Landau-Zener tunneling in a nonlinear three-level system

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We present a comprehensive analysis of the Landau-Zener tunneling of a nonlinear three-level system in a linearly sweeping external field. We find the presence of nonzero tunneling probability in the adiabatic limit (i.e., very slowly sweeping field) even for the situation that the nonlinear term is very small and the energy levels keep the same topological structure as that of the linear case. In particular, the tunneling is irregular with showing an unresolved sensitivity on the sweeping rate. For the case of fast-sweeping fields, we derive an analytic expression for the tunneling probability with stationary phase approximation and show that the nonlinearity can dramatically influence the tunneling probability when the nonlinear "internal field" resonate with the external field. We also discuss the asymmetry of the tunneling probability induced by the nonlinearity. Physics behind the above phenomena is revealed and possible application of our model to triple-well trapped Bose-Einstein condensate is discussed.

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I. INTRODUCTION

Avoided crossing of energy levels is a universal phenomenon for the quantum nonintegrable systems where the symmetry break leads to the splitting of degenerate energy levels forming a tiny energy gap. Around the avoided crossing point of the two levels the Landau-Zener tunneling (LZT) model provides an effective description for the tunneling dynamics under assumption that the energy bias of two levels undergoes a linear change with time [1]. It is a basic model in quantum mechanics and has versatile applications in quantum chemistry [2], collision theory [3], and more recently in the spin tunneling of nanomagnets [4,5], Bose-Einstein condensates (BEC) [6], and quantum computing [7], to name only a few.

LZT model has been extended to many versions taking diverse physical conditions into account: LZT problem with a time-varied sweeping rate [8], LZT model with a fast noise from the outer environment [9], LZT model with periodic modulation [10,11], and so on. Among them, LZT in a nonlinear two-level system is one of the most interesting models and attracts much attention recently [12–15]. In this model, the level energies depend on the occupation of the levels and may arise in a mean-field treatment of a many-body system where the particles predominantly occupy two energy levels. The nonlinear LZT model not only demonstrates behavior of great interest in theory but also has important applications in spin tunneling of nanomagnets [16] and a Bose-Einstein condensate in a double-well potential [12,14,17] or in an optical lattice [13,15]. However, since most of the problems of interest involve more than two energy levels, with transitions between several levels happening simultaneously [18–21], for example, BECs trapped in multiple wells [22–25], spin tunneling of nanomagnets with large spin, etc., it is naturally desirable to extend the above nonlinear tunneling to the multilevel situation.

In the present paper, we consider the simplest multilevel system—the three-level system, to investigate its complicated tunneling dynamics in the presence of nonlinearity. Because quantum transitions may happen between several levels simultaneously, the LZT in the nonlinear three-level model shows many striking properties distinguished from that of the two-level case. In the adiabatic limit we will show that, for a very small nonlinear parameter, the energy levels still keep the same topological structure as its linear counterpart, the adiabaticity breaks down manifesting the presence of a nonzero tunneling probability. This is quite different from the two-level case, where the breakdown of the adiabaticity is certainly accompanied by a topological change on the energy levels. More interestingly, the tunneling is irregular with showing an unresolved sensitivity on the sweeping rate, a phenomenon attributed to the existence of the chaotic state. In the sudden limit, we derive an analytic expression for the tunneling probability under stationary phase approximation and show that the nonlinearity can dramatically influence the tunneling probability at the resonance between the nonlinear "internal field" and the external field. We also discuss the asymmetry of the tunneling probability induced by the nonlinearity. The physical mechanism behind these phenomena is revealed and possible application of our model to triple-well trapped Bose-Einstein condensate is discussed.

The paper is organized as follows. In Sec. II we introduce our nonlinear three-level LZT model and calculate its adiabatic levels. Section III discusses LZT among the levels. Section IV gives a possible application of the model to the triple-well trapped BEC.

II. THE MODEL AND ADIABATIC LEVELS

We consider the following dimensionless Schrödinger equation:

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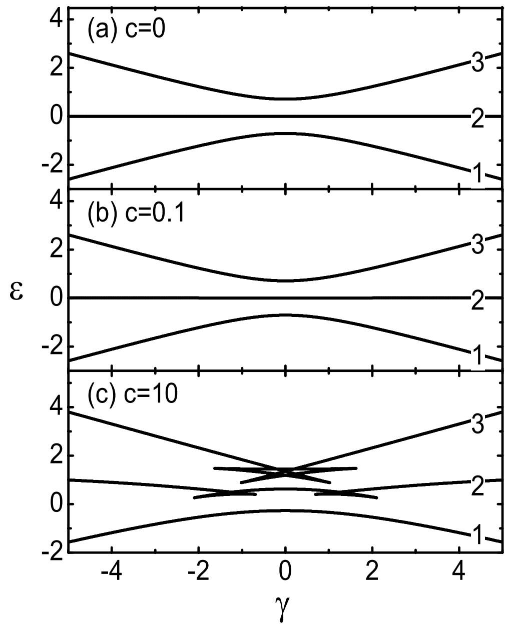

