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Many-body quantum coherence and interaction blockade in Josephson-linked Bose-Einstein condensates

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Abstract – We study many-body quantum coherence and interaction blockade in two Josephson-linked Bose-Einstein condensates. We introduce universal operators for characterizing many-body coherence without limitations on the system symmetry and total particle number N . We reproduce the results for both coherence fluctuations and number squeezing in *symmetric* systems of large N , and reveal several peculiar phenomena that may occur in *asymmetric* systems and systems of small N . For asymmetric systems, we show that, due to an interplay between asymmetry and inter-particle interaction, the coherence fluctuations are suppressed dramatically when $|E_C/E_J| \ll 1$, and both *resonant tunneling* and *interaction blockade* take place for large values of $|E_C/E_J|$, where E_C and E_J are the interaction and tunneling energies, respectively. We emphasize that the resonant tunneling and interaction blockade may allow creating single-atom devices with promising technology applications. We demonstrate that for the systems at finite temperatures the formation of self-trapped states causes an anomalous behavior.

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Utilizing the current experimental techniques, it is possible to prepare atomic Bose-Einstein condensates in which all atoms occupy the same single-particle state. To explore the quantum coherence between two such condensates, it is natural to link them through the Josephson tunneling [1], and then release them for producing interference. Atomic interferometry [2] provides an important experimental tool for the fundamental studies in atomic, optical, and quantum physics, as well as promising applications in high-precision measurements and quantum information processing [3]. By loading condensates into double-well potentials, atomic interferometers have been realized experimentally [4–7]. Furthermore, some many-body quantum effects such as coherence fluctuations [6,8], conditional tunneling [9], number squeezing [10], and long-time coherence assisted by number squeezing [11] as well as finite-temperature effects [8] have been observed in experiment. However, clear theoretical explanations for many of those effects, such as the full physical pictures of the coherence fluctuations [6,8], the resonant

tunneling and the interaction blockade [12–15] are still largely missing. Below, we present a general approach for analyzing many-body quantum coherence and interaction blockade and give a full picture of the coherence fluctuations. Particularly, our results of resonant tunnelling can qualitatively explain the conditional tunneling which has been observed recently [9].

To study quantum fluctuations in the Josephson-linked coherent systems such as Bose-Einstein condensates in double-well potentials, one has to analyze the system within the full quantum theory. Under the conditions of tight binding, the quantum system can be mapped onto a two-site Bose-Hubbard model [10]. Then, the number fluctuations can be easily analyzed by applying the techniques well developed in quantum optics [16]. Where, however, the coherence fluctuations could not be clearly depicted. Previously, the phase fluctuations have been used as a measure characterizing the coherence fluctuations. However, due to the absence of the phase operators in the Bose-Hubbard Hamiltonians, one has to introduce the quantum phase models [17,18]. The quantum phase models describe qualitatively the systems

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with small number differences, but they cannot describe the systems of large number differences and fail to maintain the hermiticity for asymmetric systems. Additionally, because the phase is measured modulo 2π , the fluctuations of phase do not provide the best quantitative characterization for the coherence fluctuations [11].

In this letter, we introduce Hermitian operators for the phase coherence rather than a relative phase, and then analyze the coherence fluctuations and interaction blockade. The coherence operators are defined with the atomic creation, annihilation, and number operators, so that they can be calculated directly with the general Bose-Hubbard models. The operational approach has two advantages: first, its validity is independent of the total particle numbers and, second, it works for both symmetric and asymmetric systems. We examine how the coherence fluctuations depend on the total number of particles, the inter-particle interaction energy, the junction tunneling energy, asymmetry, and temperature. Without loss of generality, for all calculations about the fluctuations in zero-temperature systems, we only consider the ground states of the full quantum model: the quantized Bose-Josephson junction. For the calculations about the fluctuations in finite-temperature systems, we use both the full quantum and the mean-field models.

