Maximal violation of Clauser-Horne-Shimony-Holt inequality for four-level systems

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Clauser-Horne-Shimony-Holt inequality for bipartite systems of four dimensions is studied in detail by employing the unbiased eight-port beam splitters measurements. The uniform formulas for the maximum and minimum values of this inequality for such measurements are obtained. Based on these formulas, we show that an optimal nonmaximally entangled state is about 6% more resistant to noise than the maximally entangled one. We also give the optimal state and the optimal angles which are important for experimental realization.

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I. INTRODUCTION

Recently, the inequalities [1,2] have been found that generalize the Clauser-Horne-Shimony-Holt (CHSH) inequality to systems of high dimension, which give the analytic description of previous numerical results [3]. In this Brief Report, we study the CHSH inequality for bipartite systems for four dimensions by employing the unbiased eight-port beam splitters measurements. The uniform formulas of the maximal and minimal values of this inequality are obtained. Based on these formulas, we find an optimal nonmaximally entangled state violates the inequality more strongly than the maximally entangled one, and then is about 6% more resistant to noise than the maximally entangled one. Similar to what we have pointed out for three-dimensional systems [4], we also find that the left hand of the inequality cannot be violated by the maximally entangled state. However, we find that the left hand of the inequality can be violated by some nonmaximally entangled states, and the optimal nonmaximally entangled state for the left hand of the inequality is not the optimal one for the right hand.

II. THE INEQUALITY

In this section, let us recall the Bell inequality for four dimensions obtained in Refs. [2,5]. The scenario of the inequality involves two parties: Alice can carry out two possible measurements, A_1 or A_2, on one of the particles, whereas the other party, Bob can carry out two possible measurements, B_1 or B_2, on the other one. For the composed systems of four-dimensional parties (or bipartite systems of spin $S=3/2$ particles with the relation $d=2S+1$), each measurement may have four possible outcomes: $A_1,A_2,B_1,B_2=0,1,2,3$. The joint probabilities are denoted by $P(A_i,B_j)$, which are required to satisfy the normalization condition: $\Sigma_{m,n=0}^{3}P(A_i=m,B_j=n)=1$ [6]. We define the correlation functions Q_{ij} as follows:

$$Q_{ij} \equiv \frac{1}{S}\sum_{m=0}^{3}\sum_{n=0}^{3} f^{ij}(m,n)P(A_i=m,B_j=n), \qquad (1)$$

in which $S=3/2$, and the function $f^{ij}(m,n)$ can be one of the following forms,

$$f^{ij}(m,n) = S - M(\varepsilon(i-j)(m \pm n),4), \qquad (2)$$

where $\varepsilon(x)$ is the sign function

$$\varepsilon(x) = \begin{cases} 1 & x \geqslant 0 \\ -1 & x < 0 \end{cases},$$

and $M(x,4)$ is defined as follows: $M(x,4)=(x \bmod 4)$ and $0 \leqslant M(x,4) \leqslant 3$. Then one can consider the following Bell expression:

$$I_4 = Q_{11} + Q_{12} - Q_{21} + Q_{22}. \qquad (3)$$

Especially, if taking $f^{ij}(m,n)=S-M\boldsymbol{(}\varepsilon(i-j)(m-n),4\boldsymbol{)}$, one can prove that the above expression is just the Bell expression presented in Ref. [2].

The authors of Refs. [2,5] proved that the maximum value of the above Bell expressions is 2 and minimum value of it is $-10/3$ for local variable theories. Then we get the following Bell inequality:

$$-\tfrac{10}{3} \leqslant I_4 \leqslant 2. \qquad (4)$$

However, the inequality will be violated for some entangled states by quantum predictions. For the maximally entangled state of two four-dimensional systems $\psi=\frac{1}{2}\Sigma_{j=0}^{3}|l\rangle_A|l\rangle_B$, the authors of Refs. [2,5] obtained the strongest violation of Bell expression I_4 for such a state, $I_4(QM)=\frac{2}{3}[\sqrt{2}+(10-\sqrt{2})^{1/2}]\approx 2.896\,24$. In the presence of uncolored noise the quantum state

$$\rho = (1-F)|\psi\rangle\langle\psi| + F\frac{I}{16}. \qquad (5)$$

Following Ref. [3], we define the threshold noise admixture F_{thr} (the minimal noise admixture fraction for $|\psi\rangle$) $F_{thr}=1-2/I_4(QM)$, then we get $F_{thr}\approx 0.309\,45$. This result equals to the numerical results of Ref. [3].

