Quantum resonance and antiresonance for a periodically kicked Bose-Einstein condensate in a one-dimensional box

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We investigate the quantum dynamics of a periodically kicked Bose-Einstein condensate (BEC) confined in a one-dimensional (1D) box both numerically and theoretically, emphasizing on the phenomena of quantum resonance and antiresonance. The quantum resonant behavior of BEC is different from the single particle case but the antiresonance condition ($T=2\pi$ and $\alpha=0$) is not affected by the atomic interaction. For the antiresonance case, the nonlinearity (atom interaction) causes the transition between oscillation and quantum beating. For the quantum resonance case, because of the coherence of BEC, the energy increase is oscillating and the rate is dramatically affected by the many-body interaction. We also discuss the relation between the quantum resonant behavior and the Kolmogorov-Arnold-Moser (KAM) or non-KAM property of the corresponding classical system.

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I. INTRODUCTION

Quantum systems under a periodically driving force are of great interest in varied fields of physics for their versatile applications in microscopic manipulations and control [1]. Their dynamics demonstrate many interesting behaviors, such as dynamical localization and chaos-assisted tunneling, to name a few [2–5]. Among them, quantum resonance (QR) and antiresonance (AR) are two interesting phenomena [6,7]. QR says that under a certain resonance condition, a particle acquires energy from an external force most efficiently leading to its energy increase with time in a square law. In the other limiting case, the AR case, the particle will bounce between two states and its energy shows a periodic oscillation.

QR and AR are pure quantum behaviors without classical counterparts. In the well-known kicked rotor system, given a value of the kick strength K, special resonant regimes of motion appear for periods with values $T=4\pi\frac{p}{q}$, where the integers p and q are mutually prime. Under these conditions the system regularly accumulates energy which grows quadratically in the time asymptotic [7]. The case $\frac{p}{q}=\frac{1}{2}$ presents a completely periodic behavior with period $2T$. This is the AR case. QR and AR have been observed in the atom optics imitation of the quantum kicked rotor [8]. Recent realizations of the Bose-Einstein condensate (BEC) [9] make us curious about whether or not QR and AR also exist in BEC systems and how the nonlinearity, stemming from mean field treatment of the atomic interaction [10], affects quantum resonances. Recently in Ref. [11] it has been shown that a BEC in a quasi-one-dimensional (1D) box can be achieved. This experiment provides a good condition to investigate the quantum resonances of BEC. The δ kick can be realized using counterpropagating laser beams and its spatial shape can be adjusted by phase mismatch of laser beams. The interaction strength between atoms g can be changed using a Feshbach resonance. These motivate us to study the quantum resonances of BEC under this experimental condition.

In this paper, we consider a BEC trapped in a 1D box and kicked periodically, and study how the atomic interaction affects the quantum resonant behaviors. We find that the QR and AR conditions for this system are different from the quantum kicked rotor system. For this system the AR can be only found for a special spatial shape of the δ kick when $T=2\pi$ (i.e., $\frac{p}{q}=\frac{1}{2}$), for other shapes of kick and $T=4\pi\frac{p}{q}$ ($\neq 2\pi$) the QR will be observed. Because of the coherence of BEC, for the AR case, the nonlinearity (atom interaction) causes the transition between oscillation and quantum beating; for the QR case the energy increase is oscillating and the rate is dramatically affected by the many-body interaction. We also find that for the QR case the nonlinearity (atom interaction) suppresses the sensitivity to the spatial shape of the kick. Finally, we discuss the relation between the quantum resonant behavior and the Kolmogorov-Arnold-Moser (KAM) or non-KAM property of the corresponding classical system.

This paper is organized as follows. In Sec. II, we introduce the model. In Sec. III, we show how the interaction between atoms in a BEC changes the evolution of the energy in quantum resonant cases. In Sec. IV we present our analytical results which show how the many-body interaction affects the evolution of the energy. In this section, we also show why for certain values of the parameter α there is no AR and we show that this is not related to whether the corresponding classical system is KAM or non-KAM. Finally, in Sec. V, we present our conclusions.