FIG. 1. Adiabatic energy levels at $v=1.0$: (a) linear case, (b) weak nonlinearity case of $c=0.1$, (c) strong nonlinearity case of $c=10.0$.

$$i\frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = H\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \tag{1}$$

with the Hamiltonian given by

$$H = \begin{pmatrix} \frac{\gamma}{2}+\frac{c}{4}|a_1|^2 & -\frac{v}{2} & 0 \\ -\frac{v}{2} & \frac{c}{4}|a_2|^2 & -\frac{v}{2} \\ 0 & -\frac{v}{2} & -\frac{\gamma}{2}+\frac{c}{4}|a_3|^2 \end{pmatrix}, \tag{2}$$

where v is the coupling constant between the neighboring levels; c is the nonlinear parameter; the energy bias γ is supposed to be adjusted by a linearly external field, i.e., $\gamma=\alpha t$, α is the sweeping rate; a_1, a_2, a_3 is probability amplitude in each level and the total probability $|a_1|^2+|a_2|^2+|a_3|^2$ is conserved and set to be a unit.

When the nonlinear parameter vanishes, our model reduces to the linear case and the adiabatic energy levels $\varepsilon(\gamma)=0,\pm\frac{1}{2}\sqrt{\gamma^2+2v^2}$ [Fig. 1(a)] derived by diagonalizing the Hamiltonian (2). Tunneling probability Γ_{nm} $(n,m=1,2,3)$ is defined as the occupation probability on the mth level at $\gamma\to+\infty$ for the state initially on the nth level at $\gamma\to-\infty$. For the linear case, the above system is solvable analytically and the tunneling probabilities can be explicitly expressed as [21]

$$\Gamma_{11} = \left[1-\exp\left(-\frac{\pi v^2}{2\alpha}\right)\right]^2, \tag{3}$$

$$\Gamma_{12} = 2\exp\left(-\frac{\pi v^2}{2\alpha}\right)\left[1-\exp\left(-\frac{\pi v^2}{2\alpha}\right)\right], \tag{4}$$

$$\Gamma_{13} = \exp\left(-\frac{\pi v^2}{\alpha}\right), \tag{5}$$

$$\Gamma_{22} = \left[1-2\exp\left(-\frac{\pi v^2}{2\alpha}\right)\right]^2. \tag{6}$$

The others are $\Gamma_{21}=\Gamma_{23}=\Gamma_{32}=\Gamma_{12}$, $\Gamma_{31}=\Gamma_{13}$, $\Gamma_{33}=\Gamma_{11}$ due to the symmetry of the levels.

With the presence of the nonlinear terms, we want to know how the tunneling dynamics in the above system is affected. In our discussions, the coupling parameter is set to be a unit, i.e., $v=1$. Therefore, the weak nonlinear case and the strong nonlinear case mean that $c\ll 1$ and $c\gg 1$, respectively. As to the external fields, we will consider three cases, namely, adiabatic limit, sudden limit, and moderate case, corresponding to $\alpha\ll 1$, $\alpha\gg 1$, and $\alpha\sim 1$, respectively.

Similar to the linear case, we need to analyze the adiabatic levels of the nonlinear model first. With $a_1=\sqrt{s_1}e^{i\theta_{a_1}}$, $a_2=\sqrt{1-s_1-s_2}e^{i\theta_{a_2}}$, $a_3=\sqrt{s_2}e^{i\theta_{a_3}}$, we introduce the relative phase $\theta_1=\theta_{a_1}-\theta_{a_2}$, $\theta_2=\theta_{a_3}-\theta_{a_2}$. In terms of s_1, θ_1 and s_2, θ_2, the nonlinear three-level system is cast into a classical Hamiltonian system,

$$H_e = \left(\frac{\gamma}{2}+\frac{c}{8}s_1\right)s_1 + \frac{c}{8}(1-s_1-s_2)^2 + \left(-\frac{\gamma}{2}+\frac{c}{8}s_2\right)s_2 - v\sqrt{(1-s_1-s_2)s_1}\cos\theta_1 - v\sqrt{(1-s_1-s_2)s_2}\cos\theta_2. \tag{7}$$

s_1, θ_1 and s_2, θ_2 are two pairs of canonically conjugate variables of the classical Hamiltonian system, governed by the following differential equations:

$$\dot{s}_1 = -v\sqrt{(1-s_1-s_2)s_1}\sin\theta_1, \tag{8}$$

$$\dot{\theta}_1 = \frac{\gamma}{2} - \frac{c}{4}(1-2s_1-s_2) - \frac{1-2s_1-s_2}{2\sqrt{(1-s_1-s_2)s_1}}v\cos\theta_1 + \frac{s_2}{2\sqrt{(1-s_1-s_2)s_2}}v\cos\theta_2, \tag{9}$$

$$\dot{s}_2 = -v\sqrt{(1-s_1-s_2)s_2}\sin\theta_2, \tag{10}$$

$$\dot{\theta}_2 = -\frac{\gamma}{2} - \frac{c}{4}(1-s_1-2s_2) + \frac{s_1}{2\sqrt{(1-s_1-s_2)s_1}}v\cos\theta_1 - \frac{1-s_1-2s_2}{2\sqrt{(1-s_1-s_2)s_2}}v\cos\theta_2. \tag{11}$$

The fixed points of the nonlinear classical Hamiltonian correspond to the eigenstates of the nonlinear three-level system. By setting $\dot{s}_1=\dot{s}_2=\dot{\theta}_1=\dot{\theta}_2=0$ in the equations (8)–(11) the eigenstates of the system are obtained. Accordingly, the eigenenergy is obtained by $\epsilon=H_e$, i.e., the energy levels are gained as shown in Fig. 1.