Below, we not only reproduce the pictures of coherence fluctuations and number squeezing in large-size symmetric systems which can be described by the quantum phase models, but also reveal several peculiar phenomena in small-size systems and asymmetric systems which cannot be described by the standard quantum phase models. Such as, i) due to an interplay between the system asymmetry and inter-particle interaction, single-atom resonance effects suppress the quantum fluctuations of coherence and the interaction blockade occurs between the neighboring resonant peaks; and ii) the occupation of highly excited states of degeneracy (macroscopic quantum self-trapping) causes an anomalous behavior of the coherence fluctuations at finite temperatures. These results are qualitatively consistent with the recent experiment data on many-body quantum phenomena [6,8,10,11] and finite-temperature dynamics [8]. In particular, the resonant effects are confirmed by the most recent experimental observations of the conditional tunneling [9]; the physics of the interaction blockade and resonant tunneling opens a route for designing single-atom devices for applications in sensitive metrology and information technology.

We consider the quantum model of two Josephson-linked atomic condensates with the conserved total number of particles. From the physical point of view, this situation can be realized by confining atomic condensates in a double-well potential [4–7]. Eliminating the constant terms including the total number of atoms, the system Hamiltonian

$$H = -J(a_1^\dagger a_2 + a_1 a_2^\dagger) + \frac{\delta}{2}(n_2 - n_1) + \frac{E_C}{8}(n_2 - n_1)^2 \quad (1)$$

describes a quantized Bose-Josephson junction. Here, $a_j^\dagger(a_j)$ are the atomic creation (annihilation) operators for the j -th ($j = 1, 2$) well, and n_j are the corresponding number operators. Thus, the system behavior is determined by the total number N , the charging (inter-particle interaction) energy E_C , the inter-well tunneling strength J , and the asymmetry parameter δ . It has been suggested that the symmetric systems with negative charging energies can be used to realize the Heisenberg-limited interferometry [19]. Here, we consider both symmetric and asymmetric systems with the positive charging energies.

Under the mean-field approximation, $a_j \rightarrow \sqrt{n_j}e^{i\phi_j}$ and $a_j^\dagger \rightarrow \sqrt{n_j}e^{-i\phi_j}$, the macroscopic quantum behavior (such as Josephson oscillations, self-trapping [20], and dynamical bifurcation [21]) obeys a classical non-rigid pendulum Hamiltonian,

$$H_{cl} = E_C n^2/2 + \delta n - E_J \sqrt{1 - (2n/N)^2} \cos \phi, \quad (2)$$

with the quasi-angular momentum $n = (n_2 - n_1)/2$ and the relative phase $\phi = \phi_2 - \phi_1$. Here, $E_J = NJ$ denotes the junction energy. For the symmetric systems ($\delta = 0$) with small number differences ($|2n/N| \sim 0$), one can introduce the quantum phase model via $n \rightarrow i\frac{\partial}{\partial \phi}$ and $(2n/N) \rightarrow 0$ (see refs. [17,18] and references therein). However, it cannot depict the behavior of systems with large number differences or self-trapping, and it fails to maintain hermiticity when $\delta \neq 0$.

To explore the coherence fluctuations, we introduce operators for the phase coherence. Drawing upon the quantum phase concept [22] for single-mode coherence, we define the operators for two-mode first-order coherence as

$$\begin{aligned} \cos \phi &= \frac{(a_2^\dagger a_1 + a_2 a_1^\dagger)}{\sqrt{2} \langle 2n_1 n_2 + n_1 + n_2 \rangle}, \\ \sin \phi &= \frac{i(a_2^\dagger a_1 - a_2 a_1^\dagger)}{\sqrt{2} \langle 2n_1 n_2 + n_1 + n_2 \rangle}. \end{aligned} \quad (3)$$

These two operators satisfy the condition $\langle \sin^2 \phi + \cos^2 \phi \rangle = 1$. Without considerations in total particle number conservation and inter-particle interaction, similar operators have been used to study the phase coherence of photons from different lasers [23]. So that the previous results of photons cannot be applied to systems of conserved total particle numbers and inter-particle interaction, such as the Josephson-linked condensates considered in this paper. Similarly, one can define the operators for two-mode second-order coherence, $\cos(2\phi) = [(a_2^\dagger a_1)^2 + (a_2 a_1^\dagger)^2]/K_S$ and $\sin(2\phi) = i[(a_2^\dagger a_1)^2 - (a_2 a_1^\dagger)^2]/K_S$, where the constant $K_S = \langle 2n_1 n_2 + n_1 + n_2 \rangle$. Fluctuations of the two-mode first-order coherence are quantitatively characterized by the variance

$$\Delta(\cos \phi) = \langle \cos^2 \phi \rangle - \langle \cos \phi \rangle^2. \quad (4)$$