II. THE MODEL

We focus our attention on the dynamics of a quasi-1D BEC confined in a cigar shaped trap with a pulsating potential. In the limit of validity of the mean-field treatment, this system can be described by the dimensionless nonlinear Gross-Pitaevskii equation (GPE):

$$\hat{H} = -\frac{\partial^2}{2\partial x^2} + g|\phi(x,t)|^2 + K\cos(x+\alpha)\delta_T + U(x), \quad (1)$$

where $U(x)=\infty$, for $x \leqslant 0, x \geqslant \pi$, and $U(x)=0$ elsewhere, $g = \alpha_{1D} 4\pi\hbar^2 Na/m$ is the scaled strength of nonlinear interaction, N is the number of atoms, a is the s-wave scattering length, α_{1D} is a coefficient which compensates for the loss of two dimensions [12], K is the kick strength, $\delta_t(T)$ represents $\Sigma_n \delta(t-nT)$, T is the kick period, and x denotes the position on the x axis. The variable $x \in [0,\pi]$ and α is a parameter between 0 and 2π which can be controlled by the phase mismatch of laser beams. Due to symmetries, the only important interval for the parameter α is $\alpha \in [-\pi/2, \pi/2]$. Because the BEC is unstable under kicks if the nonlinear interaction is large [13], we only study g with a maximum value of 0.5. This value is very likely in the stable region where the number of condensed particles is much bigger than the number of noncondensed ones for a long enough time that we can use the GPE to study the evolution of the wave function.

In the resonant case the energy grows in average quadratically and at the same time the number of noncondensed particle is also growing. This means that our results are valid for a limited number of kicks because after that we lose the coherence of the condensate. It could be interesting to see experimentally where this limit is. The energy of a particle is given by $\langle E\rangle = \int_0^\pi dx \left[\phi^*\left(-\frac{\partial^2}{2\partial x^2} + \frac{g}{2}|\phi|^2\right)\phi\right]$. The evolution of the wave function is given by numerical integration of Eq. (1), over a certain number of kicks using the split-operator method.

In our study, we use the ground state of the Hamiltonian $\hat{H} = -\frac{\partial^2}{2\partial x^2} + g|\phi(x,t)|^2$ (with the same boundary condition given above) as the initial condition. Due to the shape of the potential $U(x)$, $\phi(0,t)=\phi(\pi,t)=0$. The wave function $\phi(x,t)$ satisfies the normalization condition $\int_0^\pi |\phi|^2 dx = 1$. For a positive g the ground state is given by [14]

$$\phi = \sqrt{\frac{m}{g}}\left(\frac{2K(m)}{\pi}\right)\mathrm{sn}\left(2K(m)\frac{x}{\pi}|m\right), \quad (2)$$

where $K(m)$ is the elliptic complete integral of the first kind and $\mathrm{sn}(x,m)$ is the Jacobi elliptic function. The parameter m is included in the interval [0,1] and is related to g by

$$\frac{1}{g\pi}[2K(m)]^2\left(1 - \frac{E(m)}{K(m)}\right) = 1, \quad (3)$$

which comes from normalization condition. For negative values of g, the initial condition is given by [14]

