Maximal violation of the Clauser-Horne-Shimony-Holt inequality for two qutrits

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The Bell–Clauser-Horne-Shimony-Holt (BCHSH) inequality (in terms of correlation functions) of two qutrits is studied in detail by employing tritter measurements. A uniform formula for the maximum value of this inequality for tritter measurements is obtained. Based on this formula, we show that nonmaximally entangled states violate the BCHSH inequality more strongly than the maximally entangled one. This result is consistent with what was obtained by Acín *et al.* [Phys. Rev. A **65**, 052325 (2002)] using the Bell–Clauser-Horne inequality (in terms of probabilities).

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I. INTRODUCTION

Bell inequality has been proved to be not only as a tool for exposing the weirdness of quantum mechanics, but also as a more powerful resource in a number of applications, such as in quantum communication. Bell–Clauser-Horne-Shimony-Holt (BCHSH) inequality has been applied in communicating protocol (Ekert protocol) to detect the presence of the eavesdropper [1]. Furthermore, it has been found that two entangled d-dimensional systems (qudits) generate correlations that are more robust against noise than those generated by two entangled qubits [2–5]. It was suggested that the higher-dimensional entangled systems may be much superior than two-dimensional systems in quantum communication. Naturally, the extension of the protocol (involving BCHSH inequality) to higher dimension becomes an interesting problem. So, it is necessary and important to investigate the Bell inequality for higher-dimensional systems.

In an interesting paper [6], by using the Bell–Clauser-Horne inequality (in terms of probabilities) [4,5], Acín *et al.* have shown that nonmaximally entangled states violate the BCHSH inequality more strongly than the maximally entangled one. Recently, a BCHSH inequality (in terms of correlation functions) of two qutrits has been obtained [7] by searching the inequality which can give the minimal noise admixture F_{thr} for the maximally entangled states. The minimal noise admixture F_{thr} for the maximally entangled state of two qutrits has been obtained numerically by the method of linear optimization in Ref. [2] and analytically in Refs. [5,8]. The extension of the BCHSH to higher dimension is a nontrivial and interesting problem. Actually, it has been applied to quantum cryptography [9]. In this paper, we study the BCHSH inequality of two qutrits for tritter measurements by considering a class of pure states of two qutrits. A uniform formula of the maximum value of this inequality is obtained. Based on this formula, we find the states which give the maximum violation of the BCHSH inequality. This result is consistent with what was obtained by Acín *et al.* [6].

II. THE INEQUALITY

Let us consider a gedanken experiment with two observers, each measuring two observables on some state of two qutrits ρ. We denote the observables by $\hat{A}^i$ $(i=1,2)$ for the first observer (Alice) and $\hat{B}^j (j=1,2)$ for the second observer (Bob). The measurement of each observable yields three distinct outcomes denoted by a_1^i, a_2^i, and a_3^i for Alice's measurement of the observable, and b_1^j, b_2^j, and b_3^j for Bob's measurement of the observable. Specifically, the observables have the spectral decompositions $\hat{A}^i = a_1^i \hat{P}_1^i + a_2^i \hat{P}_2^i + a_3^i \hat{P}_3^i$, and $\hat{B}^j = b_1^j \hat{Q}_1^j + b_2^j \hat{Q}_2^j + b_3^j \hat{Q}_3^j$, where $\hat{P}_l^i$ $(l=1,2,3)$ and $\hat{Q}_m^j$ $(m=1,2,3)$ are mutually orthogonal projectors, respectively. The probability of obtaining the set of three numbers (a_l^i, b_m^j) in a simultaneous measurement of observables $\hat{A}^i$ and $\hat{B}^j$ on the state ρ is denoted by $P(a_l^i, b_m^j)$, which can be given by the standard formula

$$P(a_l^i, b_m^j) = \mathrm{Tr}(\rho \hat{P}_l^i \otimes \hat{Q}_m^j). \tag{1}$$

As introduced and used in Ref. [11], the correlation function $Q(\vec{\varphi}^{A_i}, \varphi^{B_j})$ (Q_{ij} for short) between Alice and Bob's measurements is

