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Environment-induced disentanglement of the Einstein–Podolsky–Rosen system

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Abstract

We study the entanglement dynamics of the Einstein, Podolsky and Rosen (EPR) system in which two spatially separated particles are coupled with their own cavity field. The environment-induced entanglement sudden death depends on the parameters of an initial entangled state and a mean photon number as well. We moreover investigate the time-evolution behaviour for both concurrence and energy transfer between the EPR particles and the environment, from which it is found that the maximal concurrence indeed corresponds to the minimal energy of the EPR system. However, their time variations are neither in one-to-one correspondence nor in step. In particular, when the state of two particles becomes disentangled, the time variation of energy transfer still exists.

Entanglement, as a nonlocal quantum correlation, is of fundamental importance in quantum physics, which has attracted much attention in recent years due to its potential application in quantum cryptography, teleportation and quantum computing [1]. The interesting relationship between entanglement and quantum phase transition has been explored [2], which may shed light upon the dramatic effects of critical many-particle systems.

Real quantum systems are inevitably influenced by their surrounding environments, which usually cause disentanglement. Many investigations have been devoted to the time evolution of entanglement with environments in order to understand the decoherence mechanism [3]. The entanglement sudden death (ESD) first proposed by Yu and Eberly is one of the most interesting observations [4] in open systems coupling with a cavity field. Subsequently, ESD followed by entanglement revival after a finite death time was demonstrated with the 'double Jaynes–Cummings' model in [5]. Liu and Chen [6] recently showed that ESD, which depends on the initial condition, may happen when two atoms are coupled to a common cavity field. It has been reported that ESD can appear remarkably even in an isolated two-qubit system with an entangled mixed initial state [7].

Although the environment is not desired in most cases, it has been shown that entanglement between two or more subsystems may be induced by their collective interactions with a common environment [8]. A stable entangled state was

also demonstrated, in which once qubits become entangled they will never be disentangled [9].

The Einstein, Podolsky and Rosen (EPR) system plays a central role in quantum communication and it is certainly of importance to study its entanglement decay induced by the environment. Two particles of the EPR system prepared initially in an entangled state are separated by a long spatial distance, and thus each particle interacts with its own environment. We find a stable ESD which depends on the initial parameter of the entangled state and the mean photon number \bar{n} of the cavity field. Since dissipation causes an energy exchange between the quantum system and its environment, it is interesting to find the relation between the entanglement and energy transfer which so far is an open topic [7, 15]. In this paper, we investigate the characteristic behaviour of concurrence and energy transfer in an EPR system with the environment which has practical importance in quantum teleportation. It is shown that the maximal concurrence (indicating a maximal coherence) corresponds to the minimal energy of the EPR system. The variations of concurrence and energy transfer do not have one-to-one correspondence [7], and moreover energy transfer exists even when the state of two EPR particles becomes stably disentangled. This observation suggests that the concurrence seems not to be directly related to the energy transfer at least for the model at hand. More work is needed towards the understanding of concurrence dynamics.

Two atoms interacting with a common cavity field have been studied extensively [4–7], and interesting results

such as ESD have been obtained. Generally speaking, the Hamiltonian for a quantum system of N identical two-level atoms collectively coupled to the same cavity field is given by [10]

$$H = H_s + H_r + H_{sr}, \quad (1)$$

where

$$H_s = \sum_{j=1}^N \frac{1}{2} \hbar \omega_A \sigma_z^j \quad (2a)$$

$$H_r = \sum_{k,\lambda} \hbar \omega_k a_{k,\lambda}^\dagger a_{k,\lambda} \quad (2b)$$

$$H_{sr} = \sum_{k,\lambda,j} \hbar (\kappa_{k,\lambda}^{j*} a_{k,\lambda}^\dagger \sigma_{j-} + \kappa_{k,\lambda}^j a_{k,\lambda} \sigma_{j+}) \quad (2c)$$