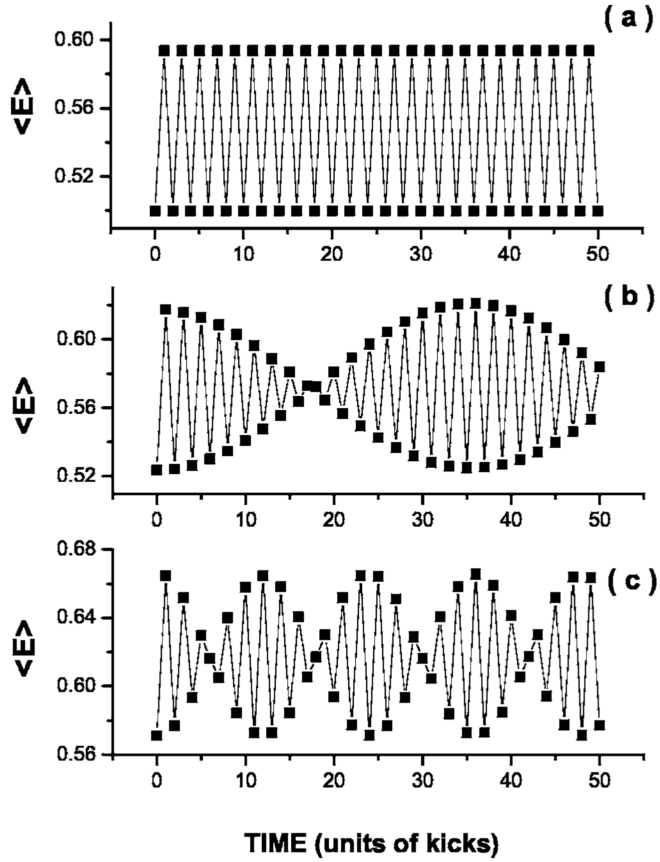

FIG. 1. Energy ($\langle E\rangle$) evolution for different values of the interaction strength g with fixed $K=0.5$. The period is fixed to $T=2\pi$ and $\alpha=0$ which corresponds to AR condition. For $g=0$ (a) the evolution of energy is perfectly periodic with period 2. For the other two cases [$g=0.1$ in (b) and $g=0.3$ in (c)], the evolution is quasiperiodic and we can see the phenomenon of beating.

$$\phi = \sqrt{\frac{m}{G}}\left(\frac{2K(m)}{\pi}\right)\mathrm{cn}\left[K(m)\left(\frac{2x}{\pi} - 1\right)|m\right], \quad (4)$$

where $\mathrm{cn}(x,m)$ is the elliptic Jacobi function cn and m and G, which is $G=-g$, are related by

$$\frac{1}{G\pi}[2K(m)]^2\left(\frac{E(m)}{K(m)} - (1-m)\right) = 1. \quad (5)$$

III. QUANTUM RESONANCE AND ANTIRESONANCE OF BEC

A. Antiresonance

For this model we find that the AR can only be observed with the condition $T=2\pi$ and $\alpha=0$ (shown in Fig. 1). This condition is different from the one of kicked rotor studied by Zhang *et al.* [13] where they discovered that the AR condition is for $T=2\pi$ and independent on the shape of kicks. In Fig. 1 one can see that if the nonlinear term is zero, the energy oscillates in time with a period $2T$. However, when the nonlinear term is nonzero, the energy oscillates in time with an amplitude that decreases gradually to zero and then

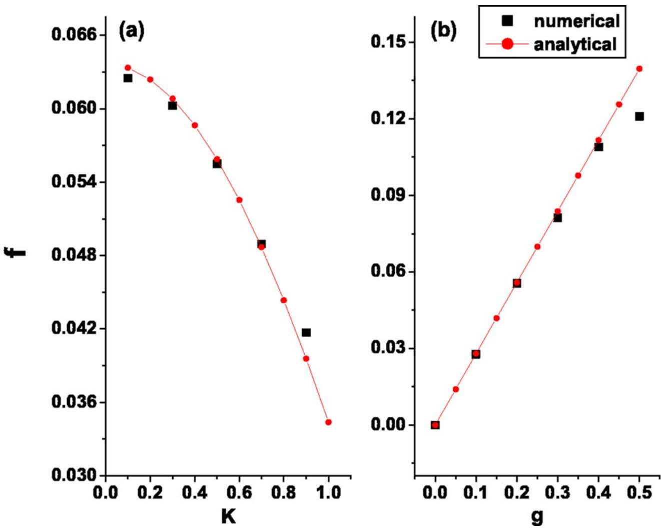

FIG. 2. (Color online) (a) Modulation frequency versus kick intensity (K) for fixed interaction value ($g=0.2$). (b) Modulation frequency versus g for fixed kick intensity ($K=0.5$). The theoretical result of Eq. (7) agrees very well with the numerical simulation.

revives, similar to the phenomena of beating in classical waves.

