frac{1}{\|\phi\|}\right)=-\frac{\phi^a}{\|\phi\|^3}. \tag{6}$$

Using the above expressions, the general tensor current can be rewritten as

$$\tilde{j}^{\mu_1\cdots\mu_{D-d}}=g_mC_d\left(\frac{1}{\sqrt{g}}\right)\epsilon^{\mu_1\cdots\mu_{D-d}\mu_{D-d+1}\cdots\mu_D}\epsilon_{a_1\cdots a_d} \partial_{\mu_{(D-d+1)}}\phi^{a_1}\partial_{\mu_{(D-d-2)}}\phi^{a_2}\cdots\partial_{\mu_D}\phi^{a_d}\sum_a\frac{\partial}{\partial\phi^a}\frac{\partial}{\partial\phi^a}(G_d(\|\phi\|)), \tag{7}$$

where C_d is a constant

$$C_d=\begin{cases} -\dfrac{1}{A(S^{d-1})d!(d-2)} & \text{for } d>2 \\ \dfrac{1}{4\pi} & \text{for } d=2, \end{cases}$$

and $G_d(\|\phi\|)$ is a generalized function

$$G_d(\|\phi\|)=\begin{cases} \dfrac{1}{\|\phi\|^{d-2}} & \text{for } d>2 \\ \ln(\|\phi\|) & \text{for } d=2. \end{cases}$$

If we define a generalized Jacobian tensor as

$$\epsilon^{a_1\cdots a_d}J^{\mu_1\cdots\mu_{D-d}}\left(\frac{\phi}{x}\right)=\epsilon^{\mu_1\cdots\mu_{D-d}\mu_{D-d+1}\mu_{D-d+2}\cdots\mu_D}\partial_{\mu_{(D-d+1)}}\phi^{a_1}\partial_{\mu_{(D-d-2)}}\phi^{a_2}\cdots\partial_{\mu_D}\phi^{a_d} \tag{8}$$

and make use of the generalized Laplacian Green function relation in ϕ space

$$\sum_a \frac{\partial}{\partial\phi^a}\frac{\partial}{\partial\phi^a}(G_d(\|\phi\|))=\begin{cases} -\dfrac{4\pi^{d/2}}{\Gamma(d/2-1)}\delta(\vec{\phi}) & \text{for } d>2 \\ 2\pi\delta(\vec{\phi}) & \text{for } d=2, \end{cases} \tag{9}$$

we obtain a δ-function like tensor current[13]

$$\tilde{j}^{\mu_1\cdots\mu_{D-d}}=g_m\delta(\vec{\phi})J^{\mu_1\cdots\mu_{D-d}}\left(\frac{\phi}{x}\right)\left(\frac{1}{\sqrt{g}}\right). \tag{10}$$

We find that $\tilde{j}^{\mu_1\cdots\mu_{D-d}}\neq 0$ only when $\phi=0$. So, it is essential to discuss the solutions of the equations

$$\phi^a(x)=0, \quad a=1,\ldots,d. \tag{11}$$

Suppose that the vector field $\vec{\phi}(x)$ possesses l isolated zeroes, according to the deduction of Ref. 13 and the implicit function theorem,[20,21] when the zeroes are regular points of ϕ-mapping, i.e., the rank of the Jacobian matrix $[\partial_\mu\phi^a]$ is d, the solution of $\vec{\phi}(x)=0$ can be parametrized by

$$x^\mu=z_i^\mu(u^1,u^2,\ldots,u^{D-d}), \quad i=1,\ldots,l, \tag{12}$$

where the subscript i represents the ith solution and the parameters $u=u(u^1,\ldots,u^{D-d})$ span a $(D-d)$-dimensional submanifold of X, denoted by N_i, which corresponds to a $\tilde{p}$-brane ($\tilde{p}=D-d-1$) with spatial $\tilde{p}$-dimension and N_i is its world volume. One sees that the tensor current $\tilde{j}^{\mu_1\cdots\mu_{D-d}}$ is not vanished only on the world volume manifolds N_i ($i=1,\ldots,l$), each of which corresponds to a $\tilde{p}$-brane. Therefore, every isolated zero of $\vec{\phi}(x)$ on X corresponds to magnetic $\tilde{p}$-branes. These ''magnetic'' $\tilde{p}$-branes had been formally discussed and were not studied based on topology theory.[8,22] Here, we must point out that the $\tilde{p}$-branes sometimes may be considered as topological defects,[11,23] in this case for our theory the vector field $\phi^a(x)$ ($a=1,\ldots,d$) may be looked upon as the generalized order parameters[19] for $\tilde{p}$-branes.