$$Q_{ij} = \sum_{l_i, m_j = 1}^{3} \alpha^{l_i + m_j} P(a_{l_i}, b_{m_j}), \tag{2}$$

where $\alpha = e^{i 2\pi/3}$. Let us define the following quantity:

$$S = \mathrm{Re}[Q_{11} + Q_{12} - Q_{21} + Q_{22}] + \frac{1}{\sqrt{3}} \mathrm{Im}[Q_{11} - Q_{12} - Q_{21} + Q_{22}]. \tag{3}$$

It can be shown [7], using the recently discovered Bell inequality for two qutrits [4], that according to local realistic theory S cannot exceed 2; i.e., $S \leqslant 2$ for local realistic theory. However, when using the quantum correlation function given in Eq. (2), $S_{\max}$ acquires the value $\frac{2}{9}(6+4\sqrt{3}) \approx 2.872\,93$ for the state $|\psi\rangle = (1/\sqrt{3}) \Sigma_i^3 |i\rangle |i\rangle$, the maximally entangled state. Following Ref. [2], we define the threshold noise admixture F_{thr} (the minimal noise admixture fraction for $|\psi\rangle$)

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$F_{thr}=1-2/S_{\max}$. Then for the maximally entangled two qutrits, we have $F_{thr}=0.30385$. For the maximally entangled two qubits, one has $F_{thr}=0.29289$. Obviously, entangled qutrits are more resistant to noise than entangled qubits [2,8].

As suggested in Refs. [7] and [4], the BCHSH inequality for two qutrits can be expressed as

$$-4\leqslant S\leqslant 2. \tag{4}$$

On the other hand, the interesting thing is the maximal F_{thr} of two qutrits obtained in Ref. [10] by the numerical linear optimization method. The authors found that the optimal nonmaximally entangled state of two qutrits is around 3% more resistant to noise than the maximally entangled one. The maximal F_{thr} is given by $F_{thr}=0.31386$ for such state (a nonmaximally state). Similar result is obtained in Ref. [6]. Obviously, the maximal violation of the inequality should be 2.914 85 for such nonmaximally entangled states.

For simplicity, we consider a gedanken experiment in which Alice and Bob's observables are defined by unbiased symmetric six-port beam splitter on the state of two qutrits,

$$|\psi\rangle=\frac{1}{\sqrt{3}}\sum_{i}^{3} a_i|i\rangle|i\rangle, \tag{5}$$

with real coefficients a_i; the kets $|i\rangle$ $(i=1,2,3)$ denote the orthonormal basis states for the qutrit. The unbiased symmetric six-port beam splitter, called tritter [12,13], is an optical device with three input and output ports. In front of every input port there is a phase shifter that changes the phase of the photon entering the given port. The observers select the specific local observables by setting appropriate phase shifts in the beams leading to the entry ports of the beam splitters. Such a process performs a unitary transformation between "mutually unbiased" bases in the Hilbert space [14–16]. The overall unitary transformation performed by such a device is given by

$$U_{ij}=\frac{1}{\sqrt{3}}\alpha^{(i-1)(j-1)}e^{i\varphi_j},\quad i,j=1,2,3, \tag{6}$$

where $\alpha=e^{i2\pi/3}$ and j denotes an input beam to the device, and i an output one; φ_j are the three phases that can be set by the local observer, denoted as $\vec{\varphi}=(\varphi_1,\varphi_2,\varphi_3)$. The transformations at Alice's side are denoted as $\vec{\varphi}^A=(\varphi_1^A,\varphi_2^A,\varphi_3^A)$, and $\vec{\varphi}^B=(\varphi_1^B,\varphi_2^B,\varphi_3^B)$ for Bob's side.