For the quantum beating case there are two frequencies: one is the frequency of kick and another is the beating frequency which is due to the coherence of the BEC and can be obtained approximately by a two-state model. As for the one in Ref. [13], for $\alpha=0$, our model can be mapped onto a two-state model [15–17]. We can write the wave function as a sum of only the ground state and the first excited state with relative population a and b (with normalization condition $|a|^2+|b|^2=1$). Defining $S_z=|a|^2-|b|^2$ as the population difference between ground state and first excited state and $-\arctan(S_y/S_x)$ as the relative phase between the two states, we can express the Hamiltonian as

$$H=-\frac{3}{4}S_z+\frac{g}{2\pi}\left[S_x^2+\frac{S_z^2}{4}\right]+\frac{K}{2}\delta_t(T)S_x. \quad (6)$$

Then, one can obtain the beating frequency of the evolution of the energy from the two-state model,

$$f_{\mathrm{beat}} \simeq \frac{g\cos(K)}{\pi}. \quad (7)$$

We can see in Fig. 2 that the theoretical approximate agrees very well with numerical calculations.

The quantum resonant behaviors of kicked BEC in 1D box are different from the one of kicked rotor studied by Zhang *et al.* [13]. For the kicked BEC in 1D box the QR behaviors can also be controlled by the spatial shape of the kick which can be adjusted by mismatch of laser beams. In our model the spatial shape is parameterized by α. Following we will show that if $\alpha\neq 0$, we will have QR even if $T=2\pi$ (AR condition of kicked rotor).

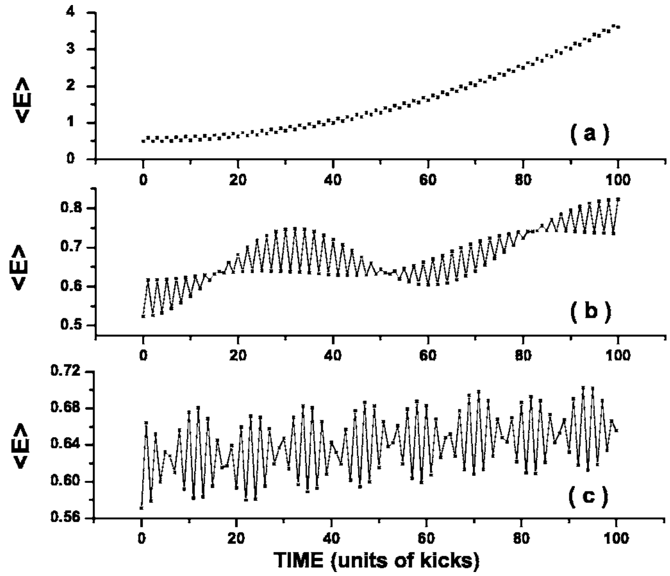

FIG. 3. Energy ($\langle E\rangle$) evolution for $K=0.5$ and $\alpha=0.1$. The interaction strength is $g=0$ for (a), $g=0.1$ for (b), and $g=0.3$ for (c). The motion is neither periodic nor quasi-periodic. In (b) and (c) we can also see the oscillation due to the nonlinear interaction.

B. Quantum resonance

For any nonzero α, the quantum behavior of the system is very different from the case of $\alpha=0$. In Fig. 3 we show the energy evolution with time (in unit of kicks) for different $g=0$ (a), $g=0.1$ (b), and $g=0.3$ (c) with $\alpha=0.1$. We can see that for $g=0$, AR does not exist anymore and there is only QR. The energy increases with time on average in square law.