For weak nonlinearity, the levels' structure is similar to its linear counterpart [Fig. 1(b)]. For strong nonlinearity [Fig. 1(c)], in the middle level a double-loop topological structure emerges and in the upper level a butterfly structure appears.

The double-loop structure is also observed in Ref. [25]. Because of these topological distortions on the energy levels, we expect that the tunneling dynamics will dramatically change.

III. LANDAU-ZENER TUNNELING

In this section we study LZT in the nonlinear three-level system both numerically and analytically. First, we consider two limit cases: adiabatic limit and sudden limit, respectively. Then we will discuss the tunneling probability in the general case and investigate the symmetry of the tunneling probability.

A. Adiabatic limit ($\alpha \ll 1$)

In the adiabatic limit, the characters of the tunneling probabilities should be entirely determined by the topology of the energy levels and the eigenstates' properties (corresponding to the stability of the fixed points in the classical Hamiltonian system), according to the adiabatic theorem [26,27]. So, we expect that, for the weak nonlinearity case, an initial state starting from any level (upper, middle or lower) will follow the levels and evolve adiabatically, as a result, no quantum transition between levels occurs; for the strong nonlinearity, an initial state from the lower level is expected to evolve adiabatically keeping stay on the ground state, leading to zero adiabatic tunneling probability, whereas for the state initially from the middle or upper level, due to the topological change of the level, it cannot move smoothly from the left side to the right side. Transition to other levels happens at the tip of the loop or butterfly. Consequently, the adiabatic tunneling probability is expected to be nonzero.

However, the above picture is only partly corroborated by our directly solving the Schrödinger equation using fourth-fifth order Runge-Kutta adaptive-step algorithm, as shown in Fig. 2.

On the one hand, Fig. 2 clearly shows, for the strong nonlinearity case, as we expect, no tunneling for the state from the lower level, but a serious adiabatic tunneling is observed for the states from the upper two levels. In particular, we find that the tunneling probability as a function of the sweeping rate shows an irregular oscillation. This oscillation is also observed by Graefe *et al.* [25]. We associate this irregularity to the chaotic state. To demonstrate it, we plot in Fig. 3 the Poincare section of the trajectories for $c=10$ before and after the tip of the butterfly structure of the upper level in Fig. 1(c). It shows that, before the tip, the eigenstate corresponds to the fixed point surrounded by the quasiperiodic orbit, therefore it is stable. As the state evolves to the right-hand tip of the butterfly, it makes contact with the chaotic sea, after that the states becomes chaotic. The characteristics of the chaos is sensitive on the parameters, therefore the chaotic state is responsible for the irregular tunneling probability exposed by Figs. 2(h) and 2(i).

On the other hand, Fig. 2 also shows that for the weak nonlinearity, even though the adiabatic levels keep the same topological structure as the linear case, there is still nonzero tunneling probability for the state started from the middle

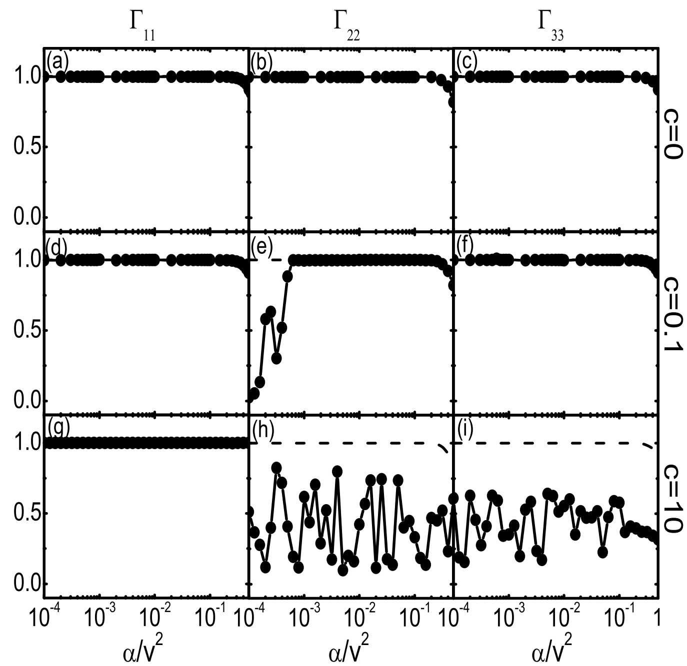

FIG. 2. The tunneling probability Γ_{11}, Γ_{22}, Γ_{33} (full circles) as functions of α for different nonlinear parameters at $v=1.0$. The dashed lines represent the results from the linear Landau-Zener model for comparison.

level. The tunneling also shows some kind of irregularity. This phenomenon is counter to our naive conjecture from observing the topological structure of the adiabatic levels.