In the following, we will discuss the inner structure of the topological tensor current $\tilde{j}^{\mu_1\cdots\mu_{D-d}}$. It can be proved that there exists a d-dimensional submanifold M in X with the parametric equation

$$x^{\mu}=x^{\mu}(v^1,\ldots,v^d), \quad \mu=1,\ldots,D, \tag{13}$$

which is transversal to every N_i at the point p_i with

$$g_{\mu\nu}\frac{\partial x^{\mu}}{\partial u^I}\frac{\partial x^{v}}{\partial v^A}\bigg|_{p_i}=0, \quad I=1,\ldots,D-d, \quad A=1,\ldots,d. \tag{14}$$

This is to say that the equations $\vec{\phi}(x)=0$ have isolated zero points on M.

As we have pointed in Refs. 14 and 15, the unit vector field defined in (2) gives a Gauss map $n:\partial M_i \to S^{d-1}$, and the generalized winding number can be given by this Gauss map

$$\begin{aligned}W_i&=\frac{1}{A(S^{d-1})(d-1)!}\int_{\partial M_i} n^*(\epsilon_{a_1\cdots a_d} n^{a_1} dn^{a_2}\wedge\cdots\wedge dn^{a_d})\\&=\frac{1}{A(S^{d-1})(d-1)!}\int_{\partial M_i}\epsilon_{a_1\cdots a_d} n^{a_1}\partial_{A_2}n^{a_2}\cdots\partial_{A_d}n^{a_d} dv^{A_2}\wedge\cdots\wedge dv^{A_d}\\&=\frac{1}{A(S^{d-1})(d-1)!}\int_{M_i}\epsilon^{A_1\cdots A_d}\epsilon_{a_1\cdots a_d}\partial_{A_1}n^{a_1}\partial_{A_2}n^{a_2}\cdots\partial_{A_d}n^{a_d} d^d v,\end{aligned} \tag{15}$$

where ∂M_i is the boundary of the neighborhood M_i of p_i on M with $p_i \notin \partial M_i$, $M_i \cap M_j=\emptyset$. Then, by duplicating the derivation of (3) from (10), we obtain

$$W_i=\int_{M_i}\delta(\vec{\phi}(v))J\left(\frac{\phi}{v}\right)d^d v, \tag{16}$$

where $J(\phi/v)$ is the usual Jacobian determinant of $\vec{\phi}$ with respect to v,

$$\epsilon^{a_1\cdots a_d}J\left(\frac{\phi}{v}\right)=\epsilon^{A_1\cdots A_d}\partial_{A_1}n^{a_1}\partial_{A_2}n^{a_2}\cdots\partial_{A_d}n^{a_d}. \tag{17}$$

According to the δ-function theory[24] and the ϕ-mapping theory, we know that $\delta(\vec{\phi}(v))$ can be expanded as

$$\delta(\vec{\phi}(v))=\sum_{i=1}^{l}\beta_i\eta_i\delta^d(\vec{v}-\vec{v}(p_i)) \tag{18}$$

on M, where the positive integer $\beta_i=|W_i|$ is called the Hopf index of the map $v\to\vec{\phi}(v)$ and $\eta_i=\mathrm{sgn}(J(\phi/v))|_{p_i}=\pm 1$ is the Brouwer degree.[14,16] One can find the relation between the Hopf index β_i, the Brouwer degree η_i, and the winding number W_i,

$$W_i=\beta_i\eta_i, \tag{19}$$

One sees that Eq. (18) is only the expansion of $\delta(\vec{\phi}(x))$ on M. In order to investigate the expansion of $\delta(\vec{\phi}(x))$ on the whole manifold X, we must expand the d-dimensional δ function of the singular point in terms of the δ function on the singular submanifold N_i which had been given in Ref. 24

$$\delta(N_i)=\int_{N_i}\delta^D(x-z_i(u))\sqrt{g_u}\,d^{(D-d)}u, \quad i=1,\ldots,l$$

in which

$$g_u = \det\left(g_{\mu\nu} \frac{\partial x^\mu}{\partial u^I} \frac{\partial x^\nu}{\partial u^J} \right), \quad I,J = 1,\ldots,(D-d). \tag{20}$$

Then, from Eq. (18), and by considering the property of the δ function, one will obtain

$$\delta(\vec{\phi}(x)) = \sum_{i=1}^{l} \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u. \tag{21}$$

Therefore, the general topological current of the $\tilde{p}$-branes can be expressed directly as

$$\tilde{j}^{\mu_1 \cdots \mu_{D-d}} = \left(\frac{1}{\sqrt{g}} \right) J^{\mu_1 \cdots \mu_{D-d}} \left(\frac{\phi}{x} \right) \sum_{i=1}^{l} \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u, \tag{22}$$

which is a new topological current theory of $\tilde{p}$-branes based on the ϕ-mapping theory.