Here, the expectation for $\cos \phi$ is

$$\langle \cos \phi \rangle = \frac{\langle a_2^\dagger a_1 + a_2 a_1^\dagger \rangle}{\sqrt{2 \langle 2n_1 n_2 + n_1 + n_2 \rangle}}, \quad (5)$$

and the expectation for $\cos^2 \phi$ is

$$\langle \cos^2 \phi \rangle = \frac{1}{2} + \frac{\langle (a_2^\dagger a_1)^2 \rangle + \langle (a_2 a_1^\dagger)^2 \rangle}{2 \langle 2n_1 n_2 + n_1 + n_2 \rangle}. \quad (6)$$

The variance $\Delta(\cos \phi)$ vanishes when two condensates becomes phase-locked. Otherwise, the variance approaches the value 1/2 when two condensates have no phase correlations. The above variance has been successfully employed for explaining the q -vortex melting [24].

For symmetric systems at zero temperature, the coherence fluctuations $\Delta(\cos \phi)$ and the number squeezing $S = \sqrt{N/\Delta N_r}$ (where $\Delta N_r = \langle N_r^2 \rangle - \langle N_r \rangle^2$ is the variance for relative numbers $N_r = n_2 - n_1$) show different parametric dependences. The coherence fluctuations grow monotonously with the ratio $E_C/E_J = E_C/(NJ)$; however, the number squeezing grows monotonously with the ratio $N^2 E_C/E_J = N E_C/J$. The behavior of the coherence fluctuations is qualitatively consistent with the experimental observations made via adjusting the separation distance [6] or the barrier height [8], which corresponds to varying E_J . Because the coherence time grows with the number squeezing [25], the different parametric dependence of the coherence fluctuations and number squeezing indicates that one can increase the coherence time without loss of coherence, by increasing the total number N or with some loss of coherence, by increasing the ratio E_C/E_J . This result justifies qualitatively the experimental approach for adjusting the coherence time via varying the barrier height (*i.e.*, controlling E_J) [11], in which the coherence time grows with the barrier height. For sufficiently large N , the parametric dependence of coherence fluctuations and number squeezing is consistent with the previous results obtained from other approaches [17,25,26]. Our operational approach also explores the finite-size effects in systems of small N . For commensurate systems (whose N 's are even integers) of very weak or strong interactions, the coherence fluctuations are almost independent of N . However, in the intermediate region of $E_C/E_J \sim 1$, the coherence fluctuations decrease with N . In fig. 1, we show the parametric dependence for the ground-state coherence fluctuations and number squeezing for symmetric systems with different total number of particles.

For asymmetric systems at zero temperature and without inter-particle interaction ($E_C = 0$), both coherence fluctuations and number squeezing increase monotonically with the asymmetry parameter δ . For small values of E_C/E_J , before the asymmetry δ starts dominating the dynamics, the coherence fluctuations can be suppressed by an interplay between asymmetry and repulsive inter-particle interaction; the suppression becomes more significant for larger values of E_C . This suppression of the