For $g\neq 0$, because of the coherence of BEC, the energy increase is oscillating and the rate is dramatically affected by the interaction term. However, though the energy oscillates, the energy on average has a quadratic increase as for the resonant case.

The behavior of the energy with the number of kicks for different values of α and g are summarized in Fig. 4, where the value of energy after 50 kicks for $K=0.5$ and different values of g and α is shown. We can see many interesting things in this figure. First of all, there is a symmetry axis at $\alpha=\pi$. For $g=0$ the value of energy is symmetric and the symmetry is broken by the nonlinear term. Opposite values of g are symmetric (considering only the kinetic energy) with respect to this axis. The energy reaches a maximum at $\alpha=\pi/2$ for negative g and at $\alpha=3\pi/2$ for positive g. The breaking of the symmetry is due to the coherence of the BEC.

From Fig. 4 it seems that, the bigger the value of g, the lesser is the effect of a small change of α. We can see this in detail in Fig. 5 where we show for $g=0$, $g=0.1$, and $g=0.3$ the effect of small values of α. For $\alpha=0.4$ only the case with $g=0.3$ is still stable, while for example for $g=0$, a tiny perturbation such as $\alpha=0.01$ is enough to make the energy increase rapidly. This shows that, the bigger the interaction is between the atoms in the BEC, the less sensitive to a variation of α is the system.

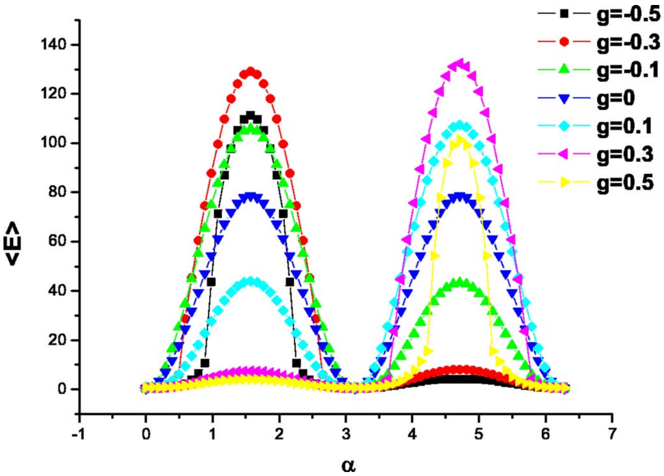

FIG. 4. (Color online) Energy ($\langle E\rangle$) evolution for different values of g with fixed $K=0.5$. The interaction strength is $g=0$ (blue), $g=\pm 0.1$ (light blue and green), $g=\pm 0.3$ (red and purple), and $g=\pm 0.5$ (black and yellow). We can see a symmetric behavior for opposite values of g as long as the strength is not too large (case $|g|=0.5$). Positive values of g reach a maximum for $\alpha=3\pi/2$ and negative for $\alpha=\pi/2$.

IV. ANALYTICAL STUDY

A. Approximate study of the influence of the interaction

In this section, we shall demonstrate that it is possible to understand analytically why the evolution of the energy has the above behavior. As it will be shown, the underlying mechanism is the interaction of the atoms in the BEC.

We approach the problem by looking for an expression of the energy to the first order in g. For small values of g we can approximate the shape of the wave function $\phi(x)$ by

$$\phi(x,T) = [e^{-iV(x)}e^{-ig|\phi|^2T/2}e^{i\nabla^2/2T}e^{-ig|\phi|^2T/2}]\phi(x,0). \qquad (8)$$

