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Nonlocal effect of a bipartite system induced by local cyclic operation

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Abstract. – The state of a bipartite system may be changed by a cyclic operation applied on one of its subsystems. The change is a nonlocal effect, and can be detected only by measuring the two parts jointly. By employing the Hilbert-Schmidt metric, we can quantify such nonlocal effects via measuring the distance between the initial and final state. We show that this nonlocal property can be manifested not only by entangled states but also by the disentangled states which are *classically correlated*. Furthermore, we study the effect for the system of two qubits in detail. It is interesting that the nonlocal effect of disentangled states is limited by $1/\sqrt{2}$, while the entangled states can exceed this limit and reach 1 for maximally entangled states.

Entanglement is a striking feature of composite quantum system, which has no classical analog. Historically, since Einstein, Podolsky, and Rosen (EPR) published their famous gedanken experiment in 1935 [1], entanglement had become a key issue in the debate about the foundations and interpretation of quantum mechanics. The appeal was changed dramatically in 1964 by John Bell's theorem [2]. Bell inequalities [3–5] bound the correlations within any local and realistic theory. According to Bell's theorem, there are some states of a composite system; when measurements are performed on the two subsystems separated in space their results are correlated in a manner which cannot be explained by local hidden variables models. For quite a long time, entanglement was widely believed to be equivalent to the violation of a Bell inequality. Whereas until 1989, Werner proved that even if Bell's inequality is satisfied by a given composite system, there is no guarantee that its state can be prepared by two distant observers who receive instructions from a common source [6]. Thereafter, it is generally recognized in the community that a quantum state of a system composed of two subsystems is called entangled if and only if it is not a separable state, *i.e.* it cannot be expressed as

$$\sigma_s = \sum_l p_l \left|\psi_A^l\right\rangle \left\langle\psi_A^l\right| \otimes \left|\psi_B^l\right\rangle \left\langle\psi_B^l\right|, \tag{1}$$

where p_l are positive real numbers and $\sum_l p_l = 1$. A separable system always satisfies Bell's inequality, but the converse is only true for pure states.

Nowadays, quantum entanglement has become not only a tool for exposing the weirdness of quantum mechanics [1,2], but also a more powerful resource in a number of applications [7–10]. One of the most important problem is the characterization and classification of mixed entangled states. The most prominent criterion for deciding whether a given state is entangled or not is known as positive partial transpose (PPT) test [11]. For systems consisting of two qubits or a qubit and a qutrit, the PPT test is the necessary and sufficient conditions for the presence of entanglement. For systems of more than three parties, or for higher-dimensions system, the PPT test is only a sufficient criterion, since there exist PPT-entangled states [12].

Entanglement witness (EW) are operators that are designed to detect the presence of entanglement in a state [12–14]. A Hermitian operator W is called entanglement witness if it has a positive expectation value for all separable states, $\mathrm{Tr}[W\sigma_s] \geq 0$, while there exists at least one state ρ for which $\mathrm{Tr}[W\rho] < 0$. Therefore, the state with negative expectation should be entangled and it is said to be detected by the witness W. Entanglement witness is an important concept and provides a very useful tool for the experimental detection of entanglement [15,16].

In this letter, we generalize the nonlocal effect manifested by the maximally entangled state in the quantum dense coding process [9] to any state of a bipartite system. By employing the Hilbert-Schmidt distance [10], we quantify this nonlocal effect. We find such nonlocal effect vanishes for product states but does not vanish for *classically correlated* states (the disentangled states which cannot be factorized) [6]. Furthermore, we investigate this effect for two qubits in detail. The interesting thing is that the nonlocal effect of disentangled states is bounded by $1/\sqrt{2}$, but for entangled states it can exceed this limit and reach 1 for maximally entangled states. Hence, the nonlocal effect can be used to detect entanglement for some states.

At first, let us remind the reader of the dense coding process. Quantum dense coding [9] enables the communication of two bits of classical information by transferring one qubit between two parties who share a maximally entangled pair. At the beginning, one party, "Alice", prepares a maximally entangled pair and sends one of the particles to another party, "Bob". Bob applies one of four possible unitary operations, and sends it back to Alice. By measuring the two particles jointly, the outcomes of these measurements tell her which of the four operations Bob applied and the corresponding two-bit classical number.