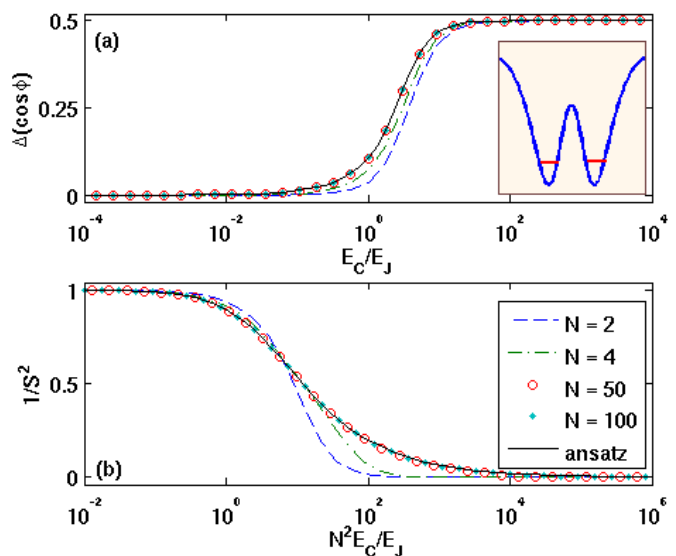


Fig. 1: Coherence fluctuations and number squeezing in symmetric Bose-Josephson junctions. (a) Coherence fluctuations $\Delta(\cos \phi)$ vs. E_C/E_J and (b) inverse square of number squeezing $1/S^2$ vs. $N^2 E_C/E_J$. Dashed lines, dash-dotted lines, circles, and dots correspond to the cases $N = 2, 4, 50$ and 100 , respectively. For sufficiently large N , the curves approach the result obtained from the Gaussian ansatz applied to the probability amplitudes (solid line).

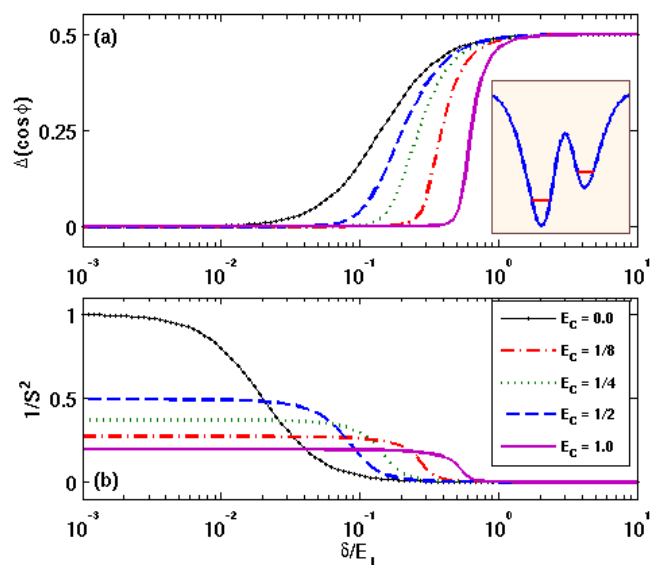


Fig. 2: Coherence fluctuations and number squeezing in asymmetric systems with $N = 100$, $E_J = 100$, and $E_C/E_J \ll 1$. (a) Coherence fluctuations vs. δ/E_J and (b) inverse square of number squeezing $1/S^2$ vs. δ/E_J .

coherence fluctuations originates from the different localization mechanisms dominated by asymmetry and repulsive inter-particle interaction. The asymmetry tends to localize all atoms in the lower well; however, the repulsive inter-particle interaction (positive charging energy) tends to localize atoms in two wells without number difference. In fig. 2, we show the coherence fluctuations and number

squeezing in asymmetric systems with $N = 100$, $E_J = 100$ and $E_C/E_J \ll 1$.

For asymmetric systems with large values of E_C/E_J at zero temperature, the coherence fluctuations can be suppressed resonantly, due to degeneracy and quasi-degeneracy of the energy levels occurring at some particular values of the ratio δ/E_C . For systems without tunneling, the eigenstate of quasi-angular momentum n has the energy $E^*(n) = \delta n + E_C n^2/2$, and so that two neighboring levels become degenerate when $E^*(n) = E^*(n+1)$. This means that, near the values of $\delta/E_C = -(n+1/2)$ for $n = (-N/2, -N/2+1, \dots, N/2-1)$, at least one atom can tunnel freely between two wells, even in the limit of very weak tunneling (*i.e.*, for very small values of E_J). Therefore, the coherence fluctuations from inter-particle interaction will be suppressed resonantly, with the resonance width depending on E_J . This is a single-atom resonance effect caused by the degeneracy and quasi-degeneracy between the states of the neighboring angular momenta n . For very small values of E_J/E_C , we perform a perturbation analysis by assuming that the perturbed ground state is a superposition of the two lowest states of the system without tunneling. Namely, the ground state $|\psi\rangle_G = \alpha_1|L_1\rangle + \alpha_2|L_2\rangle$, where $|L_1\rangle$ and $|L_2\rangle$ are the two lowest states. The two coefficients are determined by the lowest energy state of the following eigenvalue equation:

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = E \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix},$$

in which $H_{ij} = \langle L_i | H | L_j \rangle$ ($i, j = 1, 2$). The perturbation results confirm the resonant suppression of coherence fluctuations, and they show that the resonance width increases with E_J/E_C . Similarly, if we prepare a state of a quasi-angular momentum n , the correlated tunneling of an atomic pair (two-atom resonance) should correspond to the degeneracy condition $E^*(n) = E^*(n+2)$. Our results for single- and two-atom resonances explain consistently the recent observations of conditional tunneling [9]. In fig. 3, we show the coherence fluctuations in the ground states of asymmetric systems with $N = 100$, $E_J = 100$ and large values of E_C . For a very large value of the charging energy, $E_C = 800$ (green on-line lines), our numerical results differ slightly from the results calculated by the perturbation analysis. Even for a moderate value of the charging energy, $E_C = 100$, our numerical results clearly show the resonant suppression of the coherence fluctuations. Apparently, the resonance for the case $E_C = 100$ is wider than that for the case $E_C = 800$.

The single-atom resonant tunneling is a signature of interaction blockade induced by the *s*-wave scattering between atoms. It is well known that the Coulomb blockade of single-electron tunneling occurs because of the charging energy [12]. In the Bose-Josephson junction (1), the asymmetry and the inter-particle interaction play the role of the applied voltage and the charging interaction, respectively. As a result of the interaction blockade, adiabatically sweeping the asymmetry, atoms in the

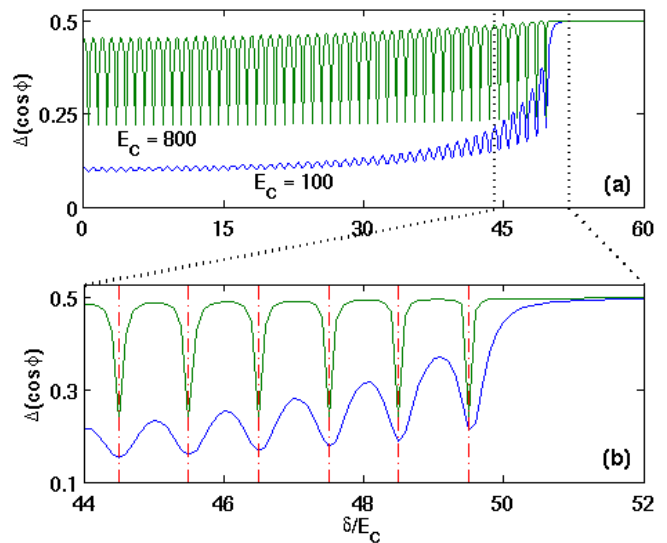


Fig. 3: Coherence fluctuations in the ground states of asymmetric systems with $N = 100$, $E_J = 100$ and large values of E_C . (a) Coherence fluctuations $\Delta(\cos \phi)$ vs. δ/E_C and (b) the enlarged region between the two dotted lines shown in (a). Blue and green on-line lines correspond to the values $E_C = 100$ and $E_C = 800$, respectively. Resonant suppression of the coherence fluctuations occurs at $\delta = -E_C(k+1/2)$ for $k = (-N/2, -N/2+1, \dots, N/2-1)$, marked by the dashed red on-line lines in (b).

ground-state tunnel through the potential barrier one by one and the coherence vs. asymmetry dependence is a set of sharp peaks between plateaus of interaction block. The interplay of the single-atom resonant tunneling and the interaction blockade allows one to create so-called single-atom devices, which would provide promising application in sensitive metrology and information technology like their electronic counterparts [14,15]. For a strongly interacting system in optical lattices with asymmetry imposed by a harmonic potential, the appearance of Mott-insulator shells [27,28] is also the results of interaction blockade. In fig. 4, we show resonant tunneling peaks and interaction blockade plateaus for a quantized Bose-Josephson junction of $N = 10$, $E_J = 10$ and $E_C = 1000, 100, 10$. In which, $\Delta(N_r)$ and $\langle N_r \rangle$ are the quantum fluctuation and expectation of the relative particle number N_r , respectively. The regular steps in $\langle N_r \rangle$ and sharp peaks in $\Delta(N_r)$ clearly show the single-atom resonant tunneling and the interaction blockade. It also shows that the interaction blockade becomes more and more significant when the interaction energy E_C increases. In the recent experiment of atoms confined in double-well lattices [9], $N = 2$ and $E_J/E_C \approx 0.2$. For the state of two atoms in the first well, $n = -1$, the single-atom (and two-atom) resonances occur at $\delta = E_C/2 = 0.78E_r$ (and $\delta = 0$) when the resonant conditions $E^*(n) = E^*(n+1)$ (and $E^*(n) = E^*(n+2)$) are satisfied. In which, the recoil energy $E_r = \hbar^2/(2m\lambda^2)$ with the mass m for a ^{87}Rb atom, the Planck's constant \hbar , and the short-lattice wavelength $\lambda = 765$ nm.