The observables measured by Alice and Bob are now defined as follows. The set of projectors for Alice's ith measurement is given by $\hat{P}_l^i=U_A^+(\vec{\varphi}^{A_i})|l\rangle\langle l|U_A(\vec{\varphi}^{A_i})$ $(l=1,2,3)$, where $U_A(\vec{\varphi}^{A_i})$ is the matrix of Alice's unbiased symmetric six-port beam splitter defined by Eq. (6). Bob's jth measurement is given by $\hat{Q}_m^j=U_B^+(\vec{\varphi}^{B_j})|m\rangle\langle m|U_B(\vec{\varphi}^{B_j})$ $(m=1,2,3)$. Then, from Eqs. (1) and (2), the correlation function for state $|\psi\rangle$ reads

$$Q_{ij}=\sum_{n,k}^{3}\sum_{l_i,m_j}^{3} a_n a_k \alpha^{l_i+m_j}(\alpha^*)^{(n-1)(l_i+m_j-2)}\times\alpha^{(k-1)(l_i+m_j-2)}e^{i(\varphi_k^{A_i}+\varphi_k^{B_j}-\varphi_n^{A_i}-\varphi_n^{B_j})}. \tag{7}$$

This shows that the results of the measurement obtained by Alice and Bob are strictly correlated.

In the following, we will investigate the BCHSH inequality (4) for the tritter measurements and give analytical discussions of above results.

III. THE MAXIMAL VIOLATION

By substituting Eq. (7) into Eq. (3), after some elaboration, we obtain

$$S=a_1a_2T_{12}+a_1a_3T_{13}+a_2a_3T_{23}, \tag{8}$$

where

$$\begin{aligned}T_{12}=\frac{1}{9}[&3\cos(\varphi_1^{A_2}-\varphi_2^{A_2}+\varphi_1^{B_1}-\varphi_2^{B_1})-3\cos(\varphi_1^{A_1}-\varphi_2^{A_1}\\&+\varphi_1^{B_1}-\varphi_2^{B_1})-3\cos(\varphi_1^{A_2}-\varphi_2^{A_2}+\varphi_1^{B_2}-\varphi_2^{B_2})\\&-\sqrt{3}\sin(\varphi_1^{A_2}-\varphi_2^{A_2}+\varphi_1^{B_1}-\varphi_2^{B_1})+\sqrt{3}\sin(\varphi_1^{A_1}-\varphi_2^{A_1}\\&+\varphi_1^{B_1}-\varphi_2^{B_1})+2\sqrt{3}\sin(\varphi_1^{A_1}-\varphi_2^{A_1}+\varphi_1^{B_2}-\varphi_2^{B_2})\\&+\sqrt{3}\sin(\varphi_1^{A_2}-\varphi_2^{A_2}+\varphi_1^{B_2}-\varphi_2^{B_2})],\end{aligned} \tag{9}$$

$$\begin{aligned}T_{13}=-\frac{1}{9}[&3\cos(\varphi_1^{A_1}-\varphi_3^{A_1}+\varphi_1^{B_1}-\varphi_3^{B_1})-3\cos(\varphi_1^{A_2}-\varphi_3^{A_2}\\&+\varphi_1^{B_1}-\varphi_3^{B_1})+3\cos(\varphi_1^{A_2}-\varphi_3^{A_2}+\varphi_1^{B_2}-\varphi_3^{B_2})\\&+\sqrt{3}\sin(\varphi_1^{A_1}-\varphi_3^{A_1}+\varphi_1^{B_1}-\varphi_3^{B_1})-\sqrt{3}\sin(\varphi_1^{A_2}-\varphi_3^{A_2}\\&+\varphi_1^{B_1}-\varphi_3^{B_1})+2\sqrt{3}\sin(\varphi_1^{A_1}-\varphi_3^{A_1}+\varphi_1^{B_2}-\varphi_3^{B_2})\\&+\sqrt{3}\sin(\varphi_1^{A_2}-\varphi_3^{A_2}+\varphi_1^{B_2}-\varphi_3^{B_2})],\end{aligned} \tag{10}$$

