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Optimal building block of multipartite quantum battery in the driven-dissipative charging

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Abstract

To take quantum advantage of collective effects in many-body system, we design an elementary block for building a quantum battery with the optimal number of atoms in a common thermal bath, which is charged collectively by a harmonic driving. Free energy is a novel tool to quantify usable energy in an open system, which includes non-preserved entropy impacts on the stored energy besides the internal energy. The interesting finding is that the free energy variation in the steady state increases non-monotonically, and reaches the maximal value at the optimal number of atoms. It ascribes to the decreasing of the internal energy and the entropy per atom with the increasing of the atoms. In particular, the elementary block with the optimal number of atoms can relax to the optimal steady state with the weak damping of the internal energy due to the strong collective driving. By comparing to each atom parallel charging independently, the optimal battery cell produces lower heat flow to the thermal bath induced by the entropy, which can not be neglected in the dissipative system. The existence of the optimal battery cell provide a guideline for designing a realizable charging scheme.

1. Introduction

Recently, diligent efforts are devoted to explore the possibility of taking advantage of quantum resources to achieve superior performances in the energy conversion and storage with the control achievement on multipartite quantum system [1–3]. Quantum battery (QB) is a quantum system for storing energy supplied by an external source. The battery exploits quantum effects for efficient charging in comparison to its classical counterpart [4–9]. A renewed effort is devoted to enhance charging of multipartite batteries in a closed system as a consequence of quantum correlations in many-body systems, which is known as collective effects [10–13].

When a multipartite battery is subjected to a common thermal bath, it gives rise to interesting quantum correlations such as the von Neumann entropy, which establishes a link to thermodynamics [14–16]. In the open charging system, a flow of heat to the bath accounting for non-preserved entropy has been overlooked. Much efforts have been devoted to investigate the QB for the energy storage in the thermal environment using different charging protocols [17–20]. An emergence of collective effects in quantum thermodynamics is attractive quantum phenomenon in dissipative systems [21–23]. Here, we focus on alternative aspect mostly unexplored, namely, free energy related to internal energy and entropy impacts on the cooperative many-body effects for the multipartite QB system. In thermodynamic charging process, free energy is an alternative tool to measure the amount of extractable work as useful energy [24, 25].

Since a harmonic driving as an external source has been proposed as an alternative powerful charging field due to the tunable driving parameters for maximal stored energy [11]. Inspired by the advantage of the

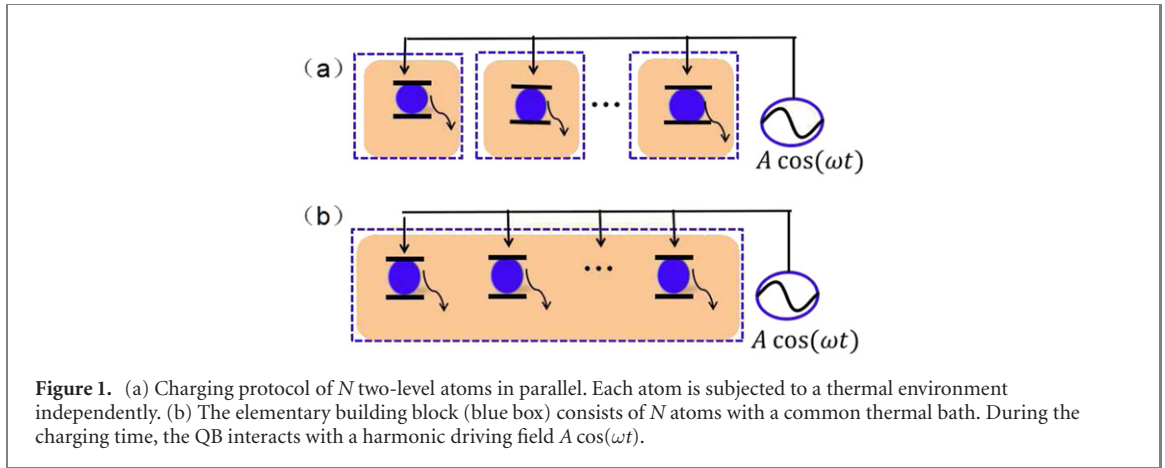


Figure 1. (a) Charging protocol of N two-level atoms in parallel. Each atom is subjected to a thermal environment independently. (b) The elementary building block (blue box) consists of N atoms with a common thermal bath. During the charging time, the QB interacts with a harmonic driving field $A \cos(\omega t)$.

collective effects and the harmonic driving, a multipartite QB in a common thermal bath, collectively coupled to a harmonic driving field, is an attractive battery model for optimal energy storage. The question is that whether collective effects in many-body system be harnessed to improve thermodynamically meaningful features in the driven-dissipative charging protocols.