In the quantum dense coding process, the subsystem of the treated particle is not changed by the local unitary operation (or in other words, the marginal statistics of measurements of the treated particle is unperturbed after the local operation applied by Bob [9]). The untreated particle is fixed all the time. So, the states of both two subsystems are not changed after the local unitary operation. However, the state of the whole system is changed after the operation applied by Bob. The shift of the state of the whole system is a nonlocal effect, since it can be observed only by measuring the two particles jointly.

Now, let us consider more general cases. Assuming Alice and Bob share a system compounded by two particles A (in Alice's hand) and B (in Bob's hand), which is in a state described by the density operator ρ. The subsystems are described by the reduced density operators, $\rho_0^A = \mathrm{tr}_B(\rho_0)$ and $\rho_0^B = \mathrm{tr}_A(\rho_0)$, respectively. Bob applies a local unitary operation U^B on the particle in his hand which satisfies

$$[\rho_0^B, U^B] = 0. \tag{2}$$

Obviously, the subsystem is not changed by such an operation. However, the whole system will not always return to its initial state, *i.e.*, $\rho_0 \neq \left(I \otimes U^B\right) \rho_0 \left(U^{B\dagger} \otimes I\right)$ in general. The change between the final and initial states cannot be detected locally. For convenience, we denote the operation satisfies condition (2) as a local cyclic operation.

To denote the difference between the initial and final states, we introduce the distance between two states [10]. Here, we employ the Hilbert-Schmidt metric, $D(\rho_1||\rho_2) = \mathrm{Tr}|\rho_1 - \rho_2|^2$, to measure the distance between quantum states ρ_1 and ρ_2, where $|X| = \sqrt{X^+X}$. The Hilbert-Schmidt metric $D(\rho_1||\rho_2) \geq 0$ with the equality saturated iff $\rho_1 = \rho_2$ [17,18]. Then, we quantify the shift between the initial state ρ_0 and the final state $\rho_f = \left(I \otimes U^B\right) \rho_0 \left(U^{B\dagger} \otimes I\right)$ by

$$d(\rho_0, U^B) = \sqrt{D(\rho_0||\rho_f)/2}. \tag{3}$$

By considering $\mathrm{Tr}(\rho_0^2) = \mathrm{Tr}(\rho_f^2)$, we can obtain

$$d(\rho_0, U^B) = \sqrt{\mathrm{Tr}(\rho_0^2) - \mathrm{Tr}(\rho_0\rho_f)}. \tag{4}$$

Obviously, $d(\rho_0, U^B(\tau)) \leq 1$ and $d(\rho_0, U^B) = 1$ only when the initial state is a pure state and it is orthonormal with the final state. In fact, for $\rho_0 = |\psi\rangle\langle\psi|$ is a pure state, we can have $d(\rho_0, U^B) = \sqrt{1 - F(\rho_0, \rho_f)}$, where $F(\rho_0, \rho_f) = \langle\psi|\rho_\tau|\psi\rangle$ is just the Bures fidelity [10].

Therefore, we have $0 \leq d(\rho_0, U^B(\tau)) \leq 1$, and the equality on the left is saturated iff $\rho_0 = \rho_f$. Hence, $d(\rho_0, U^B(\tau))$ can be used to quantify the nonlocal shift of the state induced by the local cyclic operation. For convenience, we use $d_{\max}(\rho_0)$ to denote the maximum value of $d(\rho_0, U^B)$ over all the local operations U^B which satisfy (2).

A state of a bipartite system can be written in the following form:

$$\begin{aligned} \rho_0 &= \frac{1}{N_A N_B}\left[I^A \otimes I^B + \sqrt{\frac{N_A(N_A-1)}{2}}\boldsymbol{r}^A \cdot \vec{\lambda}^A \otimes I^B + \right. \\ &\left. + \sqrt{\frac{N_B(N_B-1)}{2}} I^A \otimes \boldsymbol{r}^B \cdot \vec{\lambda}^B + \sqrt{\frac{N_A(N_A-1)N_B(N_B-1)}{4}} \beta_{ij}\lambda_i^A \otimes \lambda_j^B\right], \end{aligned} \tag{5}$$

where N_A and N_B are the dimensions of each subsystems, $\vec{\lambda}^A = (\lambda_i^A; i = 1, 2, \cdots, N_A^2 - 1)$ and $\vec{\lambda}^B = (\lambda_i^A; i = 1, 2, \cdots, N_B^2 - 1)$ are the generators of $SU(N_A)$ and $SU(N_B)$ respectively, $\boldsymbol{r}^A = (r_i^A; i = 1, 2, \cdots, N_A^2 - 1)$ and $\boldsymbol{r}^B = (r_i^B; i = 1, 2, \cdots, N_B^2 - 1)$ are two Bloch vectors, and β_{ij} are $(N_A^2 - 1)(N_B^2 - 1)$ real numbers which constructs the so-called correlation matrix $\beta = \{\beta_{ij}\}$. The states of the two subsystems are described by the following reduced density operators:

$$\begin{aligned} \rho_0^A &= \frac{1}{N_A}\left[I^A + \sqrt{\frac{N_A(N_A-1)}{2}}\boldsymbol{r}^A \cdot \vec{\lambda}^A\right], \\ \rho_0^B &= \frac{1}{N_B}\left[I^B + \sqrt{\frac{N_B(N_B-1)}{2}}\boldsymbol{r}^B \cdot \vec{\lambda}^B\right]. \end{aligned} \tag{6}$$

It is easy to see $\rho_f^B = U^B \rho_0^B U^{B+} = \rho_0^B$ for the local cyclic operation defined by (2). An interesting case is for the states of which $|\boldsymbol{r}^A| = |\boldsymbol{r}^B| = 0$. For such states, any local unitary operation is a local cyclic operation. The maximally entangled states and the Werner states [6] are belong to this case.

With the condition (2), and the trace relation $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$ for the generators of $SU(N)$, one can obtain

$$d(\rho_0, U^B) = \sqrt{\frac{(N_A-1)(N_B-1)}{N_A N_B}\left(|\beta|^2 - \sum_{i,j}\beta_{ij}\beta_{ij}^f\right)}, \tag{7}$$

in which $|\beta|^2 = \sum_{i,j} \beta_{ij}\beta_{ij}$ and β^f_{ij} are the elements of correlation matrix of the final state, which are defined by the following relations:

$$\beta^f_{ij}\lambda^A_i \otimes \lambda^B_j = \beta_{ij}\lambda^A_i \otimes U^B\lambda^B_j U^{B^+}. \tag{8}$$

In the above calculation, we have used the relation $|\beta| = |\beta^f|$. If we regard the expression $\sum_{i,j} \beta_{ij}\beta^f_{ij}$ as the inner product of two vectors, so $\sum_{i,j} \beta_{ij}\beta^f_{ij} \leq |\beta|^2$. Then we can easily prove that $d(\rho_0, U^B) \geq 0$ and $d(\rho_0, U^B) = 0$, if an only if $\rho_f = \rho_0$.

Theorem. For the state (5) of which $\beta_{ij} = \alpha r^A_i r^B_j$ $(0 \leq \alpha \leq 1)$, $d_{\max}(\rho_0) = 0$, *i.e.*, such state cannot have the nonlocal shift induced by a local cyclic operation.

Proof. From eq. (6), the condition, $[\rho^B_0, U^B] = 0$, is equivalent to

$$\boldsymbol{r}^B \cdot \vec{\lambda}^B = U^B \left(\boldsymbol{r}^B \cdot \vec{\lambda}^B\right) U^{B^+}. \tag{9}$$

If $\beta_{ij} = \alpha r^A_i r^B_j$ $(0 < \alpha \leq 1)$, the correlation matrix $\beta = \alpha \boldsymbol{r}^A \cdot \vec{\lambda}^A \otimes \boldsymbol{r}^B \cdot \vec{\lambda}^B$. Then, $\beta^f = \alpha \boldsymbol{r}^A \cdot \vec{\lambda}^A \otimes U^B \boldsymbol{r}^B \cdot \vec{\lambda}^B U^{B^+}$. From (9), we obtain $\beta^f = \beta$, *i.e.*, $\beta^f_{ij} = \alpha r^A_i r^B_j = \beta_{ij}$. Hence, $d(\rho_0, U^B) = 0$. The proof is ended.

From this theorem, we know that the nonlocal shift cannot be observed for any product state ($\rho = \rho^A \otimes \rho^B$). Hence, the effect cannot be observed for the disentangled pure states, since they are product states.