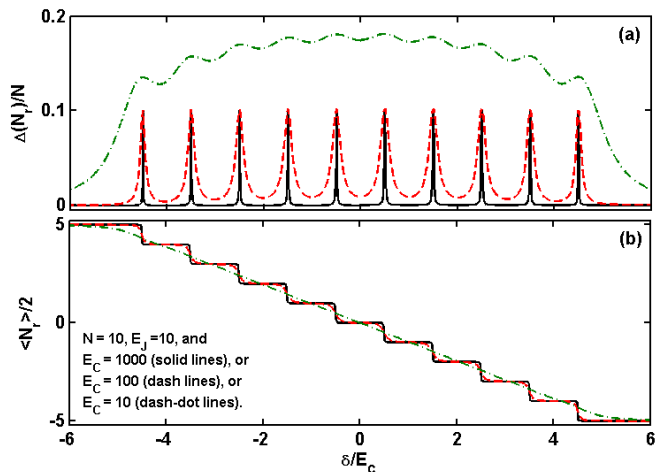


Fig. 4: Peaks of single-atom resonant tunneling and plateaus of interaction blockade. (a) The fluctuations of relative particle numbers and (b) the expectation values of half relative particle numbers *vs.* δ/E_C . Solid, dashed and dash-dotted lines correspond to $E_C/E_J = 100, 10$ and 1 , respectively.

The analysis presented above describes the system properties at zero temperature. For finite-temperature systems, assuming a canonical ensemble statistics, the equilibrium states are mixed states described by the density matrix $\rho(T) = \exp(-\beta H)/\text{Tr}[\exp(-\beta H)]$. Here, $\beta = 1/(k_B T)$, $\text{Tr}[\rho(T)] = 1$, k_B is the Boltzmann constant, and H is the Hamiltonian matrix. The thermodynamic average and variance of an arbitrary operator Q are $\langle Q \rangle_T = \text{Tr}[\rho(T)Q]$ and $\Delta(Q)_T = \text{Tr}[\rho(T)(Q - \langle Q \rangle)^2]$, respectively.

For the systems of fixed parameters, we find that the coherence fluctuations grow with temperature. However, for the systems at fixed temperature, the parametric dependence of the coherence fluctuations relies on the values of temperature. For low temperatures, the coherence fluctuations grow with E_C/E_J . This finite-temperature behavior is qualitatively consistent with the experimental data on noise thermometry with two Josephson-linked condensates of Bose atoms [8]. For higher temperatures, the coherence fluctuations grow with E_C/E_J except for the anomalous behaviors near the value $E_C/E_J \sim 4/N^2$. For every eigenstate, we numerically find that the quantum fluctuations of coherence increase with E_C/E_J . This indicates that the monotonous growth of the coherence fluctuations *vs.* E_C/E_J (except for the anomalous region) in finite-temperature systems is a signature of the quantum fluctuations.