We study an elementary block of the QB with an ensemble of two-level atoms in a common thermal bath, which are collectively charged by a harmonic driving field. We exploit the free energy to quantify the useful stored energy, which behaves non-monotonically dependent on the number of atoms N . The reason is that both of the internal energy and entropy per atom in the steady state decreases as N increase. It indicates that contributions of entropy can not be neglected in the energy storage. We find that the QB elementary with the optimal number of atoms relaxes to the optimal steady state, for which the internal energy decays weakly due to the strong driving compared to the dissipation. It is nontrivial to find the optimal elementary unit of the QB with the maximum free energy variation, which is prior to the parallel charging mode.

The paper is outlined as follows. In section 2, we propose a protocol for a multipartite battery consisting of N two-level atoms in a thermal bath, which is charged by a driving field. In section 3, we study the parallel-charging mode and the collective charging process, respectively. The internal energy, von Neumann entropy and free energy are calculated dependent on N . Finally, a brief summary is given in section 4.

2. A protocol for a multipartite QB charging

We consider an open charging system of a multipartite battery, which consists of two-level atoms coupled to an external driving as a charger to transfer energy. Figure 1(a) shows normal parallel charging strategy with independent thermal bath. Our charging protocol focus on the elementary building block (blue box), illustrated in figure 1(b), with N atoms in a common thermal bath. Atoms in each unit are collectively charged by a harmonic field. The total Hamiltonian consists of the QB-system part H_s and the interacting part H_I as

$$H = H_s + H_I, \quad (1)$$

$$H_s = \omega_0 J_z, \quad H_I = A \cos(\omega t) J_x, \quad (2)$$

where $J_\alpha = \sum_i^N \sigma_i^\alpha / 2$ ($\alpha = x, y, z$) is the collective operator of N two-level atoms with the energy level splitting ω_0 . A and ω are the driving amplitude and the modulated frequency. The Hamiltonian H is semi-classical Dicke model [26, 27]. Such systems can be described by the Dicke states $|J, m\rangle$ ($m = -J, \dots, J$), which are eigenstates of J^2 and J_z .

In the practical application, the QB are coupled also to the environment, which is modeled as the thermodynamics dissipation. We consider that the QB system weakly interacts with a thermal bath [28]. So each angular momentum J sector approximately evolves independently. We consider the Hilbert space in the maximum angular momentum sector $J = N/2$. And quantum correlations emerge in the symmetric Dicke states. Initially, N atoms decouple with the driving field. The initial state of the N atoms is the Gibbs thermal state $\rho_s^N(0) = e^{-\beta H_s} / Z$ with the partition function $Z = \sum_{m=-N/2}^{N/2} e^{-m\omega_0\beta}$.

3. Driven-dissipative dynamics for the collective charging process

The QB system evolution involves energy transferred from the coherent driving field and energy dissipation into the thermal thermal. To measure the amount of energy stored in the QB, the importance is to define the usable energy as extractable work. In the normal charging without thermal environment, the internal energy change of QB can be utilized in the later retraction. Yet, only part of the internal energy can be extracted in the charging process within the thermal environment. In the open system, von Neumann entropy quantifies the correlations in the QB system [29, 30], and is defined as $S(\rho_s^N) = -\text{Tr}(\rho_s^N \ln \rho_s^N)$. The entropy is not preserved in the evolution due to dissipation, which is different from the preserved entropy in a closed system. The corresponding heat flow to the thermal bath induced by the entropy is given by $Q = k_B TS$, which plays a negative contribution in the energy storage. Thus, the useful energy stored in the QB is measured by the free energy

$$F(\rho_s^N) = E(\rho_s^N) - k_B TS(\rho_s^N), \quad (3)$$

where $E(\rho_s^N) = \text{Tr}(H_s \rho_s^N)$ is the internal energy of the QB system.