If we define a Lagrangian as

$$L = \sqrt{\frac{1}{(D-d)!} g_{\mu_1 \nu_1} \cdots g_{\mu_{(D-d)} \nu_{(D-d)}} \tilde{j}^{\mu_1 \cdots \mu_{D-d}} \tilde{j}^{\nu_1 \cdots \nu_{D-d}}}, \tag{23}$$

which is just the generalization of Nielsen's Lagrangian,[25] from the above deductions, we can prove that

$$L = \left(\frac{1}{\sqrt{g}} \right) \delta(\vec{\phi}(x)). \tag{24}$$

Then, the action takes the form

$$S = \int_X L \sqrt{g} d^D x = \int_X \delta(\vec{\phi}(x)) d^D x. \tag{25}$$

By substituting the formula (21) into (25), we obtain an important result,

$$S = \int_X \sum_{i=1}^{l} \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u \, d^D x = \sum_{i=1}^{l} \beta_i \eta_i \int_{N_i} \sqrt{g_u} d^{(D-d)} u, \tag{26}$$

i.e.,

$$S = \sum_{i=1}^{l} \eta_i S_i, \tag{27}$$

where $S_i = \beta_i \int_{N_i} \sqrt{g_u} d^{(D-d)} u$. This is just the generalized Nambu action for multi-$\tilde{p}$-branes ($\tilde{p} = D - d - 1$), which is the straightforward generalization of Nambu action for the string world-sheet action.[26] Here this action for multi-$\tilde{p}$-branes is obtained directly by ϕ-mapping theory, and it is easy to see that this action is just Nambu action for multistrings when $D - d = 2$.[13]

III. THE GAUGE FIELD CORRESPONDING TO THE TOPOLOGICAL CURRENT

In this section, we will study the antisymmetric tensor gauge field corresponding to the topological tensor current presented in Sec. II. We know that p-branes naturally act as the ''electric'' source of a rank $p+2$ field strength

$$F = dA, \tag{28}$$

where A is a $(p+1)$-form as the tensor gauge potential and satisfies the gauge transformation

$$A \rightarrow A + d\Lambda_p .$$

From Eq. (28), one has the Bianchi identity

$$dF \equiv 0, \tag{29}$$

and the ''electric'' current density associated with the source can be expressed as

$$j^{\mu_1 \cdots \mu_{p+1}} = \nabla_\mu F^{\mu \mu_1 \cdots \mu_{p+1}}. \tag{30}$$

Just as the usual Maxwell's equation, we know that Eqs. (28)–(30) imply the presence of an ''electric'' charge, i.e., p-branes, but no ''magnetic'' source.[11]

Now, let us discuss the case when there exists the ''magnetic'' source. For this case, one must introduce another $(p+2)$-form G for the magnetic source, and the field strength F must be modified to

$$F = dA + G, \tag{31}$$

which is the generalized field strength including the contribution of the ''magnetic'' source, i.e., ''magnetic'' branes: $\tilde{p}$-branes with $\tilde{p} = D - p - 4$.

To obtain the explicit expression for G, let us consider that the current density corresponds to the magnetic source which is given by

$$\tilde{j}^{\mu_1 \cdots \mu_{\tilde{p}+1}} = \nabla_\mu {}^*F^{\mu \mu_1 \cdots \mu_{\tilde{p}+1}}. \tag{32}$$

Using (31) and (32), we obtain

$$\tilde{j}^{\mu_1 \cdots \mu_{(\tilde{p}+1)}} = \frac{1}{\sqrt{g}} \partial_\mu \left(\sqrt{g} \frac{\epsilon^{\mu \mu_1 \cdots \mu_{\tilde{p}+1} \mu_{\tilde{p}+2} \cdots \mu_{D-1}}}{\sqrt{g}} G_{\mu_{\tilde{p}+2} \cdots \mu_{D-1}} \right). \tag{33}$$