Mapping this model on a periodic ring, we can predict the value of the energy after a certain number of kicks. In general a model with $x\in[0,\pi]$ and an infinite square potential can be mapped into a model with $x\in[0,2\pi]$ and periodic boundary conditions. The initial condition $\tilde{\phi}(x,0)$ is, for $0\leq x<\pi$, $\tilde{\phi}(x,0)=\phi(x,0)/\sqrt{2}$, and for $\pi\leq x<2\pi$, $\tilde{\phi}(x,0)=\phi(2\pi-x,0)/\sqrt{2}$, where $\phi(x,0)$ is the initial condition for the model with $x\in[0,\pi]$. The kick $\tilde{V}(x)$ is given, for $0\leq x<\pi$, by $\tilde{V}(x)=V(x)$ and, for $\pi\leq x<2\pi$ by $\tilde{V}(x)=V(2\pi-x)$, where $V(x)$ in this case is $K\cos(x+\alpha)$. This mapping could be done for any potential.

The evolution of the wave function, after an even number ($N=2M$) of kicks, is given by

$$\tilde{\phi}(x,NT) = e^{-iM(\tilde{V}+\tilde{V}_\pi)}\left(\frac{1+e^{-i2g\,\sin^2(x)}}{2}\right)^N\tilde{\phi}(x,0), \qquad (9)$$

where $\tilde{V}_\pi=\tilde{V}(x+\pi)$. In this way we can compute the energy which is given by

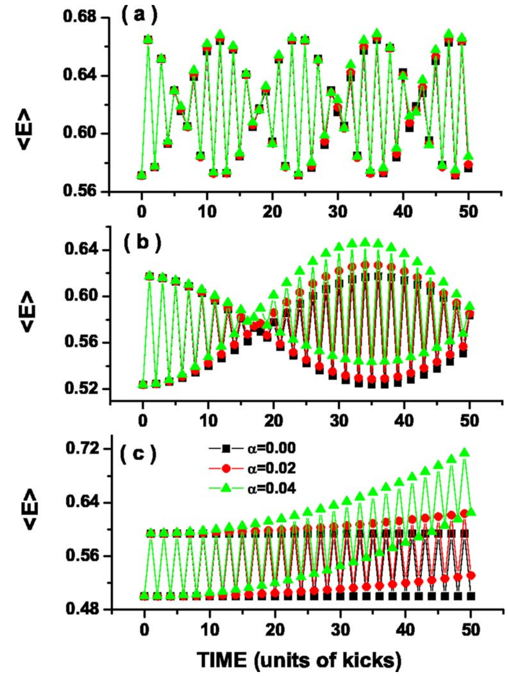

FIG. 5. (Color online) Energy ($\langle E\rangle$) as function of the number of kicks for different values of g and $\alpha=0$ (black), $\alpha=0.02$ (red), and $\alpha=0.04$ (green). The interaction strength is $g=0$ (c), $g=0.1$ (b), and $g=0.3$ (a). The bigger g is the more stable is the motion to changes of the parameter α.

$$E(N) = \frac{N^2K^2}{8}\sin^2(\alpha) - \frac{16}{15\pi}g\sin\alpha + \frac{1}{2} + \frac{3}{16\pi}g. \qquad (10)$$

To obtain this result, we have used $\phi(x,0)=\sqrt{\frac{2}{\pi}}\sin x$ which is a very good approximation in the case of small values of g here studied.

Equation (10) approximates the value of the energy after an even number N of kicks. We can see in Fig. 6 how good the approximation is. From Eq. (10) we can see if $\alpha=0$ the energy is independent of the number of kicks N and if $\alpha\neq 0$ the energy increases with the number of kicks in square law.

Moreover it is possible to see in Eq. (10) why for $g=0$ the behavior is completely symmetric and also where the symmetry for opposite values of g comes from. However, it is not possible to understand the behavior for bigger values of g in this way. For example we can see in Fig. 4 that for $g=\pm 0.5$ the maximum is smaller than for $g=\pm 0.3$. It is also not possible to show the oscillation of the energy due to the nonlinear term, but for our purpose of understanding the average growth of g for different values of α, the method shown in this paper is good, at least for small values of g.