To explain this unusual phenomenon, we need to make a detailed analysis on the property of the fixed points of the classical system Hamiltonian (7), corresponding to the eigenstates of the middle level.

We plot quantity s_1 as the function of γ in Figs. 4(a) and 4(b), we see the adiabatic evolution of the eigenstate breaks down around $\gamma=-2$ due to the nonlinearity [Fig. 4(b)]. This adiabaticity breakage is caused by the change on the property of the fixed point corresponding to the eigenstate of the middle level. This is revealed by investigating the Hamiltonian-Jaccobi matrix obtained by linearizing the nonlinear equations (8)–(11) at fixed points,

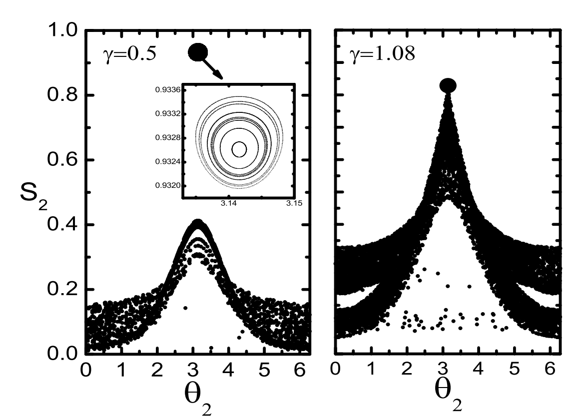

FIG. 3. Poincare section of the trajectories for $c=10$ before and after the tip of the butterfly structure of the upper level in Fig. 1(c).

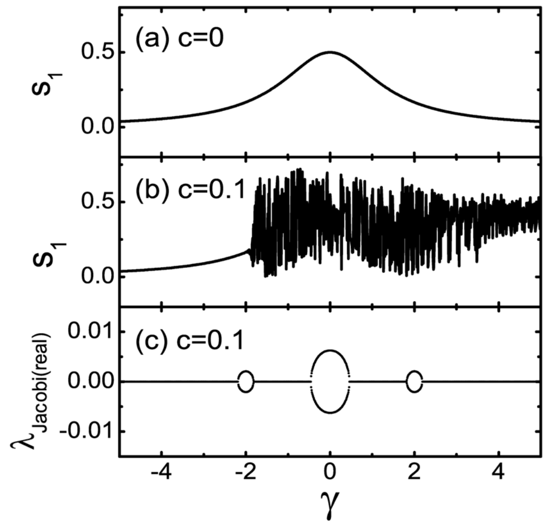

FIG. 4. The variety of s_1 with γ when the eigenstate $(0,1,0)^T$ evolves adiabatically at $v=1.0$, $\alpha=0.0001$. (a) Linear case. (b) Nonlinear case at $c=0.1$. (c) The real parts of the eigenvalues of H_J.

$$H_J=\begin{pmatrix} -\frac{\partial^2 H_e}{\partial s_1\partial\theta_1} & -\frac{\partial^2 H_e}{\partial^2\theta_1} & -\frac{\partial^2 H_e}{\partial s_2\partial\theta_1} & -\frac{\partial^2 H_e}{\partial\theta_2\partial\theta_1} \\ \frac{\partial^2 H_e}{\partial^2 s_1} & \frac{\partial^2 H_e}{\partial\theta_1\partial s_1} & \frac{\partial^2 H_e}{\partial s_2\partial s_1} & \frac{\partial^2 H_e}{\partial\theta_2\partial s_1} \\ -\frac{\partial^2 H_e}{\partial s_1\partial\theta_2} & -\frac{\partial^2 H_e}{\partial\theta_1\partial\theta_2} & -\frac{\partial^2 H_e}{\partial s_2\partial\theta_2} & -\frac{\partial^2 H_e}{\partial^2\theta_2} \\ \frac{\partial^2 H_e}{\partial s_1\partial s_2} & \frac{\partial^2 H_e}{\partial\theta_1\partial s_2} & \frac{\partial^2 H_e}{\partial^2 s_2} & \frac{\partial^2 H_e}{\partial\theta_2\partial s_2} \end{pmatrix}. \tag{12}$$

We solve the eigenvalues of H_J for different γ and plot our results in Fig. 4(c). These eigenvalues can be real, complex or pure imaginary. Only pure imaginary eigenvalues correspond to the stable fixed point, others indicate the unstable ones. In Fig. 4(c), we can see the eigenvalues are complex numbers (i.e., their real parts are not zero) around $\gamma=0,\pm 2$. The corresponding fixed points are unstable. For other regions, the eigenvalues of H_J are pure imaginary. Therefore, even though no topological structure changes on the level structures, the instability of the fixed point corresponding to the middle level leads to the breakdown of the adiabaticity manifesting the irregular nonzero tunneling probability exposed by Fig. 2(e) in the adiabatic limit.