III. THE MAXIMAL VIOLATION

Here, as in the previous works, we restricted our analysis to unbiased eight-port beam splitters [7,8], more specifically to Bell multiport beam splitters [3]. Unbiased Bell $2d$-port

TABLE I. The first set of vertices, in which $i \neq j$.

$\{T_{ij}\}$	$T_{[ab]}$	$T_{[ac]}$	$T_{[ad]}$	$T_{[bc]}$	$T_{[bd]}$	$T_{[cd]}$
	Γ_1	Γ_2	Γ_3	Γ_3	Γ_2	Γ_3
	$-\Gamma_1$	$-\Gamma_2$	$-\Gamma_3$	Γ_3	Γ_2	Γ_3
	$-\Gamma_1$	Γ_2	Γ_3	$-\Gamma_3$	$-\Gamma_2$	Γ_3
	Γ_1	$-\Gamma_2$	Γ_3	$-\Gamma_3$	Γ_2	$-\Gamma_3$
	Γ_1	Γ_2	$-\Gamma_3$	Γ_3	$-\Gamma_2$	$-\Gamma_3$
	Γ_1	$-\Gamma_2$	$-\Gamma_3$	$-\Gamma_3$	$-\Gamma_2$	Γ_3
	$-\Gamma_1$	Γ_2	$-\Gamma_3$	$-\Gamma_3$	Γ_2	$-\Gamma_3$
	$-\Gamma_1$	$-\Gamma_2$	Γ_3	Γ_3	$-\Gamma_2$	$-\Gamma_3$

beam splitters have the property that a photon entering into any single port (out of d) has equal chances to exit from any output port. In addition, for Bell $2d$-port beam splitters the elements of their unitary transition matrix $\mathbf{U}^d$ are solely powers of the dth root of unity $\gamma_d = \exp(i2\pi/d)$, namely, $\mathbf{U}_{ij}^d = (1/\sqrt{d})\gamma_d^{(i-1)(j-1)}$.

Let us now imagine spatially separated Alice and Bob who perform the Bell-type experiment via the eight-port beam splitters on the state

$$|\phi\rangle = \frac{1}{2}\sum_{i=0}^{3} a_i |i\rangle_A |i\rangle_B, \tag{6}$$

with real coefficients a_i, where, e.g., $|i\rangle_A$ describes a photon in mode i propagating to Alice. One has ${}_x\langle i|i'\rangle_x = \delta_{ii'}$, with $x=A,B$. The overall unitary transformation performed by such a device is given by

$$U_{ij}^4 = \tfrac{1}{2}\gamma_4^{ij} e^{i\varphi_j}, \quad i,j = 0,1,2,3, \tag{7}$$

where $\gamma_4 = e^{i\pi/2}$ and j denotes an input beam to the device, and i an output one; φ_j are the four phases that can be set by the local observer, denoted as $\vec{\varphi} = (\varphi_0, \varphi_1, \varphi_2, \varphi_3)$. The transformations at Alice's side are denoted as $\vec{\varphi}^A = (\varphi_0^A, \varphi_1^A, \varphi_2^A, \varphi_3^A)$ and $\vec{\varphi}^B = (\varphi_0^B, \varphi_1^B, \varphi_2^B, \varphi_3^B)$ for Bob's side. In this way the local observable is defined. The quantum prediction for the joint probability $P^{QM}(A_i = m, B_j = n)$ to detect a photon at the mth output of the multiport A and another one at the nth output of the multiport B calculated for the state (6) is given by

$$P^{QM}(A_i = m, B_j = n) = \sum_{k=0}^{3}\sum_{l=0}^{3} a_k a_l \gamma_4^{(k-l)(m+n)} e^{i(\varphi_k^{A_i} + \varphi_k^{B_j} - \varphi_l^{A_i} - \varphi_l^{B_j})}. \tag{8}$$