For the charging dynamics, energy exchanges between the different subparts. The change of the energy is given by $\Delta E(t) = E(\rho_s^N(t)) - E(\rho_s^N(0))$, which is associate with the energy transferred from the driving field and the energy flow into the bath. Meanwhile, the change of th entropy is given by $\Delta S = S(\rho_s^N(t)) - S(\rho_s^N(0))$. So the difference in the free energy $\Delta F = \Delta E - k_B T \Delta S$ measures the useful energy stored in the QB. At zero temperature $T = 0$ or with no dissipation, the free energy change is equivalent to the internal energy, $\Delta F(\rho_s^N) = \Delta E(\rho_s^N)$. Here we only consider the situation with one thermal bath during the charging and later retraction process with the same temperature. The similar definition of the useful work is well discussed in the quantum thermodynamic resource discussions [31].

3.1. Parallel charging

We first consider the QB system in the parallel charging in figure 1(a). Each two-level atom couples to a distinct thermal bath at the temperature T . For $N = 1$ QB system, Liouvillian master equation describes the individual driven-dissipative dynamics

$$\begin{aligned} d\rho_s^1/dt = & -i \left[\frac{\omega_0}{2} \sigma_z + \frac{A}{2} \cos(\omega t) \sigma_x, \rho_s^1 \right] + \gamma [n(T) + 1] (2\sigma_- \rho_s^1 \sigma_+ - \{\sigma_+ \sigma_-, \rho_s^1\}) + \gamma n(T) \\ & \times (2\sigma_+ \rho_s^1 \sigma_- - \{\sigma_- \sigma_+, \rho_s^1\}), \end{aligned} \quad (4)$$

where the loss rate γ fixes the timescale of the dissipation process, and the mean occupation number of thermal photons at an inverse temperature $\beta = (k_B T)^{-1}$ is $n(T) = [\exp(\omega\beta) - 1]^{-1}$.

Initially, the QB system is in the Gibbs thermal state, $\rho_s^1(0) = e^{-\beta H_s} / Z$ with the partition function $Z = \text{Tr}(e^{-\beta H_s})$. The initial internal energy is $E_1(0)/\omega_0 = (e^{-\beta\omega_0/2} - e^{\beta\omega_0/2}) / (2Z)$. The dynamics of the internal energy is obtained approximately by neglecting the fast oscillation terms (see appendix A)

$$\begin{aligned} \frac{E_1(t)}{\omega_0} = & -\frac{\gamma^2 \chi}{2\gamma^2 \chi^2 + A^2} \left\{ 1 - \frac{e^{-\Gamma t}}{2\chi\gamma^2} \left\{ [(2\chi\gamma^2(1 + \alpha\chi) + \alpha A^2) \cos(\Omega t) \right. \right. \\ & \left. \left. - \frac{\gamma}{2\Omega} [2\gamma^2 \chi^2(1 + \alpha\chi) + A^2(4 + \alpha\chi)] \sin(\Omega t) \right\} \right\}, \end{aligned} \quad (5)$$