The above instability mechanism occurs for any smaller nonlinear perturbation. Let us make some analytic deduction as follows. Note that the fixed points of equations (8)–(11) can be accurately calculated if $c=0$: $s_1^0=s_2^0=\frac{1}{2+\gamma^2}$, $\theta_1^0=0$, $\theta_2^0=\pi$ for $\gamma>0$, and $\theta_1^0=\pi$, $\theta_2^0=0$ for $\gamma<0$. By employing the perturbation theory using c as a small parameter, we can get the fixed points for small c: $s_1^0=\frac{1}{2+\gamma^2}-\frac{(1-\gamma^2)^2}{4(2+\gamma^2)}c\gamma$, $s_2^0=\frac{1}{2+\gamma^2}+\frac{(1-\gamma^2)^2}{4(2+\gamma^2)}c\gamma$, $\theta_1^0=0$, $\theta_2^0=\pi$ for the nonlinear case. Substituting them into Eq. (12), we can obtain the eigenvalues of H_J by solving the following quartic equation:

$$(64+1280\gamma^4)x^4+(64+c^2+1344\gamma^2)x^2+(16+c^2+352\gamma^2)=0.$$

The useful quadratic discriminant is $\Delta=4096\gamma^4-2432c^2\gamma^2+(c^4-128c^2)$. In the linear case, $c=0$, $\Delta=4096\gamma^4$ is always larger than zero, which means that the solutions for x are pure imaginary, thus the fixed points are stable. For small c, $\lim_{\gamma\to 0}\Delta<0$, the real part of the solutions $\sim c/16$, while the imaginary part $\sim\sqrt{2}/2$. As a result, the fixed point corresponding to the middle level becomes unstable around $\gamma=0$ for any small nonlinearity, implying the breakdown of the adiabatic evolution of states on the middle level.

B. Sudden limit ($\alpha \gg 1$)

The sudden limit corresponds to nonadiabatic LZT. The tunneling probability does not relate much to the structure of the levels. In this limit a weak nonlinearity does not affect the tunneling probability, however, a strong nonlinearity can dramatically influence the tunneling dynamics.

In this limit, we can derive the analytical expression of the tunneling probabilities using the stationary phase approximation (SPA). As a demonstration, we concentrate on the middle level, i.e., to calculate Γ_{22} which is equal to $1-\Gamma_{21}-\Gamma_{23}$. Because of the large sweeping rate α, a quantum state would stay on the initial level most of the time. Thus the amplitudes a_1 and a_3 in the Schrödinger equation (1) remain small and $|a_2|\sim 1$ all the time. A perturbation treatment of the problem becomes adequate.

We begin with the variable transformation,

$$a_1=a_1'\exp\left[-i\int_0^t\left(\frac{\gamma}{2}+\frac{c}{4}|a_1|^2\right)dt\right], \tag{13}$$

$$a_2=a_2'\exp\left[-i\int_0^t\left(\frac{c}{4}|a_2|^2\right)dt\right], \tag{14}$$

$$a_3=a_3'\exp\left[-i\int_0^t\left(-\frac{\gamma}{2}+\frac{c}{4}|a_3|^2\right)dt\right]. \tag{15}$$

As a result, the diagonal terms in the Hamiltonian are transformed away, and the evolution equations of a_1', a_2', a_3' become

$$\frac{da_1'}{dt}=-\frac{v}{2i}a_2'\exp\left[i\int_0^t\left(\frac{\gamma}{2}+\frac{c}{4}(|a_1|^2-|a_2|^2)\right)dt\right],$$

$$\frac{da_2'}{dt}=-\frac{v}{2i}a_1'\exp\left[i\int_0^t\left(-\frac{\gamma}{2}+\frac{c}{4}(|a_2|^2-|a_1|^2)\right)dt\right]-\frac{v}{2i}a_3'\exp\left[i\int_0^t\left(\frac{\gamma}{2}+\frac{c}{4}(|a_2|^2-|a_3|^2)\right)dt\right],$$

$$\frac{da_3'}{dt}=-\frac{v}{2i}a_2'\exp\left[i\int_0^t\left(-\frac{\gamma}{2}+\frac{c}{4}(|a_3|^2-|a_2|^2)\right)dt\right].$$

We need to calculate the above integrals self-consistently. Due to the large α, the nonlinear term in the exponent gen-

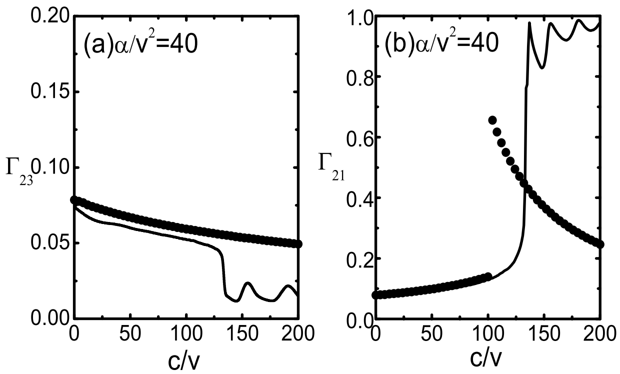

FIG. 5. Comparison between our analytic results using SPA (full circles and crosses) and the numerical integration of the Schrödinger equation (1) (solid lines).