It is well known that some disentangled mixed states are able to exhibit non-locality [6], which are the so-called *classically correlated* states. A property of this nonlocal effect is that the effect can be observed for the disentangled states which are *classically correlated.* A state ρ is classically correlated if it can be expressed as

$$\rho = \sum_l^M p_l \left|\psi^l_A\right\rangle \left\langle\psi^l_A\right| \otimes \left|\psi^l_B\right\rangle \left\langle\psi^l_B\right|, \tag{10}$$

with $M > 1$, where p_l are positive real numbers and $\sum_l^M p_l = 1$. Denoting $\boldsymbol{r}^{A_l,B_l}$ as the Bloch vectors corresponding to $\left|\psi^l_A\right\rangle \left\langle\psi^l_A\right|$ and $\left|\psi^l_B\right\rangle \left\langle\psi^l_B\right|$, respectively, the Bloch vectors for such state are $\boldsymbol{r}^A = \sum_l^M p_l \boldsymbol{r}^{A_l}$ and $\boldsymbol{r}^B = \sum_l^M p_l \boldsymbol{r}^{B_l}$, and $\beta_{ij} = \sum_l^M p_l r^{A_l}_i r^{B_l}_j$. Then from eq. (8) we can see that, for a local operation U^B satisfying (9), $\beta^f_{ij} \neq \beta_{ij}$ unless $M = 1$ (or $\boldsymbol{r}^{A_1} = \boldsymbol{r}^{A_2} = \cdots = \boldsymbol{r}^{A_M}$). Therefore, this nonlocal effect can be observed for the disentangled states which are *classically correlated.*

To make the above discussion more clear, we study this nonlocal effect for two qubits in detail. For qubits, it is common to choose the generators of $SU(2)$ as Pauli matrices, *i.e.*, $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The unitary operation applied on the subsystem can be expressed as $U^B = e^{i\frac{\varphi}{2}\boldsymbol{u}\cdot\vec{\sigma}}$ where $\boldsymbol{u}$ is a unit vector. At first, we discuss the case for pure states. For notational convenience, we assume the initial state as follows:

$$|\psi\rangle = k_1 |00\rangle + k_2 |11\rangle, \tag{11}$$

with $|k_1|^2 + |k_2|^2 = 1$. For $|k_1| = |k_2| = \frac{\sqrt{2}}{2}$, the state is a maximally entangled state. The states of two subsystems are

$$\rho_A = \rho_B = \frac{1}{2}(\left[I + \left(|k_1|^2 - |k_2|^2\right)\sigma_3\right]). \tag{12}$$

It is easy to prove that the unitary operation which satisfies (2) can be expressed as

$$U^B = e^{i\varphi/2\sigma_3}. \tag{13}$$

Then, $\rho_f = |\psi_f\rangle \langle\psi_f|$ with $|\psi_f\rangle = I \otimes U^B|\psi\rangle$. From eq. (4), we obtain

$$d(\psi, U^B) = 2|k_1 k_2 \sin\varphi/2|. \tag{14}$$

Obviously, $d_{\max}(\psi) = 2|k_1k_2|$, which just equals the degree of entanglement for pure state of two qubits suggested in refs. [19–21]. The definition of the entanglement degree is consistent with the violation of Bell's inequality. The optimal form of Bell's inequality for the entangled qubits is known as the Clauser-Horne-Shimony-Holt (CHSH) inequality [3]. It has been shown by Gisin [22] that any entangled pure state of qubit pairs can violate the CHSH inequality and the maximum violation is $B_{\max}(\psi) = 2\sqrt{1+4|k_1k_2|^2}$. Obviously, $B_{\max}(\psi) = 2\sqrt{1+d^2_{\max}(\psi)}$. Therefore, the nonlocal effect can be used to quantify the entanglement of pure states of qubit pairs.

Although disentangled states may have such nonlocal effect, the maximum value for disentangled states is bounded and this boundary can be exceeded by entangled states. For the disentangled states expressed by eq. (10), $\beta_{ij} = \sum_l^M p_l r_i^{A_l} r_j^{B_l}$. Then, one can have $|\beta|^2 \leq 1$, since $|\boldsymbol{r}^{A_l}| = |\boldsymbol{r}^{B_l}| = 1$. On the other hand, $\sum_{i,j} \beta_{ij}\beta^f_{ij} \geq -|\beta|^2$ for qubit [23]. Therefore, from eq. (7) we can immediately obtain

$$d_{\max} \leq \frac{1}{\sqrt{2}} \tag{15}$$

for the states which are *classically correlated*. Therefore, the nonlocal shift of disentangled states is bounded by $1/\sqrt{2}$.

From (14) we know that the shifts of entangled states can exceed this limit and reach 1 for maximally entangled states. It is interesting that the entangled states violate the *classically correlated* states by the factor $\sqrt{2}$, which consists with the CHSH inequality.

Any state violating the inequality (15) is entangled. Therefore, the nonlocal effect can be employed to detect entanglement of some states. On the other hand, because $d_{\max} = 0$ for the product states, we can use this nonlocal effect to identify product states.