B. Classical dynamical properties and quantum resonance

The study of quantum systems whose classical counterpart is non-KAM already showed interesting results. In both the classical and quantum cases, the non-KAM systems [18–21] demonstrate quite different behavior from the KAM

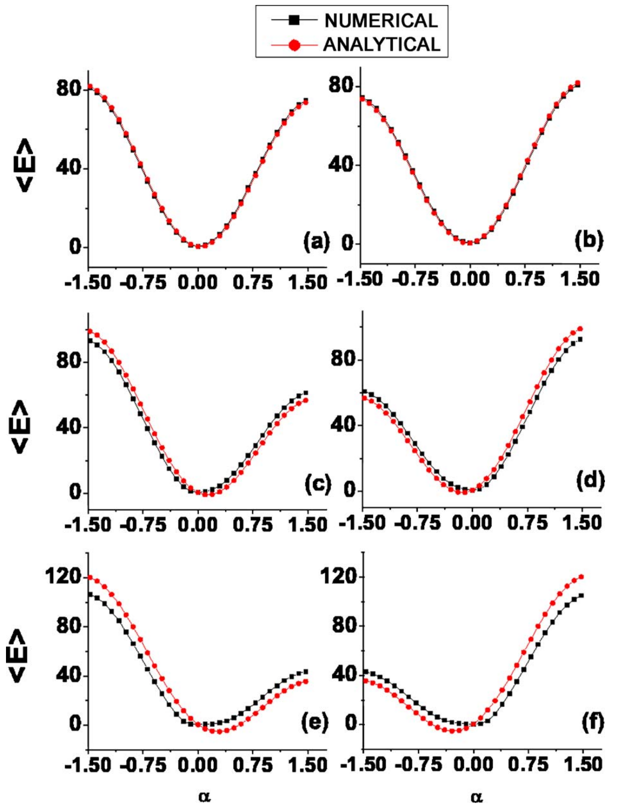

FIG. 6. (Color online) Evolution of the energy ($\langle E\rangle$) for different values of the interaction strength g and $K=0.5$ after 50 kicks for different values of α. The numerical (black) curve is compared to the analytical ones given by Eq. (10) (red). The red curve approximates the numerical results. The values of the interaction strength are $g=0.01$ (a), $g=-0.01$ (b), $g=0.05$ (c), $g=-0.05$ (d), $g=0.1$ (e), $g=-0.1$ (f).

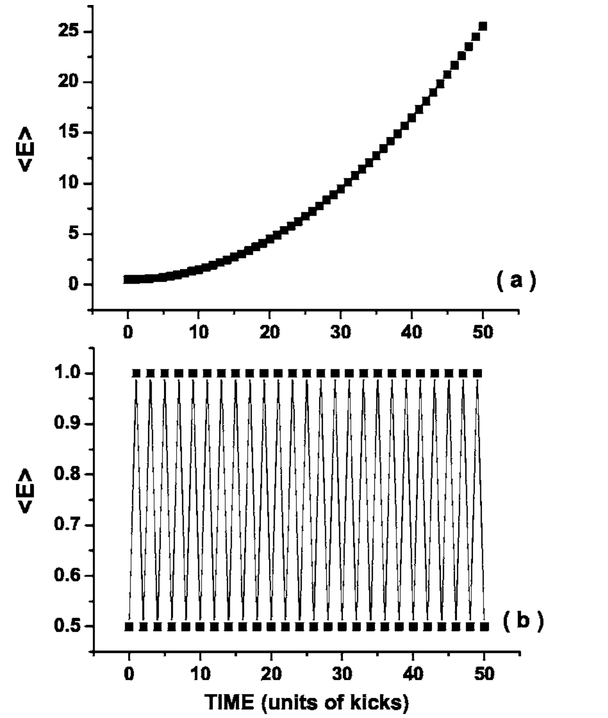

FIG. 7. (a) Energy ($\langle E\rangle$) evolution for a kick of type $V(x)=K\cos(2x)$. $T=4\pi\frac{1}{2}$ and $K=0.1$. We do not have AR even though the system is KAM. (b) Evolution of the energy ($\langle E\rangle$) with the number of kicks for a kick of the kind $V(x)=\pi/2-x$ for $0\leqslant x<\pi$ and $V(x)=-3\pi/2+x$ for $\pi\leqslant x<2\pi$. The system is non-KAM but we have AR.