with the dipole coupling constants

$$\kappa_{k,\lambda}^j = -ie^{ik \cdot \mathbf{r}_j} \sqrt{\frac{\omega_k}{2\hbar\epsilon_0 V}} \hat{\mathbf{e}}_{k,\lambda} \cdot \mathbf{d}_{21}, \quad (3)$$

where σ_z^j and $\sigma_\pm^j = \frac{1}{2}(\sigma_x^j \pm i\sigma_y^j)$ are the Pauli operators of the j th atom with the energy level spacing ω_A and $a_{k,\lambda}$ ($a_{k,\lambda}^\dagger$) is the annihilation (creation) cavity-field operator of mode k (field frequency ω_k) and polarization index λ . \mathbf{r}_j denotes the spatial position of the j th atom, V is the quantization volume and $\hat{\mathbf{e}}_{k,\lambda}$ is the polarization unit vector. ϵ_0 and d_{21} represent the vacuum permittivity and atomic dipole matrix element, respectively. We define the operators $\Gamma_j^\dagger = \sum_{k,\lambda} \kappa_{k,\lambda}^{j*} a_{k,\lambda}^\dagger$, $\Gamma_j = \sum_{k,\lambda} \kappa_{k,\lambda}^j a_{k,\lambda}$, which become in interaction picture

$$\tilde{\Gamma}_j^\dagger = \sum_{k,\lambda} \kappa_{k,\lambda}^{j*} a_{k,\lambda}^\dagger e^{i\omega_k t}, \quad (4a)$$

$$\tilde{\Gamma}_j = \sum_{k,\lambda} \kappa_{k,\lambda}^j a_{k,\lambda} e^{-i\omega_k t}. \quad (4b)$$

The master equation of a reduced density operator for the N -atom system can be written in the Born approximation as [10]

$$\begin{aligned} \dot{\rho} = & - \sum_{j,l=1}^N \int_0^t dt' \{ [\sigma_{j-} \sigma_{l-} \tilde{\rho}(t') - \sigma_{l-} \tilde{\rho}(t') \sigma_{j-}] \\ & \times e^{-i\omega_A(t+t')} \langle \tilde{\Gamma}_j^\dagger(t) \tilde{\Gamma}_l^\dagger(t') \rangle_r + [\sigma_{j+} \sigma_{l+} \tilde{\rho}(t') \\ & - \sigma_{l+} \tilde{\rho}(t') \sigma_{j+}] e^{i\omega_A(t+t')} \langle \tilde{\Gamma}_j(t) \tilde{\Gamma}_l(t') \rangle_r \\ & + [\sigma_{j-} \sigma_{l+} \tilde{\rho}(t') - \sigma_{l+} \tilde{\rho}(t') \sigma_{j-}] e^{-i\omega_A(t-t')} \\ & \times \langle \tilde{\Gamma}_j^\dagger(t) \tilde{\Gamma}_l(t') \rangle_r + [\sigma_{j+} \sigma_{l-} \tilde{\rho}(t') - \sigma_{l-} \tilde{\rho}(t') \sigma_{j+}] \\ & \times e^{i\omega_A(t-t')} \langle \tilde{\Gamma}_j(t) \tilde{\Gamma}_l^\dagger(t') \rangle_r \} + \text{h.c.}, \end{aligned} \quad (5)$$

where $\langle \rangle_r$ are called bath correlation functions. One can obtain in principle the time evolution of the density operator if correlation functions are known. We emphasize that equation (5) is the master equation of N atoms interacting with the same cavity field. Since the spatial correlation comes from the spatial phase of a coupling constant in equation (3), each atom will feel statistically an independent cavity field, if the atoms are separated from each other by a long distance

compared with the wavelength $\lambda_A = \frac{2c\pi}{\omega_A}$. In this case, the cavity-field correlation functions vanish for $j \neq l$. Thus, equation (5) reduces to the master equation of N atom coupled independently to N independent cavity fields. In the Schrödinger picture, we have [10]