It has been pointed out in Sec. II that the current density of the ''magnetic'' branes is a topological current given by Eq. (3), which can be rewritten as

$$\tilde{j}^{\mu_1 \cdots \mu_{D-d}} = \frac{g_m}{A(S^{d-1})(d-1)!} \left(\frac{1}{\sqrt{g}} \right) \partial_{\mu_{(D-d+1)}} (\epsilon^{\mu_1 \cdots \mu_{D-d} \mu_{D-d+1} \mu_{D-d+2} \cdots \mu_D}$$
$$\times \epsilon_{a_1 a_2 \cdots a_d} n^{a_1} \partial_{\mu_{(D-d-2)}} n^{a_2} \cdots \partial_{\mu_D} n^{a_d}), \tag{34}$$

where $(D-d) = \tilde{p}+1$, i.e., $\tilde{p} = D - d - 1$. Comparing Eq. (33) to (34), we can obtain

$$G_{\mu_1 \cdots \mu_{d-1}} = \frac{(-1)^{(D-d)} g_m}{A(S^{d-1})(d-1)!} \epsilon_{a_1 a_2 \cdots a_d} n^{a_1} \partial_{\mu_1} n^{a_2} \cdots \partial_{\mu_{d-1}} n^{a_d}, \tag{35}$$

and

$$G = \frac{(-1)^{(D-d)} g_m}{A(S^{d-1})(d-1)!} \epsilon_{a_1 a_2 \cdots a_d} n^{a_1} dn^{a_2} \wedge \cdots \wedge dn^{a_d}. \tag{36}$$

Of equal interest is the ''magnetic'' charge carried by the multi $\tilde{p}$-branes, which is given by

$$Q^M = \int_\Sigma \tilde{j}^{\mu_1 \cdots \mu_{\tilde{p}+1}} \sqrt{g} \, d\sigma_{\mu_1 \cdots \mu_{\tilde{p}+1}} \tag{37}$$

where Σ is a d-dimension $(d=p+3)$ hypersurface in X, while $d\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}}$ is the convariant surface element[27] of Σ. From (32) and (37), it is easy to prove that

$$Q^M=\int_{\partial\Sigma}F,$$

where $\partial\Sigma$ is the boundary of Σ and a $(p+2)$-dimension hypersurface. Substituting (22) into (37), we have

$$Q^M=g_m\int_{\Sigma}J^{\mu_1\cdots\mu_{\tilde{p}+1}}\left(\frac{\phi}{x}\right)\sum_{i=1}^{l}\beta_i\eta_i\int_{N_i}\delta^D(x-z_i(u))\sqrt{g_u}d^{(D-d)}u\,d\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}}, \tag{38}$$

from (8), and the relation

$$\frac{1}{(\tilde{p}+1)!}\epsilon^{\mu_1\cdots\mu_{\tilde{p}+1}\nu_1\cdots\nu_d}d\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}}=dx^{\nu_1}\wedge\cdots\wedge dx^{\nu_d},$$

expression (38) can be rewritten as

$$Q^M=g_m\int_{\phi(\Sigma)}\sum_{i=1}^{l}\beta_i\eta_i\int_{N_i}\frac{1}{\sqrt{g}}\delta^D(x-z_i(u))\sqrt{g_u}d^{(D-d)}u\,d^{(d)}\phi. \tag{39}$$

Since on the singular submanifold N_i we have

$$\phi^a(x)|_{N_i}=\phi^a(z_i^1(u),\ldots,z_i^D(u))\equiv 0, \tag{40}$$

this leads to

$$\partial_\mu\phi^a\frac{\partial x^\mu}{\partial u^I}\bigg|_{N_i}=0. \tag{41}$$

Using this expression, one can prove

$$J^{\mu_1\cdots\mu_{D-d}}\left(\frac{\phi}{x}\right)\bigg|_{\vec{\phi}=0}=\frac{\sqrt{g}}{\sqrt{g_u}}\epsilon^{I_1\cdots I_{(D-d)}}\frac{\partial x^{\mu_1}}{\partial u^{I_1}}\cdots\frac{\partial x^{\mu_{(D-d)}}}{\partial u^{I_{(D-d)}}}. \tag{42}$$

Then we obtain a useful formula

$$d^{(d)}\phi\sqrt{g_u}d^{(D-d)}u=\sqrt{g}d^Dx. \tag{43}$$

By making use of the above formula and (39), we finally get

$$Q^M=g_m\sum_{i=1}^{l}\beta_i\eta_i\int_X\delta^D(x-z_i(u))d^Dx=g_m\sum_{i=1}^{l}\beta_i\eta_i. \tag{44}$$

Equation (44) shows that the ith brane carries the ''magnetic'' charge $Q_i^M=g_m\beta_i\eta_i=g_mW_i$, which is topologically quantized and characterized by Hopf index β_i and Brouwer degree η_i, the winding number W_i of the ϕ mapping.