and

$$\begin{aligned}T_{23}=-\frac{1}{9}[&3\cos(\varphi_2^{A_1}-\varphi_3^{A_1}+\varphi_2^{B_1}-\varphi_3^{B_1})-3\cos(\varphi_2^{A_2}-\varphi_3^{A_2}\\&+\varphi_2^{B_1}-\varphi_3^{B_1})+3\cos(\varphi_2^{A_2}-\varphi_3^{A_2}+\varphi_2^{B_2}-\varphi_3^{B_2})\\&-\sqrt{3}\sin(\varphi_2^{A_1}-\varphi_3^{A_1}+\varphi_2^{B_1}-\varphi_3^{B_1})+\sqrt{3}\sin(\varphi_2^{A_2}-\varphi_3^{A_2}\\&+\varphi_2^{B_1}-\varphi_3^{B_1})-2\sqrt{3}\sin(\varphi_2^{A_1}-\varphi_3^{A_1}+\varphi_2^{B_2}-\varphi_3^{B_2})\\&-\sqrt{3}\sin(\varphi_2^{A_2}-\varphi_3^{A_2}+\varphi_2^{B_2}-\varphi_3^{B_2})]\end{aligned} \tag{11}$$

are three continuous functions of 12 angles $\vec{\varphi}^{A_i}$ and $\vec{\varphi}^{B_j}$ $(i,j=1,2)$. So, S is the continuous function of the twelve variables. The points which satisfy

$$\frac{\partial S}{\partial \varphi_j^{\Lambda_i}}=0 \quad (\Lambda=A,B;\ i=1,2;\ \text{and}\ j=1,2,3) \tag{12}$$

are the critical points of the function S. According to the theory of extreme points of continuous functions, we know that the extreme points belong to the critical points of the function. So, we can extract the maximum and minimum of S from the critical points by comparing the value of S among the critical points, since the maximum and minimum points must be one of the extreme points.

On the other hand, we can know that $|t_{12}|\leqslant\frac{4}{3}, |t_{13}|\leqslant\frac{4}{3}$, and $|t_{23}|\leqslant\frac{4}{3}$. However, the above three formulas are strongly correlated, so t_{12}, t_{13}, and t_{23} cannot reach their maximum value at the same time. It happens that when one of t_{12}, t_{13}, and t_{23} reaches its maximum value $\frac{4}{3}$, the others can reach their submaximum value $4/3\sqrt{3}$. If we consider t_{12}, t_{13}, and t_{23} as three coordinates, then they can form a complicated polyhedron. The polyhedral vertices are the points where t_{12}, t_{13}, and t_{23} reach their extreme values.

Lemma. For the formula $G=\Sigma_{i=1}^{N}\xi_i R_i$, where ξ_i are N real parameters, the maximum (minimum) points of G must be on the boundary of the region formed by R_i for any ξ_i.

Proof. Giving $G^0=\Sigma_{i=1}^{N}\xi_i R_i^0$, if R_i^0 $(i=1,2,\ldots,N)$ are in the inner region formed by R_i, we can always have $G=G^0+\Sigma_{i=1}^{N}\xi_i\Delta R_i$, in which ΔR_i are infinitesimal values satisfying $\xi_i\Delta R_i>0$ $(i=1,2,\ldots,N)$, so that $G>G^0$; or ΔR_i are infinitesimal values satisfying $\xi_i\Delta R_i<0$ $(i=1,2,\ldots,N)$, so that $G<G^0$. So, we can know that the maximum (minimum) points of G can only find on the boundary.

Theorem. The maximum and minimum values of S for a given state (5) must be found at the vertices of polyhedron formed by t_{ij} $(i\neq j;\ i,j=1,2,3)$.