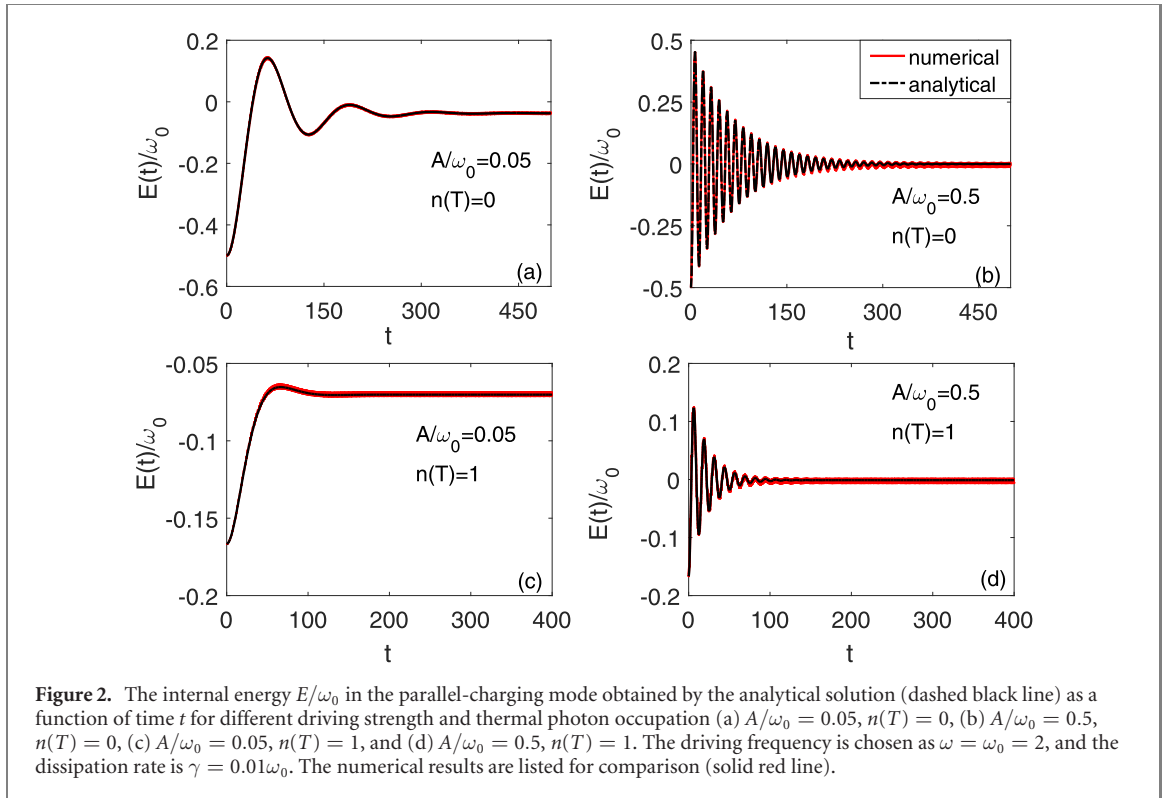
where $\alpha = \text{Tr}[\sigma_z \rho(0)]$, and the oscillation Rabi frequency is $\Omega = \sqrt{A^2 - \gamma^2 \chi^2 / 4}$ with $\chi = [1 + 2n(T)]$. Dissipation effects are reflected in the decoherent relaxation rate $\Gamma = 3\gamma\chi/2$, which results in fast damping for high temperature T . The driven system performs exponentially damped Rabi oscillations between the states $|g\rangle$ and $|e\rangle$, tending towards a steady state

$$\frac{E_1(t \rightarrow \infty)}{\omega_0} = -\frac{\gamma^2 \chi}{2\gamma^2 \chi^2 + A^2}. \quad (6)$$

In the absence of driving as $A \rightarrow 0$, it covers $E_1(t \rightarrow \infty)/\omega_0 \rightarrow -1/2$. In the opposite limit, when A becomes very large compared to γ , the internal energy reaches its maximal value $E_1(t \rightarrow \infty)/\omega_0 \rightarrow 0$ in the steady state, which is so-called the optimal steady state. The corresponding population is shared almost equally between the ground and excited states in the long time limit.

Especially, at zero temperature $T = 0$, equation (5) reduces into

$$\frac{E_1(t)}{\omega_0} = \frac{-\gamma^2}{2\gamma^2 + A^2} \left[1 + \frac{A^2}{2\gamma^2} e^{-3\gamma t/2} \left(\cos \Omega t + \frac{3\gamma}{2\Omega} \sin \Omega t \right) \right]. \quad (7)$$



For a strong coherent driving, E_1 in the steady state approaches to its maximum 0.

We calculate $E_1(\rho_s^1)$ by solving equation (4) numerically for the resonance $\omega = \omega_0$ in figure 2. The analytical solutions in equation (5) agree well with numerical ones. It is observed that a larger driving amplitude A results in better charging of the QB even in presence of dissipation in figures 2(b) and (d). As expected, higher temperature T with a larger $n(T)$ induces faster relaxation dynamics to reach the steady thermal state in figures 2(c) and (d). All curves present a damped oscillatory behaviors, and the amplitude is modulated by the exponential decay dictated by the decoherent relaxation given in equation (5). As the driving amplitude is very large compared to the dissipation rate, $A/\gamma = 50$, the system can relaxed to the optimal steady state with the maximum internal energy $E_1(t \rightarrow \infty)/\omega_0 \rightarrow 0$. It indicates that strong driving can transfer energy to the QB system to suppress the dissipation, resulting in maximizing the internal energy.