For convenience, we use the definition of the correlation functions Q_{ij} with the function $f^{ij} = S - M[\varepsilon(i-j)(m+n), d]$. By substituting Eq. (8) into Eq. (3), after some elaboration, we can obtain

$$I_4 = \sum_{k<l} a_k a_l T_{kl}, \quad (k = 0,1,2; l = 1,2,3), \tag{9}$$

where T_{kl} are six continuous functions of 16 angles $\vec{\varphi}^{A_i}$ and $\vec{\varphi}^{B_j}(i,j=1,2)$. Let us define $\varphi_{ab}^{ij} = \varphi_a^{A_i} - \varphi_b^{A_i} + \varphi_a^{B_j} - \varphi_b^{B_j}$, then we can list these six functions as follows:

$$T_{01} = \tfrac{1}{6}\{[\cos(\varphi_{01}^{11}) - \cos(\varphi_{01}^{21}) - \cos(\varphi_{01}^{12}) + \cos(\varphi_{01}^{22})] - [\sin(\varphi_{01}^{11}) + \sin(\varphi_{01}^{21}) + \sin(\varphi_{01}^{12}) + \sin(\varphi_{01}^{22})]\}, \tag{10}$$

$$T_{02} = -\tfrac{1}{6}[\cos(\varphi_{02}^{11}) - \cos(\varphi_{02}^{21}) + \cos(\varphi_{02}^{12}) + \cos(\varphi_{02}^{22})], \tag{11}$$

$$T_{03} = \tfrac{1}{6}\{[\cos(\varphi_{03}^{11}) - \cos(\varphi_{03}^{21}) - \cos(\varphi_{03}^{12}) + \cos(\varphi_{03}^{22})] + [\sin(\varphi_{03}^{11}) + \sin(\varphi_{03}^{21}) + \sin(\varphi_{03}^{12}) + \sin(\varphi_{03}^{22})]\}, \tag{12}$$

$$T_{12} = \tfrac{1}{6}\{[\cos(\varphi_{12}^{11}) - \cos(\varphi_{12}^{21}) - \cos(\varphi_{12}^{12}) + \cos(\varphi_{12}^{22})] - [\sin(\varphi_{12}^{11}) - \sin(\varphi_{12}^{21}) + \sin(\varphi_{12}^{12}) + \sin(\varphi_{12}^{22})]\}, \tag{13}$$

$$T_{13} = -\tfrac{1}{6}[\cos(\varphi_{13}^{11}) - \cos(\varphi_{13}^{21}) + \cos(\varphi_{13}^{12}) + \cos(\varphi_{13}^{22})], \tag{14}$$

and

$$T_{23} = \tfrac{1}{6}\{[\cos(\varphi_{23}^{11}) - \cos(\varphi_{23}^{21}) - \cos(\varphi_{23}^{12}) + \cos(\varphi_{23}^{22})] - [\sin(\varphi_{23}^{11}) - \sin(\varphi_{23}^{21}) + \sin(\varphi_{23}^{12}) + \sin(\varphi_{23}^{22})]\}. \tag{15}$$

We can know that $|T_{kl}| \leqslant [(10-\sqrt{2})^{1/2}(2+3\sqrt{2})/21]$. However, they are strongly correlated, so T_{kl} cannot reach their maximum value at the same time. As has been proved in Ref. [4], the maximum (minimum) values of I_4 can only be found on the vertices of the polyhedron formed by T_{kl}. There are three sets of the vertices of the polyhedron. By denoting $\Gamma_1 = [(10-\sqrt{2})^{1/2}(2+3\sqrt{2})/21] \approx 0.87104$, $\Gamma_2 = \sqrt{2}/3 \approx 0.4714$, and $\Gamma_3 = [(10-\sqrt{2})^{1/2}(4-\sqrt{2})/21] \approx 0.3608$, we list them in Tables I–III.

In these tables, we have used the stipulations, e.g., $T_{[ab]} = T_{ab}$ for $a<b$ or $T_{[ab]} = T_{ba}$ for $b<a$. As has been proved in Ref. [4], for each vertices of the polyhedron one can obtain an extreme value of the I_4, and the maximum (or minimum) value can be obtained by comparing the values among them.

Assuming $\{A_i, (i=0,1,2,3)\} = \{|a_0|, |a_1|, |a_2|, |a_3|\}$, where "=" means the equality of two sets, and A_i are in decreasing order, i.e., $A_0 \geqslant A_1 \geqslant A_2 \geqslant A_3$. For convenience, we denote $\Gamma_1 = [(10-\sqrt{2})^{1/2}(2+3\sqrt{2})/21]$, $\Gamma_2 = \sqrt{2}/3$, and $\Gamma_3 = [(10-\sqrt{2})^{1/2}(4-\sqrt{2})/21]$. Let us define

$$B_1(|\phi\rangle) = (A_0A_1)\Gamma_1 + (A_0A_2 + A_1A_3)\Gamma_2 + (A_0A_3 + A_1A_2 + A_2A_3)\Gamma_3, \tag{16}$$

and

$$B_2(|\phi\rangle) = (A_0A_1)\Gamma_3 + (A_0A_2 + A_1A_3)\Gamma_2 + (A_0A_3 + A_1A_2 - A_2A_3)\Gamma_1. \tag{17}$$

Then, the maximum value of I_4 for $|\phi\rangle$ must be

$$I_4^{\max}(|\phi\rangle) = \max[B_1(|\phi\rangle), B_2(|\phi\rangle)]. \tag{18}$$

TABLE II. The second set of vertices, in which $i \neq j$.