It is not difficult to observe this nonlocal effect by using the following Bell-type experiment. Under the transformation $U^B = e^{i\frac{\varphi}{2}\boldsymbol{u}\cdot\vec{\sigma}}$, we can get

$$\sigma_i^f = U^B \sigma_i U^{B\dagger} = \cos\varphi\sigma_i + \epsilon_{ijk} u_j \sin\varphi\sigma_k + 2\sin^2\frac{\varphi}{2} u_i \boldsymbol{u}\cdot\vec{\sigma}. \tag{16}$$

In fact, $\vec{\sigma}^f = (\sigma_1^f, \sigma_2^f, \sigma_3^f)$ is just another set of Pauli matrices. From (8), we have

$$\beta^f_{ij}\sigma_i \otimes \sigma_j = \beta_{ij}\sigma_i \otimes \sigma^f_j. \tag{17}$$

Let us perform the measurements either A_1 or A_2 on one particle, and either B_1 or B_2 on the other, where $A_1 = \boldsymbol{n}^1\cdot\vec{\sigma}$, $A_2 = \boldsymbol{n}^2\cdot\vec{\sigma}$, $B_1 = \boldsymbol{m}^1\cdot\vec{\sigma}$, and $B_2 = \boldsymbol{m}^2\cdot\vec{\sigma}$. Let $E(A, B)$, denote the quantum expectation value of the product AB. We define F as

$$F = E(A_1, B_1) + E(A_1, B_2) + E(A_2, B_1) - E(A_2, B_2), \tag{18}$$

which is just the CHSH expression [3]. Let us introduce the measurement matrix T as $T_{ij} = (n_i^1 + n_i^2)m_j^1 + (n_i^1 - n_i^2)m_j^2$, $i, j = 1, 2, 3$. We can obtain the quantum expectation of F for the initial state ρ_0,

$$F(\rho_0, T) = \sum_{i,j} \beta_{ij} T_{ij}. \tag{19}$$

Then, for the final state ρ_f, if one chooses the measurements, $A_1^f = A_1$, $A_2^f = A_2$, $B_1^f = \boldsymbol{m}^1 \cdot \vec{\sigma}^f$, and $B_2^f = \boldsymbol{m}^2 \cdot \vec{\sigma}^f$, we can prove that

$$F(\rho_0, T) = F(\rho_f, T^f), \tag{20}$$

in which T^f is the measurement matrix corresponding to the measurement settings for the final state.

At first, we let $B_1 = \sigma_1$, and $B_2 = \sigma_2$ be fixed, and then change the settings for A_1 and A_2 to find the maximal value $F_{\max}(\rho_0, T)$ for the initial state. We can obtain the optimal settings $\overline{A}_1$ and $\overline{A}_2$.

Secondly, we apply measurements on the final state. At this time, we let $A_1^f = \overline{A}_1$ and $A_2^f = \overline{A}_2$ be fixed. Then we vary the settings for the others. From the above discussion, we can see that $F(\rho_f, T^f)$ will reach its maximal value (which must equal $F_{\max}(\rho_0, T)$), if $B_1^f = \sigma_1^f$, and $B_2^f = \sigma_2^f$. So, from the relations between Pauli matrices, one can get σ_3^f. Hence, from (7) and (17) we can obtain β^f and the $d(\rho_0, U^B)$ immediately.

In conclusion, we have investigated nonlocal effects for the bipartite system induced by local cyclic operations of one of its subsystem. We employ the Hilbert-Schmidt distance to measure the nonlocal effect. Such nonlocal shifts vanish for product states, but do not vanish for disentangled states that are only *classically correlated*. Therefore this nonlocal effect can be used to classify the disentangled states. For qubit pairs, we show that the nonlocal shift of disentangled states is limited by $1/\sqrt{2}$, while the shifts of entangled states can exceed this limit and reach 1 for maximally entangled states. Hence, the nonlocal effect can be used as a sufficient condition of detecting the entanglement.

In fact, the nonlocality is due to the existence of correlations in compound quantum systems, which is a more general notion than entanglement. It is well known that the local operations on the subsystem of the compound quantum system in the distance labs paradigm can produce nonlocal consequences. In this letter attention was focused on the nonlocal properties caused by the local operations which do not make the subsystem change. Such nonlocal property is not equivalent to entanglement in general. We hope that such nonlocal property, especially, the fact that nonlocal property is implied by disentangled states, will draw much more the attention of physicists to the study of the nonlocality and entanglement of quantum systems.

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