erally gives a rapid phase oscillation, which makes the integral small. The dominant contribution comes from the stationary point t_0 of the phase around which we have

$$a_1' = -\frac{v}{2i}\int_{-\infty}^{t} dt \exp\left[i\int_0^t \left(\frac{\gamma}{2} + \frac{3c}{4}|a_1|^2 - \frac{c}{4}\right)dt\right], \quad (16)$$

$$\frac{\gamma}{2} + \frac{3c}{4}|a_1|^2 - \frac{c}{4} = \alpha_1(t-t_0), \quad (17)$$

with

$$\alpha_1 = \frac{\alpha}{2} + \frac{3c}{4}\left(\frac{d|a_1|^2}{dt}\right)_{t_0}. \quad (18)$$

Since $|a_1|^2 = |a_1'|^2$, then we have

$$|a_1|^2 = \left(\frac{v}{2}\right)^2 \left| \int_{-\infty}^{t} dt \exp\left(\frac{i}{2}\alpha_1(t-t_0)^2\right)\right|^2. \quad (19)$$

This expression can be differentiated and evaluated with results at time t_0. A standard Fresnel integral with the result $\left(\frac{d|a_1|^2}{dt}\right)_{t_0} = \left(\frac{v}{2}\right)^2\sqrt{\frac{\pi}{\alpha_1}}$ is obtained. Combining this with the relation (15), we come to a closed equation for α_1,

$$\alpha_1 = \frac{\alpha}{2} + \frac{3c}{4}\left(\frac{v}{2}\right)^2\sqrt{\frac{\pi}{\alpha_1}}. \quad (20)$$

For given α, c, v, α_1 the above equation can be obtained. The tunneling probability is

$$\Gamma_{23} = |a_1|^2_{+\infty} = \frac{\pi v^2}{2\alpha_1}. \quad (21)$$

The alliance of Eqs. (20) and (21) gives the analytic expression on the tunneling probability Γ_{23} in the sudden limit. Compared with our numerical simulation it shows good agreement at $c/v<130$, $c/v>130$ a clear deviation is observable [Fig. 5(a)]. It is due to the resonance between the "internal field" and the external field leads to the invalidity of our assumption $|a_2|\sim 1$, as we show later.

Similarly, to calculate Γ_{21}, we consider the following equations:

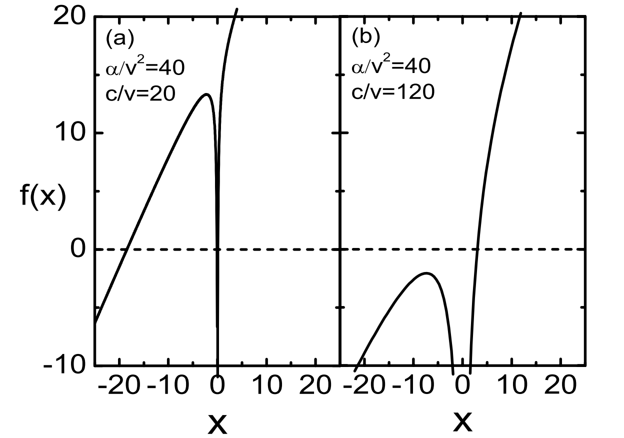

FIG. 6. The plot of function $f(x)=x+\frac{\alpha}{2}-\frac{3cv^2}{16}\sqrt{\frac{\pi}{|x|}}$.

$$a_3' = -\frac{v}{2i}\int_{-\infty}^{t} dt \exp\left[i\int_0^t \left(-\frac{\gamma}{2} + \frac{3c}{4}|a_3|^2 - \frac{c}{4}\right)dt\right], \quad (22)$$

$$-\frac{\gamma}{2} + \frac{3c}{4}|a_3|^2 - \frac{c}{4} = \alpha_3(t-t_0), \quad (23)$$

$$\alpha_3 = -\frac{\alpha}{2} + \frac{3c}{4}\left(\frac{v}{2}\right)^2\sqrt{\frac{\pi}{|\alpha_3|}}. \quad (24)$$

Differently, in this case we may have three stationary phase points that are solutions of Eq (24) when $c < \frac{8}{27}\sqrt{\frac{6}{\pi}}\frac{\alpha^{3/2}}{v^2}$, but only one solution otherwise, as demonstrated in Fig. 6. We denote them as α_{31}, α_{32}, α_{33} from smallest to largest. For small c, α_{31} is around $-\alpha/2$, and the other two solutions are located at the two sides of the origin. In this case, we simply take $\alpha_3 = \alpha_{31}+\alpha_{32}+\alpha_{33}$.