The useful energy stored of N atoms in the parallel charging mode by N copies of such single-atom QB is N times the free energy stored in the single-atom QB. In contrast to the parallel-charging mode, we are interested in the coherent charging of the atom ensemble, for which the stored energy is expected to scale nonlinearly with N .

3.2. An optimal elementary unit for collective charging

We now investigate the maximum stored energy when the N atoms are coupled to the common thermal bath in figure 1(b), so-called collective charging mode. In a realistic dissipation system of an ensemble of atoms, Liouvillian master equation is generally described by a sum over local channels for each atom,

$$\frac{d\rho_s(t)^N}{dt} = -i[H, \rho_s^N(t)] + \gamma[n(T) + 1]L[\sigma_-]\rho_s^N + \gamma n(T)L[\sigma_+]\rho_s^N, \quad (8)$$

where decoherence acts locally on each atom with the same rate γ via the Lindbladian $L[\sigma_{\pm}]\rho_s^N = \sum_{n=1}^N \sigma_{\pm}^n \rho_s^N (\sigma_{\pm}^n)^{\dagger} - (\sigma_{\pm}^n)^{\dagger} \sigma_{\pm}^n \rho_s^N / 2 - \rho_s^N (\sigma_{\pm}^n)^{\dagger} \sigma_{\pm}^n / 2$. The dimension of the Hilbert space is 2^N , which is intractable as N increases. For a collective process of an ensemble of atoms [19, 32, 33], there is identical coupling between all the atoms in the ensemble and the common bath. A decoherence model does respect the particle symmetry and preserves symmetric collective states, which are invariant under the permutation of particle labels. It thus is reasonable to consider the atom ensemble collective couplings to the environment in symmetric atomic dynamics. It has become common practice to study decoherence in spin ensemble by approximating the above equation by its associated collective process $L[J_{\pm}]\rho_s^N = J_{\pm} \rho_s^N J_{\pm}^{\dagger} - J_{\pm}^{\dagger} J_{\pm} \rho_s^N / 2 - \rho_s^N J_{\pm}^{\dagger} J_{\pm} / 2$. For the collective decoherence process, the open system of the QB dynamics is

governed by the master equation,

$$\frac{d\rho_s(t)^N}{dt} = -i[H, \rho_s^N(t)] + \gamma[n(T) + 1]L[J_-]\rho_s^N + \gamma n(T)L[J_+]\rho_s^N. \quad (9)$$

If all operators are collective, then the symmetric collective states span an invariant subspace, which cannot create coherence between different J sectors. Since we focus on symmetric Dicke state $|J, m\rangle$ and prepare the initial Gibbs thermal state in the angular momentum sector $J = N/2$. The system preserves the $N + 1$ -dimensional Hilbert space under such collective dissipation dynamics.

The initial energy is obtained as $E(0) = \omega_0 \text{Tr}[J_z \rho_s^N(0)] = \omega_0 \sum_{m=-N/2}^{N/2} m e^{-m\beta\omega_0} / Z$, which decreases to the lowest energy $-\omega_0/2$ as N increases.

It is interesting to analyze the useful stored energy in the steady state dependent on N . We use the Hosltein–Primakoff transformation in terms of auxiliary bosonic operators b^\dagger and b : $J_z = b^\dagger b - N/2$, $J_+ = b^\dagger \sqrt{N - b^\dagger b}$ and $J_- = \sqrt{N - b^\dagger b} b$. Equations of motion for mean values of the observables can be expressed as $\frac{d\langle O \rangle}{dt} = \text{Tr}(O \frac{d\rho_s^N}{dt})$, which gives explicitly as

$$\frac{d\langle b \rangle}{dt} = -i\omega_0 \langle b \rangle - iA\sqrt{N} - \frac{1}{2}\gamma N \langle b \rangle, \quad (10)$$

$$\frac{d\langle b^\dagger b \rangle}{dt} = -iA\sqrt{N}(\langle b^\dagger \rangle - \langle b \rangle) - \gamma N \langle b^\dagger b \rangle + \gamma N n(T). \quad (11)$$