$$\begin{aligned} \dot{\rho} = & -i\frac{1}{2}\omega_A \sum_{j=1}^N [\sigma_{jz}, \rho] + \frac{\gamma}{2}(\bar{n}+1) \sum_{j=1}^N (2\sigma_{j-} \rho \sigma_{j+} \\ & - \sigma_{j+} \sigma_{j-} \rho - \rho \sigma_{j+} \sigma_{j-}) + \frac{\gamma}{2}\bar{n} \sum_{j=1}^N (2\sigma_{j+} \rho \sigma_{j-} \\ & - \sigma_{j-} \sigma_{j+} \rho - \rho \sigma_{j-} \sigma_{j+}), \end{aligned} \quad (6)$$

which can also be used to describe the Dicke super-radiance and superfluorescence [11] and is the starting point of our investigation of entanglement dynamics.

For a two-qubit EPR system coupled with an environment, the concurrence $C(\rho(t))$ [12] is a good tool for characterizing quantum entanglement. In order to be specific, the initial mixed state in basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ is of the form

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix}$$

with parameters given by $\rho_{11} = \rho_{44} = \frac{1}{3}(1-F)$, $\rho_{22} = \rho_{33} = \frac{1}{6}(1+2F)$ and $\rho_{23} = \rho_{32} = \frac{1}{6}(1-4F)$ [6]. This class of mixed states including, particularly, Bell states as well as the well-known Werner mixed states [13] has been investigated in various physical systems, for example, the J-C model [4, 6, 7], spin chain [14], etc.

Based on equation (6), the time evolution of corresponding matrix elements can be found as

$$\rho_{11}(t) = \frac{1}{3(1+2\bar{n})^2} [\chi + \chi\bar{n} + 3\bar{n}\sqrt{\chi} + (\chi+3)\bar{n}^2 - F\chi(1+2\bar{n})^2], \quad (7)$$

$$\begin{aligned} \rho_{22}(t) = \rho_{33}(t) = & \frac{1}{6(1+2\bar{n})^2} [3\sqrt{\chi} + 6\bar{n}(1+\bar{n}) \\ & + 2\chi(-1-\bar{n}-\bar{n}^2 + F(1+2\bar{n})^2)], \end{aligned} \quad (8)$$

$$\begin{aligned} \rho_{44}(t) = & \frac{1}{3(1+2\bar{n})^2} [\chi(1+\bar{n}+\bar{n}^2) - 3\sqrt{\chi}(1+\bar{n}) \\ & + 3(1+\bar{n})^2 - F(1+2\bar{n})^2], \end{aligned} \quad (9)$$

$$\rho_{23}(t) = \frac{1}{6}(1-4F)\sqrt{\chi}, \quad (10)$$

where $\chi = e^{-2(1+2\bar{n})t\gamma}$, $\bar{n} = \frac{e^{-\hbar\omega/\kappa_B T}}{1-e^{-\hbar\omega/\kappa_B T}}$ with the Boltzmann constant κ_B and temperature T .

The concurrence of state $\rho(t)$ is obtained as $C(t) = 2 \max\{0, |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}\}$, which as a function of time is plotted in figure 1. Figure 1 clearly shows the ESD taking place at a certain time period called decay time after which two EPR particles are essentially disentangled. Entanglement decay becomes faster with an increasing mean photon number \bar{n} for a given initial state. Comparing figures 1(a) and (b) and figures 1(c) and (d), one can find that concurrence depends on the initial parameter F , which increases with F , and so does