Then

$$a_3' = -\frac{v}{2i}\int_{-\infty}^{t} dt \exp\left(i\int_0^t \frac{\alpha_3}{2}(t-t_0)^2 dt\right). \quad (25)$$

The tunneling probability is

$$\Gamma_{21} = |a_3|^2_{+\infty} = \left|\frac{\pi v^2}{2\alpha_3}\right|. \quad (26)$$

The alliance of Eqs. (24) and (26) will give the approximate solution of the Γ_{21}. Compared with our numerical simulation it shows a good agreement at $c/v<110$, whereas for $c/v>110$ a clear deviation is observed [Fig. 5(b)].

What happens around $c/v=110$ that leads to the breakdown of our stationary phase approximation? The reason is the resonance between the "internal field" and the external field. Let us recall the exponent in the integrand of Eq. (22), we find the effective sweeping rate should be the difference between the change rate of the "internal field" (i.e., $|a_3|$) and the sweeping rate of the external field. At $c/v=110$, we find the two frequencies become almost identical, leading to the invalidity of SPA assumption of rapid phase oscillation. This resonance is accompanied by the bifurcation of the stationary

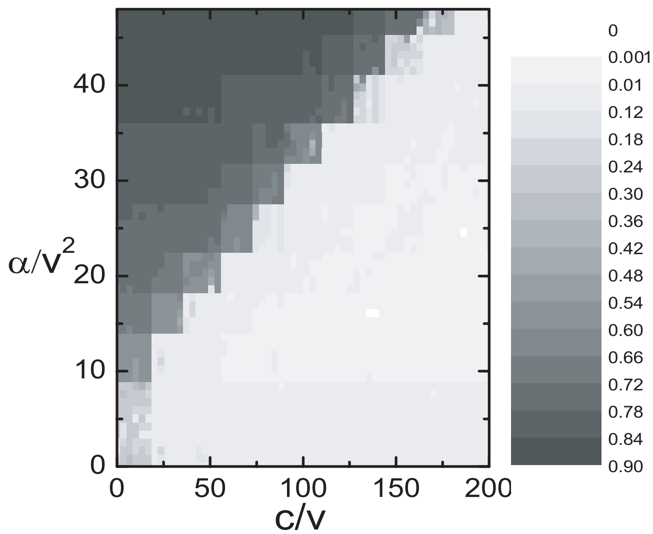

FIG. 7. (Color online) The contour plot of tunneling probability Γ_{22} as the functions of the scaled sweeping rate and nonlinearity.

phase points. Crossing $c/v=110$ we observe that the number of stationary phase points changes from three to one, as shown in Fig. 6. The resonance breaks the SPA leading to a serious transition from level 2 to level 1, consequently, at $c/v>130$, our assumption $|a_2|\sim 1$ becomes invalid, and our approximation on the Γ_{23} from SPA is no longer good as shown in Fig. 5(a).

C. General property of the nonlinear tunneling probability

The nonlinear tunneling probability as the function of the two scaled quantities α/v^2 and c/v show many unusual properties. Taking mid-level tunneling Γ_{22}, for example, we make a large numerical exploration for a wide range of parameters, to demonstrate the general property of the nonlinear tunneling probability in Fig. 7. In general, increasing the sweeping rate will reduce the probability of tunneling to the upper or lower level and the positive nonlinearity usually suppresses the probability of the state's staying in the middle level, because of that the nonlinearity with positive c can be regarded as a kind of repulsive potential. This repulsive self-interaction makes particles tend to transition to a lower level more easily, and this transition becomes more serious at the occurrence of the resonance between the "internal field" and the external field. The occurrence of the resonance is clearly exposed by the boundary between the white regime and the dark regime in Fig. 7. In the white regime, due to the resonance, the nonlinearity dramatically changes the tunneling probability.

The other issue we want to address is the symmetry. The nonlinearity makes levels deform and therefore break the symmetry between the upper level and lower level; consequently, the relations $\Gamma_{21}=\Gamma_{23}=\Gamma_{32}=\Gamma_{12}$, $\Gamma_{31}=\Gamma_{13}$, $\Gamma_{33}=\Gamma_{11}$ that hold in the linear case, break in the presence of the nonlinearity. For our three-level system, the symmetry breaking is clearly exposed by Fig. 8 showing the tunneling probability Γ_{nm} as the functions of α/v^2 for $c=10$, $c=40$. In the linear case, we have $\Gamma_{21}=\Gamma_{23}=\Gamma_{32}=\Gamma_{12}$, however, with the