IV. CONCLUSION

In this paper the ϕ-mapping theory is introduced to study the $\tilde{p}$-branes theory, which is a development of our former theories of magnetic monopoles and topological strings. We present a new topological tensor current of magnetic multi-$\tilde{p}$-branes and discuss the inner structure of this current in detail. It is shown that every isolated zero of the vector field $\vec{\phi}$ (i.e., order parameters) is just corresponding to a magnetic brane, $\tilde{p}$-brane ($\tilde{p}=D-d-1$). The generalized Nambu action for multi-$\tilde{p}$-branes can be obtained directly in terms of this topological current. The topological structure of the charges carried by $\tilde{p}$-branes shows that the magnetic charges are topologically quantized and labeled by the Hopf index and Brouwer degree, the winding number of the ϕ mapping. The theory formulated in this paper is a new concept for topological $\tilde{p}$-branes based on the ϕ-mapping theory.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China and Doctoral Science Foundation of China.

[1] J. H. Schwarz, ''Lectures on Superstring and *M*-theory Dualities,'' hep-th/9607201.
[2] E. Witten, Nucl. Phys. B **443**, 85 (1995).
[3] P. K. Townsend, Phys. Lett. B **350**, 184 (1995).
[4] C. M. Hull, Nucl. Phys. B **468**, 113 (1996).
[5] P. Orland, Nucl. Phys. B **205**, 107 (1982).
[6] J. Scherk and J. H. Schwarz, Phys. Lett. **52**, 347 (1974); J. H. Schwarz, Phys. Rep. **89**, 223 (1982).
[7] Y. Nambu, Phys. Rep., Phys. Lett. **23**, 250 (1976).
[8] R. I. Nepomechie, Phys. Rev. D **31**, 1921 (1985).
[9] A. Strominger, Nucl. Phys. B **343**, 167 (1990).
[10] C. M. Hull, Nucl. Phys. B **509**, 216 (1998).
[11] M. J. Duff, R. R. Khuri, and J. X. Lu, Phys. Rep. **256**, 213 (1995).
[12] Y. S. Duan and M. L. Ge, Sci. Sin. **11**, 1072 (1979); G. H. Yang and Y. S. Duan, Int. J. Theor. Phys. **37**, 2435 (1998).
[13] Y. S. Duan and J. C. Liu, in *Proceedings of Johns Hopkins Workshop 11*, edited by Y. S. Duan *et al.* (World Scientific, Singapore, 1988).
[14] Y. S. Duan and X. H. Meng, J. Math. Phys. **34**, 1149 (1993); Y. S. Duan, S. Li, and G. H. Yang, Nucl. Phys. B **514**, 705 (1998).
[15] Y. S. Duan and L. B. Fu, J. Math. Phys. **39**, 4343 (1998).
[16] Y. S. Duan, H. Zhang, and S. Li, Phys. Rev. B **58**, 125 (1998).
[17] Y. S. Duan, S. L. Zhang, and S. S. Feng, J. Math. Phys. **35**, 4463 (1994); Y. S. Duan, G. H. Yang, and Y. Jiang, Int. J. Mod. Phys. A **58**, 513 (1997).
[18] Y. S. Duan and S. L. Zhang, Int. J. Eng. Sci. **28**, 689 (1990); **29**, 153 (1991); **29**, 1593 (1991); **30**, 153 (1992).
[19] Y. S. Duan, H. Zhang, and L. B. Fu, Phys. Rev. E **59**, 528 (1999).
[20] E. Goursat, A Course in Mathematical Analysis Vol. 1 (transl. E. R. Hedrick,1904).
[21] L. V. Toralballa, *Theory of Functions* (Merrill, Columbus, OH, 1963).
[22] C. Teitelboim, Phys. Lett. **B167**, 69 (1986).
[23] M. C. Diamantini, ''Topological defects in gauge theories of open *p*-branes,'' hep-th/960790.
[24] J. A. Schouten, *Tensor Analysis for Physicists* (Clarendon, Oxford, 1951).
[25] H. B. Nielsen, and P. Olesen, Nucl. Phys. B **57**, 367 (1973).
[26] Y. Nambu, ''Lectures prepared for the Copenhagen Summer Symposium, 1970'' (unpublished).
[27] A. Ashtekar and R. O. Hansen, J. Math. Phys. **19**, 1543 (1978).