Proof. We know that the maximum points of S belong to the critical points of S. For the critical points in the inner region formed by (t_{12},t_{13},t_{23}), from the Lemma we know that the value of such critical points must be less than some values of S on the boundary, so they cannot be the maximum points of S. For the same reason, if the critical point on the boundary (except for vertices), we can know that the value of S on this point must be less than S on one of the vertices on this boundary. Then, the maximum value of S must be only found on the vertices of the region formed by (t_{12},t_{13},t_{23}). In analogy with the above discussion, the minimum value of S can also be found on the vertices.

To find out the maximum (minimum) value, we have to calculate the vertices of the polyhedron formed by t_{ij}. For convenience, we denote T_1 as one of $\{t_{12},t_{13},t_{23}\}$, T_2 as one of $\{t_{12},t_{13},t_{23}\}/\{T_1\}$, and T_3 as one of $\{t_{12},t_{13},t_{23}\}/\{T_1,T_2\}$; where $\{\}/\{\}$ means division of sets, namely, if $T_1=t_{12}$, then $T_2\in\{t_{12},t_{13},t_{23}\}/\{t_{12}\}=\{t_{13},t_{23}\}$, and so on. In the following, we list the vertices formed by the maximum and submaximum of t_{ij} (it is enough),

$$(|T_1|,|T_2|,|T_3|)=\left(\frac{4}{3},\frac{4}{3\sqrt{3}},\frac{4}{3\sqrt{3}}\right) \quad \text{for} \quad T_1T_2T_3>0, \tag{13}$$

and

$$(|T_1|,|T_2|,|T_3|)=\left(\frac{4}{3},\frac{4}{3},\frac{4}{3}\right) \quad \text{for} \quad T_1T_2T_3<0. \tag{14}$$

Comparing the value of S among these points, we can obtain the maximum and minimum values of S for state (5). Assuming $\{K_i(i=1,2,3)\}=\{|a_1a_2|,|a_1a_3|,|a_2a_3|\}$, where "=" means the equality of two sets, and K_i are in decreasing order, i.e., $K_1\geqslant K_2\geqslant K_3$, let us define

$$S_1(|\psi\rangle)=\frac{4}{3}K_1+\frac{4}{3\sqrt{3}}(K_2+K_3) \tag{15}$$

and

$$S_2(|\psi\rangle)=\frac{4}{3}(K_1+K_2-K_3). \tag{16}$$

Then, we can know that the maximum value of S must be

$$S_{\max}(|\psi\rangle)=\max[S_1(|\psi\rangle),S_2(|\psi\rangle)]. \tag{17}$$

From Eqs. (15) and (16), we know that $S_2(|\psi\rangle)\geqslant S_1(|\psi\rangle)$ only for $K_3/K_2\leqslant 2-\sqrt{3}$. If taking $\Sigma_i a_i^2=3$ into account, one can prove that when $\max(|a_1|,|a_2|,|a_3|)\geqslant(6+3\sqrt{3})^{1/2}/2=1.673\,03$, $S_2(|\psi\rangle)\geqslant S_1(|\psi\rangle)$. Let us define $A_{\max}=\max(|a_1|,|a_2|,|a_3|)$, finally we obtain that

$$S_{\max}(|\psi\rangle)=\begin{cases}\frac{4}{3}K_1+(4/3\sqrt{3})(K_2+K_3), & A_{\max}\leqslant\frac{(6+3\sqrt{3})^{1/2}}{2}\\ \frac{4}{3}(K_1+K_2-K_3), & A_{\max}>\frac{(6+3\sqrt{3})^{1/2}}{2}.\end{cases} \tag{18}$$

We can also prove that the minimum of S is

$$S_{\min}(|\psi\rangle)=-\frac{4}{3}(K_1+K_2+K_3). \tag{19}$$

Obviously one can easily find that for maximally entangled state $|\psi\rangle=(1/\sqrt{3})\Sigma_i^3|i\rangle|i\rangle$ (i.e., $a_i=1$), we have $S_{\max}=\frac{2}{9}(6+4\sqrt{3})$ and $S_{\min}=-4$, which are the same as the results obtained in Refs. [4,6,10,17].