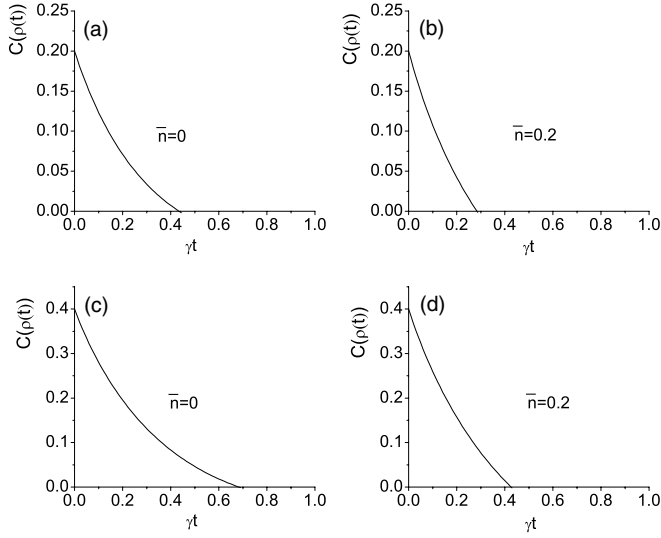


Figure 1. Concurrence as a function of the rescaled time $t\gamma$, (a) with $F = 0.6$, $\bar{n} = 0$, (b) $F = 0.6$, $\bar{n} = 0.2$, (c) $F = 0.7$, $\bar{n} = 0$, (d) $F = 0.7$, $\bar{n} = 0.2$.

the behaviour of entanglement obtained in the present paper in agreement with the result of [6]. The concurrence versus parameter F for a given time $t\gamma$ and mean photon number \bar{n} is plotted in figure 2, indicating a threshold value of parameter F only above which concurrence begins to appear (i.e. the quantum correlation between two EPR particles starts to build) and increases monotonically with the parameter F . The mean photon number \bar{n} dependence of entanglement is also shown in figure 2.

Since the dissipation-induced decoherence causes energy exchange between the quantum system and its environment, it is desirable to establish a link between the entanglement and energy transfer [7, 15]. Cui *et al* [7] have argued that the variations of average energy $\langle H_s \rangle$ of the quantum system ($N = 2$) and concurrence are almost in step and suggest a general relation between time evolutions of entanglement and energy transfer. The variations of concurrence $C(\rho(t))$ and energy $\langle H_s \rangle = \rho_{11}(t) - \rho_{22}(t)$ as a function of $t\gamma$ for our EPR system are shown in figures 3(a) and (b) with $F = 0.7$.

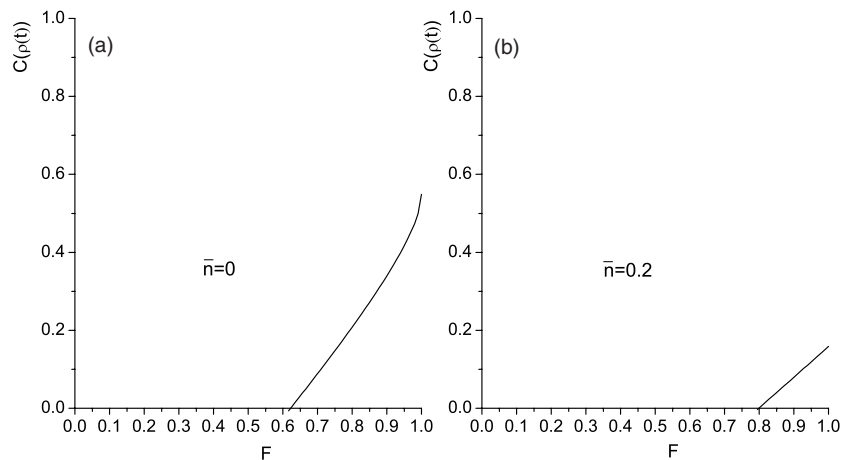


Figure 2. Concurrence as a function of F for $t\gamma = 0.6$, and $\bar{n} = 0$ (a) and 0.2 (b) respectively.