In the long time limit, according to stationary condition $d\langle b \rangle/dt = 0$, one obtains the stable value $\langle b(t \rightarrow \infty) \rangle = A\sqrt{N}/(-\omega_0 + i\gamma N/2)$. In the limit $N \rightarrow \infty$, the internal energy per atom $E/(N\omega_0) = \langle b^\dagger b \rangle/N - \frac{1}{2}$ in the steady state is given analytically as

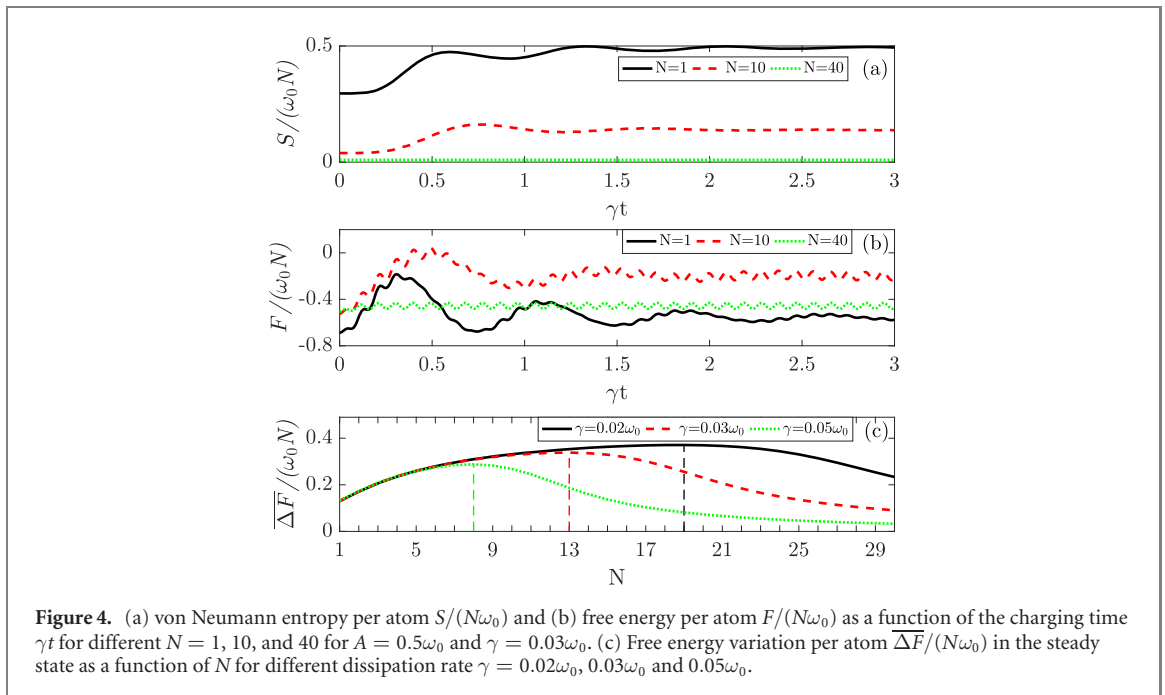
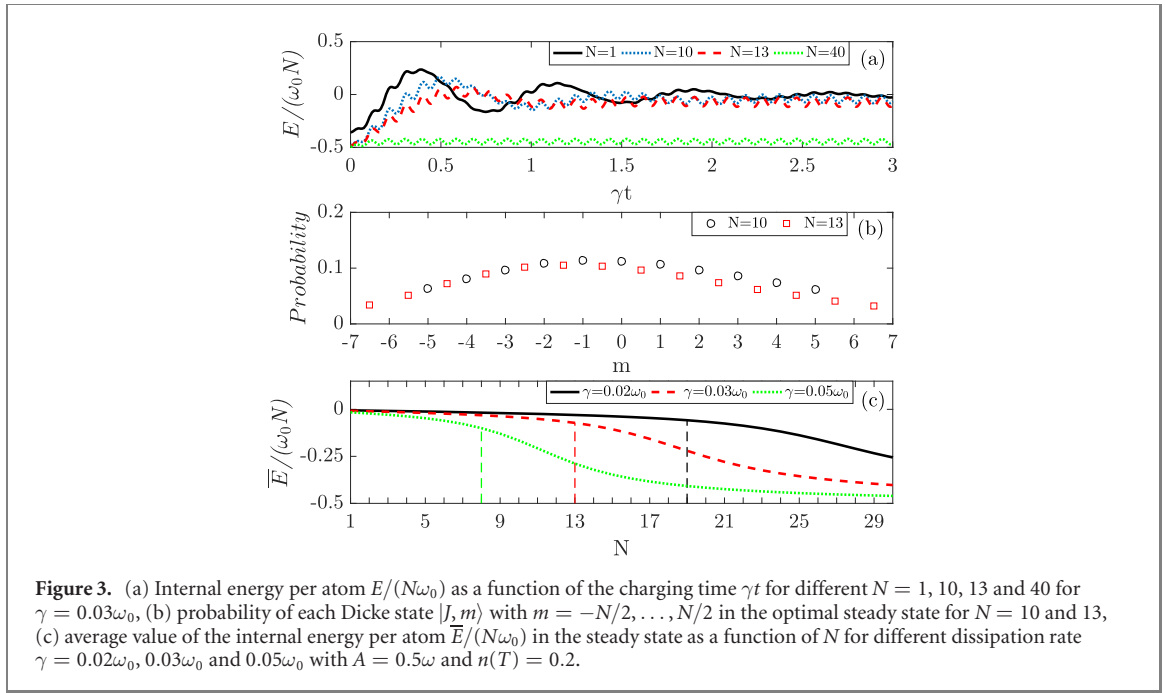
$$\frac{E(t \rightarrow \infty)}{N\omega_0} = -\frac{1}{2} + \frac{A^2}{\omega_0^2 + \gamma^2 N^2/4} + \frac{n(T)}{N}. \quad (12)$$