$\{T_{ij}\}$	$T_{[ab]}$	$T_{[ac]}$	$T_{[ad]}$	$T_{[bc]}$	$T_{[bd]}$	$T_{[cd]}$
	$-\Gamma_1$	$-\Gamma_2$	$-\Gamma_1$	$-\Gamma_1$	$-\Gamma_2$	Γ_3
	Γ_1	Γ_2	Γ_1	$-\Gamma_1$	$-\Gamma_2$	Γ_3
	Γ_1	$-\Gamma_2$	$-\Gamma_1$	Γ_1	Γ_2	Γ_3
	$-\Gamma_1$	Γ_2	$-\Gamma_1$	$-\Gamma_1$	$-\Gamma_2$	$-\Gamma_3$
	$-\Gamma_1$	$-\Gamma_2$	Γ_1	$-\Gamma_1$	Γ_2	$-\Gamma_3$
	$-\Gamma_1$	Γ_2	Γ_1	Γ_1	Γ_2	Γ_3
	Γ_1	$-\Gamma_2$	Γ_1	Γ_1	$-\Gamma_2$	$-\Gamma_3$
	Γ_1	Γ_2	$-\Gamma_1$	$-\Gamma_1$	Γ_2	$-\Gamma_3$

From Eq. (18), one can immediately get that for the maximally entangled states, i.e., $|a_i|=1$ $(i=0,1,2,3)$, the maximum value of I_4 for such states are

$$I_4^{\max} = \Gamma_1 + 2\Gamma_2 + 3\Gamma_3 = \tfrac{2}{3}(\sqrt{2} + \sqrt{10-\sqrt{2}}). \tag{19}$$

This is just the result obtained in Refs. [2,5,9], and gives $F_{thr} \approx 0.309\,45$ which equals to the numerical results of Ref. [3].

Consider a_i as variables, we can obtain the maximal value of $I_4^{\max}$, denoted as $\overline{I}_{\max}$. The value of Eq. (18) should be maximum for $A_0 = A_1$ and $A_2 = A_3$. One can easily find that $B_1(|\phi\rangle) > B_2(|\phi\rangle)$ for any state. For this case, by calculating the extreme value of $B_1(|\phi\rangle)$ with $A_0 = A_1$ and $A_2 = A_3$, after some elaboration, we get

$$\overline{I}_{\max} = \overline{A}_+^2\Gamma_1 + 2\overline{A}_+\overline{A}_-(\Gamma_2+\Gamma_3) + \overline{A}_-^2\Gamma_3, \tag{20}$$

where

$$\overline{A}_\pm = \mathbf{[}1 \pm \mathbf{(}\tfrac{1}{791}\{357 + 7\sqrt{2} - 20(10-\sqrt{2})^{1/2} - 58[2(10-\sqrt{2})]^{1/2}\}\mathbf{)}^{1/2}\mathbf{]}^{1/2}$$

with $\overline{A}_+ = A_0 = A_1$ and $\overline{A}_- = A_2 = A_3$. We then have $\overline{S}_{\max} \approx 2.9727$,and the threshold amount of noise is about $F_{thr} \approx 0.3272$, which was also obtained in Ref. [9] by calculating the maximum eigenvalue of the Bell operator [10]. One can see this optimal nonmaximally entangled state is about 6% more resistant to noise than the maximally entangled one. One can check the above results with the following optimal angles for $a_i > 0$ $(i=0,1,2,3)$:

$$\vec{\varphi}^{A_1} = \left(0, \frac{\pi}{6}, -\pi, \frac{4\pi}{9}\right), \quad \vec{\varphi}^{A_2} = \left(0, -\frac{5\pi}{9}, \frac{5\pi}{9}, -\frac{\pi}{3}\right); \tag{21}$$

and

$$\vec{\varphi}^{B_1} = \left(0, -\frac{\pi}{2}, \frac{13\pi}{18}, -\frac{11\pi}{18}\right), \quad \vec{\varphi}^{B_2} = \left(0, \frac{7\pi}{36}, -\frac{27\pi}{36}, -\frac{7\pi}{18}\right). \tag{22}$$