In Fig. 1, we give the comparison between the theoretical results and the numerical calculations obtained by multi-random-search optimization method, which shows a perfect agreement; (a) for $S_{\max}$ and (b) for $S_{\min}$, in which a_1 changes in region $[-\sqrt{3},\sqrt{3}]$, $a_2=\sqrt{(3-a_1^2)\varepsilon}$, and $a_3=\sqrt{(3-a_1^2)(1-\varepsilon)}$ $(0\leqslant\varepsilon\leqslant 1)$. One can find some inflection points in Fig. 1(a), for example, at the point $a_1=1$ when $\varepsilon=0.5$. These inflexion points are due to the discontinuous change of K_1, the maximum value among $|a_1a_2|,|a_1a_3|$, and $|a_2a_3|$, e.g., for $\varepsilon=0.5$, $K_1=a_2a_3=[(3-a_1^2)/2]$ when $a_1\leqslant 1$, but when $a_1>1$, $K_1=a_1a_2=a_1\sqrt{(3-a_1^2)/2}$. On the

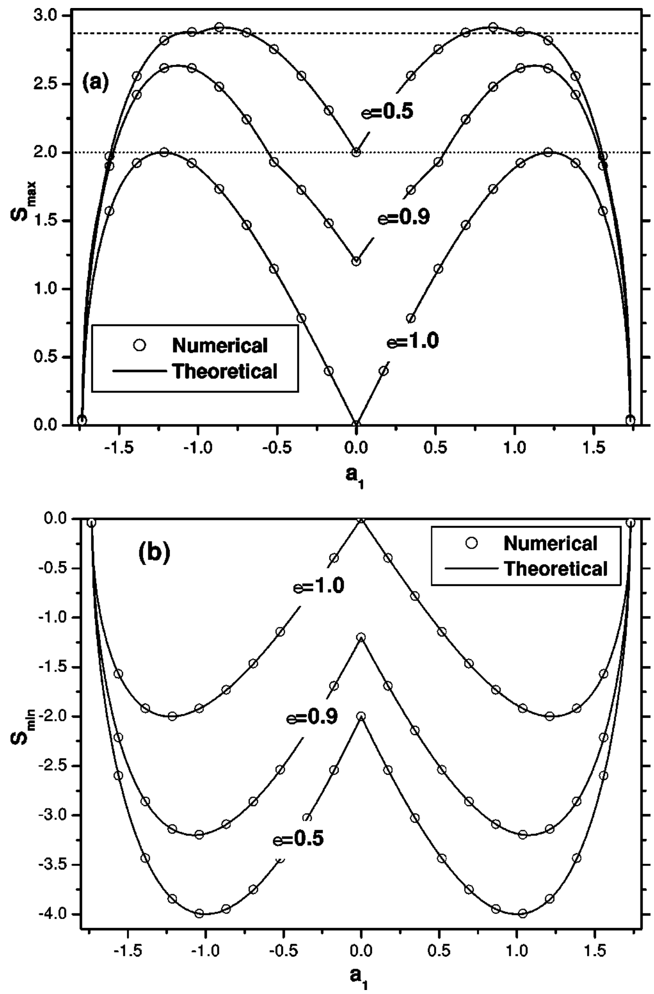

FIG. 1. (a). The maximal value of the inequality for tritter measurements, S_{max}, for the state given by Eq. (5), where a_1 changes in region $[-\sqrt{3},\sqrt{3}]$; $a_2=\sqrt{(3-a_1^2)\varepsilon}$; and $a_3=\sqrt{(3-a_1^2)(1-\varepsilon)}$, $0\leqslant\varepsilon\leqslant1$. The solid lines are theoretical results, circles are numerical dates, the dotted line shows the maximal value predicted by the local realistic theory, and the dashed line marks the value of the maximally entangled states. (b) The minimal value of the inequality, S_{min}.

other hand, we can see from Fig. 1(a) that the maximally entangled states are not the states that give the maximal violation of the Bell inequality.