It is noted that the driving amplitude is strengthened proportional to $A\sqrt{N}$, which competes to the dissipative rate γN . In the limit $N \rightarrow \infty$, the dissipation rate γN is dominated, and $E/(N\omega_0)$ in equation (12) decreases rapidly to the lowest energy $-1/2$. From the analytical solutions, the internal energy per atom is expected to decrease from 0 to $-1/2$ as N increases.

For the QB system with finite atoms, we calculate the internal energy by solving equation (9) numerically in figure 3(a). As N increases up to 13, the internal energy per atom decreases slightly and almost tends to the maximal value $E(t \rightarrow \infty)/(N\omega_0) \rightarrow 0$ in the steady state. It is called the optimal steady state with almost the maximum internal energy. The reason is that the effective driving strength is larger than the dissipation rate for small values of N , $A\sqrt{N} \gg \gamma N$, and can transfer energy to suppress the internal energy flow to the bath. In the optimal steady state, populations for $N = 10$ and 13 are shared almost equally between arbitrary two symmetric Dicke states $|J, m\rangle$ and $|J, -m\rangle$ in figure 3(b), respectively. However, for $N = 40$, the dissipation is dominated, and the average internal energy per atom decays to $-1/2$. Consequence, the average internal energy per atom in the steady state $\bar{E}/(N\omega_0)$ decreases dependent on the system size N in figure 3(c). However, $\bar{E}/(N\omega_0)$ decreases very weakly for small values of N . It indicates that the QB system with small number atoms can achieve the optimal steady state with the small reduction of the internal energy in the end of charging process.

Besides the internal energy, a heat flow to the thermal environment induced by the entropy cannot be neglected in the free energy. In the absence of dissipation, von Neumann entropy of the QB system is preserved and unchanged. In the open system, the entropy is not preserved in the evolution in figure 4(a), which induces the heat dissipated to the bath. It is observed that the entropy of each atom decreases as N increases due to the increasing of dissipation. It demonstrates that the heat flow to the bath induced by the entropy is lower than that for the parallel charging mode with $N = 1$.