Using the mean-field Hamiltonian (2) with the same parameters, we estimate the coherence fluctuations with the classical Boltzmann theory. For systems of low temperatures and very small values of E_C/E_J , the classical estimations show excellent consistence with the quantum ones. Otherwise, the classical results greatly differ from the quantum ones. This means that the mean-field Hamiltonian works good for systems of low temperatures and

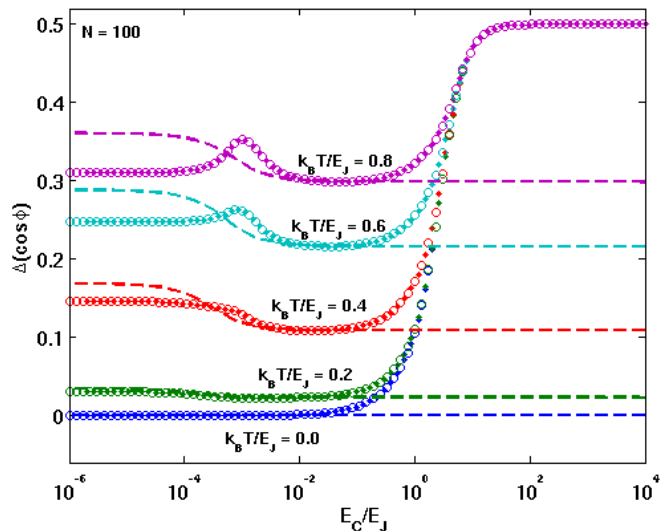


Fig. 5: Quantum and thermal fluctuations of the phase coherence *vs.* E_C/E_J for symmetric systems of $N = 100$ and different values of temperature. Circles and dots correspond to the cases $E_J = 1.0$ and $E_J = 0.5$, respectively. Dashed lines are estimated with the classical Boltzmann theory.

very small values of E_C/E_J . Because the mean-field treatment uses the coherent-state approximation and only the low-energy states for systems of small E_C/E_J are close to coherent states, the excellent consistence for systems of low temperature and small values of E_C/E_J is a natural result of the mean-field approximation.

In fig. 5, we show the quantum and thermal fluctuations of the phase coherence *vs.* E_C/E_J for symmetric systems of $N = 100$ and different values of T . In which, the dashed lines are obtained from the mean-field model (2) and the other data are calculated from the full quantum model (1). The region near $E_C/E_J \geq 4/N^2$ is the region of degenerated highest-excited states which corresponds to the macroscopic quantum self-trapping in mean-field models [20,21]¹. The anomalous behavior of the coherence fluctuations in this region is caused by the occupation of the high-excited states of twofold degeneracy when the temperature becomes sufficiently high. We also calculate the averaged energy spacing for different parameters, and the results show a minimum in the crossover region between normal and self-trapping regions. For fixed and sufficiently high temperatures, the populations of high-excited states in this crossover region will be larger than those in other regions, and then the coherence fluctuations show an anomalous peak.

¹Writing the pure states for the quantum system (1) as $|\Psi\rangle = \sum_{n=-N/2}^{N/2} C(n)|n=N_r/2\rangle$ the density distributions $|C(n)|^2$ *vs.* E_C/E_J for the highest-excited and sub-highest-excited states can be calculated. Indeed, the classical π -phase stationary states in the mean-field Hamiltonian (2) correspond to the extrema (minima or maxima) of $|C(n)|^2$. In which, the stable and unstable fixed points correspond to the maxima and minima, respectively. The details of the correspondence between quantum degeneracy and classical bifurcation will be published elsewhere.

In conclusion, we have studied the mechanisms of coherence fluctuations and interaction blockade in two Josephson-linked Bose-Einstein condensates with repulsive interaction. Our results for the coherence fluctuations and their resonant suppression are qualitatively consistent with several recent experimental studies of coherence fluctuations [6,8], conditional tunneling of single atoms and atom pairs [9], number squeezing [10], and squeezing-assisted long-time coherence [11]. We have revealed how the fluctuations depend on the system parameters and temperature. The different parametric dependences of the coherence fluctuations and the number squeezing provide a route for controlling the coherence time experimentally by adjusting the barrier height (junction energy) [11]. In particular, we have shown that a competition between the potential asymmetry and inter-particle interaction leads to the resonant tunneling and interaction blockade. The resonant effects explain consistently the recent observations of the conditional tunneling of single atoms and atom pairs [9]. More promisingly, the mechanisms of resonant single-atom tunneling and interaction blockade provide an alternative approach for designing single-atom devices for applications in sensitive metrology and information technology. At higher temperatures, an anomalous behavior of the coherence fluctuations is an indicative of the energy level degeneracy corresponding to the macroscopic quantum self-trapping effect [20,21].

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