one. For instance, in classical KAM systems, as the external or driven parameter is increased, the invariant curves gradually break up. Local chaos evolves into global chaos and diffusion takes place. In a non-KAM system, there are no such invariant curves for any small external or driven parameters. Quantum mechanically, the quantum interference suppresses the classical diffusion leading to a so-called exponential localization [2], while in a non-KAM system, the localization becomes power law, or in other words, there is no localization [18].

In our case, it is interesting to notice that the system that we study presents AR when the classical counterpart is KAM (for $\alpha=0$), and that there is QR when the classical equivalent system is non-KAM (for $\alpha\neq 0$). We would like to understand whether the properties of the quantum system and its classical equivalent (AR with KAM and QR with non-KAM) are related.

To understand this we start showing a general result which is valid for a generic periodically kicked system with $x\in[0,2\pi]$ and the kick is given by a generic $V(x)$. Now, starting from Ref. [7] it is easy to see that $\phi(x)_{N+1}=\exp[-iV(x)]\phi(x+\pi)_N$. So after two kicks with a period $T=2\pi$ we have that

$$\phi(x)_{N+2}=\exp\{-i[V(x)+V(x+\pi)]\}\phi(x)_N. \tag{11}$$

From this point it is easy to derive that the condition for AR is $T=2\pi$ and

$$V(x)+V(x+\pi)=C. \tag{12}$$

This result, after doing the mapping discussed in Sec. III, can be applied to our model which, as it should be emphasized, is characterized by an infinite well and $x\in[0,\pi]$. For $\alpha\neq 0$ or $\alpha\neq\pi$, the mapping on the circle of the kick $\tilde{V}$ does not follow Eq. (12). $\tilde{V}(x)+\tilde{V}(x+\pi)\neq C$, but it is a function of x.

It is now obvious that AR is not related to whether the corresponding classical system is KAM or non-KAM, but due to Eq. (12). We can have KAM systems without AR and non-KAM systems with AR. This is shown also in Fig. 7 where the kick is given either by $V(x)=K\cos(2x)$ (KAM) and we see QR, either by $V(x)=\pi/2-x$ for $0\leqslant x<\pi$ and $V(x)=-3\pi/2+x$ for $\pi\leqslant x<2\pi$ and we have AR.

V. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have investigated the quantum dynamics of a periodically kicked Bose-Einstein condensate confined in a 1D box both numerically and theoretically, emphasizing on the phenomena of QR and AR. We find that the atomic interaction does not affect the AR condition (or QR condition). However, the resonant behaviors of BEC is different from the single particle case. For the AR case, the nonlinearity (atom interaction) causes the transition from oscillation to quantum beating. For the QR case, because of the coherence of BEC, the energy increase is oscillating and the rate is dramatically affected by the interaction between atoms. And, the rate at which the energy increases in the system depends on the atom interaction. The interaction breaks the symmetric evolution of the energy for different values of α around $\alpha=\pi$. We have also found that, the stronger the interaction between atoms, the more stable the system is to small changes of α. This means that for bigger interaction the AR behavior will be much more stable to errors in the matching of the kick-generating lasers and the trap. We also discussed the relation between the quantum resonant behavior and the KAM or non-KAM property of the corresponding classical system.

We would like to emphasize the fact that the system that we studied can be realized with current experimental techniques. In this way with the same experimental set-up it could be possible to observe the phenomenon which we have shown, such as quantum beating, and the phenomenon of the destruction of AR. The phenomenon of quantum beating can be used to measure the value of the interaction of the atoms, and the breaking of AR can be used to see if the laser matches with the trap.

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