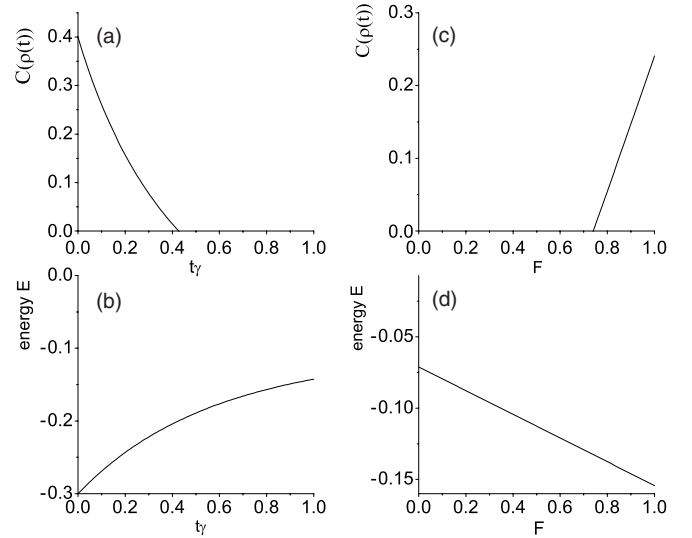


Figure 3. The concurrence $C(\rho(t))$ (a) and energy $\langle H_s \rangle$ (b) as functions of $t\gamma$ with $F = 0.7$, $\bar{n} = 0.2$. $C(\rho(t))$ (c) and $\langle H_s \rangle$ (d) versus F with $t\gamma = 0.5$.

The concurrence $C(\rho(t))$ and energy $\langle H_s \rangle$ versus F is plotted in figures 3(c) and (d), respectively, with $t\gamma = 0.5$. It is indeed observed that the maximal concurrence corresponds to the minimal value of energy $\langle H_s \rangle$. However, the variations of energy $\langle H_s \rangle$ and concurrence $C(\rho(t))$ are not in one-to-one correspondence and not in step either. With increase of energy $\langle H_s \rangle$, the concurrence $C(\rho(t))$ decreases until it completely vanishes in figures 3(a) and (b). It seems that the concurrence decay is induced by information exchange between the EPR system and the cavity field. At the same time, the EPR system absorbs energy from the cavity field. Particularly for the parameter value $F = 0.25$ corresponding to $\rho_{23}(t) = 0$, the state ρ becomes disentangled independently of time, i.e. $C(\rho(t)) = 0$, while the variation of energy $\langle H_s \rangle$ still exists as seen from figure 4. A critical energy value E_c has been proposed in [7] for generation of ESD, below which the state becomes disentangled, namely ESD appears. However, this seems not to be the case for our EPR system because ESD here can happen only above this critical value E_c . It is still

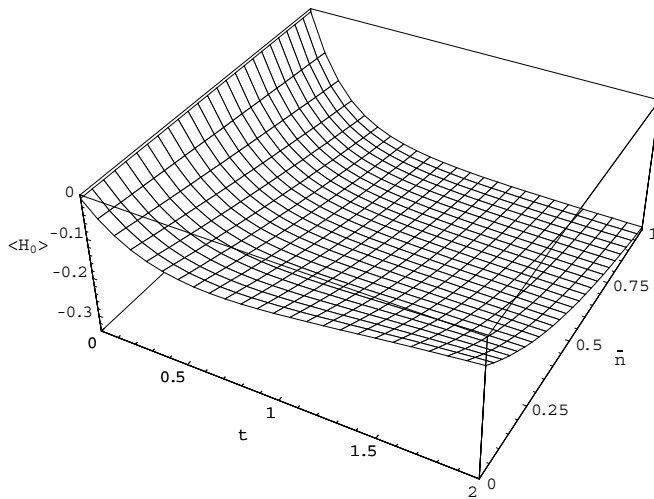


Figure 4. $\langle H_s \rangle$ as a function of \bar{n} and time t , with $F = 0.25$ and $\gamma = 1$.

an open problem to find a definite connection between the concurrence and energy transfer.

In conclusion, we have studied the time evolution of entanglement for two EPR particles in cavity fields. It was found that disentanglement depends on the initial entangled state and mean photon number \bar{n} as well. Comparing the time evolution behaviours for both concurrence and energy transfer, we concluded that the maximal concurrence indeed corresponds to the energy minimum of a quantum system while its time variations are neither in one-to-one correspondence nor in step. In particular, when the state of two particles becomes disentangled with vanishing concurrence, the time evolution of energy transfer still exists.

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