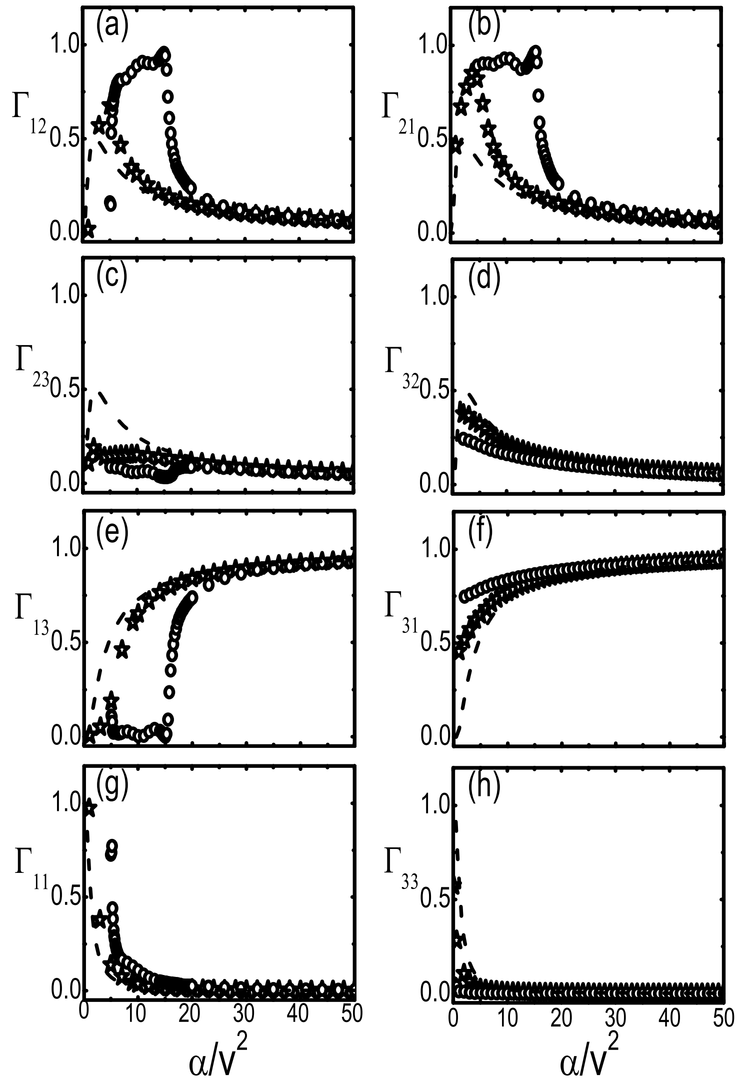

FIG. 8. Γ_{nm} as the function of α for $c=10$ (open pentacles), $c=40$ (open circles) at $v=1.0$. Dashed line denotes the linear case for comparison.

presence of the nonlinear, Γ_{12}, Γ_{21} increases whereas the Γ_{23}, Γ_{32} decreases. A similar thing happens for Γ_{31}, Γ_{13} and Γ_{33}, Γ_{11}. The above symmetry breaking may be observed experimentally [28].

IV. CONCLUSION AND APPLICATION

In conclusion, we have made a comprehensive analysis of the Landau-Zener tunneling in a nonlinear three-level system, both analytically and numerically. Many tunneling properties are demonstrated and behind the dynamical mechanism is revealed.

Our model can be directly applied to the triple-well trapped BEC and explains the tunneling dynamics between the traps [24,25]. In a triple trap $v(r)$, a BEC is described by Gross-Pitaevskii equation (GPE) $i\hbar\frac{\partial\Psi(r,t)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi(r,t)+[v(r)+g_0|\Psi(r,t)|^2]\Psi(r,t)$ under the mean-field approximation, where $g_0=\frac{4\pi\hbar^2 aN}{m}$, m is the atomic mass and a is the scattering length of the atom-atom interaction. The wave function $\Psi(r,t)$ of GPE is the superposition of three wave functions describing the condensate in each trap [12], i.e., $\Psi(r,t)=\psi_1(t)\phi_1(r)+\psi_2(t)\phi_2(r)+\psi_3(t)\phi_3(r)$. When we study the tunneling of three weakly coupled BEC in traps 1, 2, and 3, the dynamics of the system is described by the nonlinear Schrödinger equation with the Hamiltonian,

$$H=\begin{pmatrix} E_1^0+c_1|\psi_1|^2 & -K_{12} & 0 \\ -K_{12} & E_2^0+c_2|\psi_2|^2 & -K_{23} \\ 0 & -K_{23} & E_3^0+c_3|\psi_3|^2 \end{pmatrix}, \quad (27)$$

where $E_\alpha^0=\int\left[\frac{\hbar^2}{2m}\left|\nabla\phi_\alpha\right|^2+v(r)\left|\phi_\alpha\right|^2\right]dr$ $(\alpha=1,2,3)$ is the ground-state energy for each trap. $c_\alpha=\int g_0|\phi_\alpha|^4dr$ $(\alpha=1,2,3)$ stands for atom-atom interaction, i.e., nonlinear parameter. $K_{12}=-\int\left[\frac{\hbar^2}{2m}\nabla\phi_1\nabla\phi_2+v(r)\phi_1\phi_2\right]dr$ is the coupling matrix element between traps 1 and 2. $K_{23}=-\int\left[\frac{\hbar^2}{2m}\nabla\phi_2\nabla\phi_3+v(r)\phi_2\phi_3\right]dr$ is the coupling matrix element between traps 2 and 3. For simplicity, we only consider the case that these two coupling matrix elements are the same and there is no coupling between traps 1 and 3, i.e., $K_{12}=K_{23}=K$, $K_{13}=0$. The energy bias can be adjusted by tilting the trapping well and the nonlinearity can be adjusted by the Feshbach resonance technique. We hope our theory will stimulate the experiment in this direction.

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