On the other hand, we can also calculate the minimum value of I_4. Let us define

$$S_1(|\phi\rangle) = -(A_0A_1 + A_0A_3 + A_1A_2)\Gamma_1 - (A_0A_2 + A_1A_3)\Gamma_2 + (A_2A_3)\Gamma_3, \tag{23}$$

and

$$S_2(|\phi\rangle) = -2(A_0A_1 + A_0A_3 + A_1A_2 + A_2A_3)/3 - (A_0A_2 + A_1A_3)/3. \tag{24}$$

The minimum value of I_4 for $|\phi\rangle$ should be

$$I_4^{\min}(|\phi\rangle) = \min[S_1(|\phi\rangle), S_2(|\phi\rangle)]. \tag{25}$$

Then, for the maximally entangled state, the minimum value of I_4 is

$$I_4^{\min} = -\tfrac{10}{3}. \tag{26}$$

One sees that the maximally entangled state does not violate the left hand of the inequality (3). However, for a nonmaximally entangled state with $A_0 = A_1 = K_+$ and $A_2 = A_3 = K_-$ where

$$K_\pm = \mathbf{[}1 \pm \mathbf{(}\tfrac{1}{791}\{357 - 7\sqrt{2} - 80(10-\sqrt{2})^{1/2} + 6[2(10-\sqrt{2})]^{1/2}\}\mathbf{)}^{1/2}\mathbf{]}^{1/2} \tag{27}$$

we find the minimum value of $I_4^{\min}$, denoted as $\overline{I}_{\min}$:

$$\overline{I}_{\min} = -K_+^2\Gamma_1 - 2K_+K_-(\Gamma_1+\Gamma_2) + K_-^2\Gamma_3 \approx -3.464\,24. \tag{28}$$

Obviously, such states violate the left hand of the inequality.

TABLE III. The third set of vertices, in which $i \neq j$.

$\{T_{ij}\}$	$T_{[ab]}$	$T_{[ac]}$	$T_{[ad]}$	$T_{[bc]}$	$T_{[bd]}$	$T_{[cd]}$
	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$
	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$
	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
	$-\frac{2}{3}$	$-\frac{12}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$

IV. DISCUSSION

In summary of the present paper, we study the CHSH inequality for $d=4$ in details on the Bell-type experiment via the eight-port beam splitters which is realizable for nowadays technique. We give the analytic formulas of the maximum and minimum values of this inequality for such an experimental consideration. The maximal violations we obtained are the same as Ref. [9]. We also give the optimal state and the optimal angles which are important for experimental realization.

It is well known that for bipartite systems of two dimensions, the CHSH inequality is symmetry. For any entangled state the inequality is violated symmetrically, and will be maximally violated by the maximally entangled states. However, for the higher-dimensional systems, namely d-dimensional systems $(d>2)$, the inequality is asymmetry (see in Refs. [2,5]),

$$\frac{-2(d+1)}{(d-1)} \leqslant I_d \leqslant 2. \tag{29}$$

The authors of Refs. [2,5] studied the violation of the right hand of the above inequality for maximally entangled states and reproduced the results of previous numerical works [3]. For $d=3$, one can immediately obtain that the left hand of the above inequality is -4, which can never be violated by any state [2,5]. In Ref. [4], the authors have shown that the minimal values of I_3 for maximally entangled states is just -4. They also found that an optimal nonmaximally entangled state violates the inequality more strongly than the maximally entangled one. For $d=4$, we also find that the left hand of the inequality can be violated by some nonmaximally entangled states, and the optimal nonmaximally entangled state for the left hand of the inequality is not the optimal one for the right hand.

In fact, the relations (29) and (3) define two inequalities, namely, the right ones and the left ones. The right inequalities are optimal and tight but the left ones are not tight. The asymmetry of the CHSH inequalities is due to the asymmetry of Hilbert space for higher-dimensional systems [11]. For the systems of two dimensions, the Hilbert space is symmetry, so the CHSH inequalities are symmetry as well. But for the higher-dimensional systems, the Hilbert space is asymmetry, so the inequalities which are optimal for the right hand will be not optimal for the left. In other words, we cannot find an inequality for higher-dimensional systems which is optimal for both sides.

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