Consider a_i as variables; we can obtain the maximal value of S_{max} (denoted as $\overline{S}_{max}$), by calculating the extreme value of Eq. (18), and after some elaboration we get

$$\overline{S}_{max}=1+\sqrt{\frac{11}{3}}, \tag{20}$$

when

$$\{|a_1|,|a_2|,|a_3|\}=\left\{\left[\frac{3}{2}\left(1-\sqrt{\frac{3}{11}}\right)\right]^{1/2},\sqrt{\frac{3-a_1^2}{2}},\sqrt{\frac{3-a_1^2}{2}}\right\}. \tag{21}$$

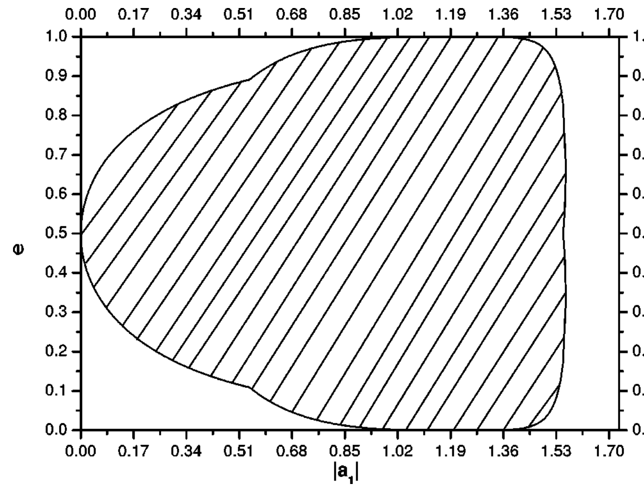

FIG. 2. The figure shows the states that violate the inequality for tritter measurements. The states in the shaded region violate the inequality.

One sees that for this value the threshold amount of noise is about $F_{thr}=0.3139$, which is the same as what has been obtained in recent calculations [6,10,17]. So, this result gives another evidence for inequality (4).

On the other hand, we can also calculate the minimum value of S_{min}, denoted as $\overline{S}_{min}$,

$$\overline{S}_{min}=-4 \quad \text{for} \quad \{|a_1|,|a_2|,|a_3|\}=\{1,1,1\}. \tag{22}$$

Then, we can know that

$$0\leqslant S_{max}\leqslant 1+\sqrt{\frac{11}{3}}, \quad -4\leqslant S_{min}\leqslant 0. \tag{23}$$

Obviously, for tritter measurements, the left hand of inequality (4) would never be violated, and the right hand only be violated by some of pure states. We can easily find the states that violate the inequality for tritter measurements from formula (18). In Fig. 2, we show the states described by $[a_1,a_2=\sqrt{(3-a_1^2)\varepsilon},a_3=\sqrt{(3-a_1^2)(1-\varepsilon)}]$, which violate the inequality for tritter measurements. The states which violate the inequality are in the shadow region; the states of which $S_{max}=2$ are on the boundary of the shadow region; the states in other regions cannot violate the inequality for tritter measurements.

We should add here that some similar calculations as well as some equivalence results were obtained by Cereceda [17], where the author compared some of the two-qutrit inequalities and investigated them in detail.

IV. DISCUSSION

In the above discussion we only concentrate on tritter measurements which can be easily carried out for technology used nowadays [12]. By studying the BCHSH inequality of two qutrits in detail, we give formulas of the maximum and minimum values of this inequality, and obtain the states which give the maximal violation of the BCHSH inequality. The maximal violation we obtained is the same as Refs. [6,10].

Indeed, one should use general measurements to study the problem of maximizing the Bell violation for a state, or in other words, for some states the tritter measurements are not optimal.

So, some states do not violate the inequality using tritter measurements, but may violate the inequality when general measurements are taken into account [17]. For example, for the state with $|a_1|=1.56$ and $\varepsilon=0.5$, $S_{\max}=1.964$ for tritter measurements, which does not violate the inequality; but if we employ the following measurements: $\hat{P}_l^i=U_A^+(\vec{\varphi}^{A_i})|x_l\rangle\langle x_l|U_A(\vec{\varphi}^{A_i})$ $(l=1,2,3)$ and $\hat{Q}_m^j=U_B^+(\vec{\varphi}^{B_j})|x_m\rangle\langle x_m|U_B(\vec{\varphi}^{B_j})$ $(m=1,2,3)$, where $|x_1\rangle=(1/\sqrt{2})[|1\rangle+|2\rangle]$, $|x_2\rangle=(1/\sqrt{2})[|1\rangle-|2\rangle]$, and $|x_3\rangle=|3\rangle$ are orthonormal bases, we can obtain $S_{\max}=2.0132$ (violates the inequality).

However, by employing tritter measurements, it can reveal many important properties of Bell inequality of entangled two qutrits. For instance, for the maximally entangled state $|\psi\rangle=(1/\sqrt{3})\Sigma_i^3|i\rangle|i\rangle$ and the states that maximally violate the inequality, the tritter measurements are optimal, and based on such entangled qutrit pairs a cryptographic protocol has been presented more recently [9] by employing tritter measurements.

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[1] A.K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[2] D. Kaszlikowski, P. Gnaciński, M. Żukowski, W. Miklaszewski, and A. Zeilinger, Phys. Rev. Lett. **85**, 4418 (2000).
[3] T. Durt, D. Kaszlikowski, and M. Żukowski, Phys. Rev. A **64**, 024101 (2001).
[4] D. Kaszlikowski, L.C. Kwek, J.L. Chen, M. Żukowski, and C.H. Oh, Phys. Rev. A **65**, 032118 (2002).
[5] D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).
[6] A. Acín, T. Durt, N. Gisin, and J.I. Latorre, Phys. Rev. A **65**, 052325 (2001).
[7] J.-L. Chen, D. Kaszlikowski, L.C. Kwek, and C.H. Oh, Mod. Phys. Lett. A **17**, 2231 (2002).
[8] J.-L. Chen, D. Kaszlikowski, L.C. Kwek, C.H. Oh, and M. Żukowski, Phys. Rev. A **64**, 052109 (2001).
[9] D. Kaszlikowski, D.K.L. Oi, M. Christandl, K. Chang, A. Ekert, L.C. Kwek, and C.H. Oh, Phys. Rev. A **67**, 012310 (2003).
[10] D. Kaszlikowski, D. Gosal, E.J. Ling, L.C. Kwek, M. Żukowski, and C.H. Oh, Phys. Rev. A **66**, 032103 (2002).
[11] C. Mattle, M. Michler, H. Weinfurter, A. Zeilinger, and M. Żukowski, Appl. Phys. B: Lasers Opt. **B60**, S111 (1995).
[12] A. Zeilinger, H.J. Bernstein, D.M. Greenberger, M.A. Horne, and M. Żukowski, in *Quantum Control and Measurement*, edited by H. Ezawa and Y. Murayama (Elsevier, Amsterdam, 1993); A. Zeilinger, M. Żukowski, M.A. Horne, H.J. Bernstein, and D.M. Greenberger, in *Quantum Interferometry*, edited by F. DeMartini and A. Zeilinger (World Scientific, Singapore, 1994).
[13] M. Żukowski, A. Zeilinger, and M.A. Horne, Phys. Rev. A **55**, 2564 (1997).
[14] J. Schwinger, Proc. Natl. Acad. Sci. U.S.A. **46**, 570 (1960).
[15] I.D. Ivanovic, J. Phys. A **14**, 3241 (1981).
[16] W.K. Wootters, Found. Phys. **16**, 391 (1986).
[17] J.L. Cereceda, e-print quant-ph/0212117.