Since the internal energy and entropy per atom decreases in the steady state as N increases. Thus the free energy increases firstly for small values of N , then decreases for large values of $N = 40$ in figure 4(b). In the ending of the charging process, the useful energy stored in the QB is quantified by the free energy variation in the steady state $\overline{\Delta F}/(N\omega_0)$, which is defined as $\int_{\Delta t} [F(\rho(t \rightarrow \infty)) - F(\rho(0))] dt / (N\omega_0 \Delta t)$. It yields non-monotonic relations between the free energy variation and N in figure 4(c). A peak with the maximal value of $\overline{\Delta F}/(N\omega_0)$ is observed at an optimal number of atoms N_{op} . It is interesting to find that the system with N_{op} atoms can approximately relax to the optimal steady state with $E/(N\omega_0) \rightarrow 0$ in figure 3(c). As the ratio of A/γ increases, the optimal number N_{op} shifts to a larger value. In the absence of the dissipation, the stored energy increases linearly with N [11]. Here, we propose an efficient elementary unit of the QB for the maximum stored energy, which consists of N_{op} atoms in a common thermal bath in figure 1(b). In such



optimal QB elementary, the system can relax to the optimal steady state with the small damping of the internal energy and with lower heat dissipated to the bath by comparing to the parallel charging mode.

4. Conclusion

We have considered the multipartite battery comprising of an ensemble of two-level atoms in a thermal environment, which is charged by an external harmonic driving. For each atom charging in parallel, the system can relax to the optimal steady state with the maximal value of the internal energy by the analytical solution. Meanwhile, it produces heat dissipated to the bath induced by the non-preserved entropy. In the collective charging process, the internal energy and entropy per atom decreases as N increases, and it leads to non-monotonic relations between the free energy variation per atom and N . Consequence, we find an elementary unit of the QB with the optimal number of atoms to reach the maximum free energy variation in the ending of the charging process. The optimal QB elementary can relax to the optimal steady state with the small damping in the internal energy due to competition of coherent driving and decoherent

dissipation. Moreover, the heat induced by the entropy in the optimal QB elementary is lower than that for the parallel charging mode. In comparison to classical thermodynamics, our driven-dissipative charging protocol sheds new light on physically realizable charging schemes in open system.

Acknowledgments

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Analytical solutions for single-atom battery

The master equation of the single-atom battery system is given by

$$\begin{aligned} d\rho_s/dt = & -i\frac{\omega_0}{2}[\sigma_z, \rho_s] - i\frac{A}{4}(e^{i\omega t} + e^{-i\omega t})[\sigma_x, \rho_s] + \gamma[n(T) + 1](2\sigma_- \rho_s \sigma_+ - \{\sigma_+ \sigma_-, \rho_s\}) \\ & + \gamma n(T)(2\sigma_+ \rho_s \sigma_- - \{\sigma_- \sigma_+, \rho_s\}). \end{aligned} \quad (A1)$$

We perform a rotating-frame transformation using $U = \exp(i\omega t \sigma_z/2)$ to give

$$\begin{aligned} d\rho'_s/dt = & -i\frac{\omega_0 - \omega}{2}[\sigma_z, \rho'_s] - i\frac{A}{4}(e^{i\omega t} + e^{-i\omega t})[e^{i\omega t} \sigma_+ + e^{-i\omega t} \sigma_-, \rho'_s] \\ & + \gamma[n(T) + 1](2\sigma_- \rho'_s \sigma_+ - \{\sigma_+ \sigma_-, \rho'_s\}) \\ & + \gamma n(T)(2\sigma_+ \rho'_s \sigma_- - \{\sigma_- \sigma_+, \rho'_s\}), \end{aligned} \quad (A2)$$

where $\rho'_s = U\rho_s U^\dagger$. When the driving strength A is much smaller than the two-level energy ω_0 on resonance with the QB system $\omega_0 = \omega$, it is reasonable to making a rotating-wave approximation by ignoring fast oscillating terms. The Bloch equations are derived as

$$\langle \dot{\sigma}_z \rangle_\tau = i\frac{A}{2}(\langle \sigma_- \rangle_\tau - \langle \sigma_+ \rangle_\tau) - 2\gamma[\langle \sigma_z \rangle_\tau(2n(T) + 1) + 1], \quad (A3)$$

$$\langle \dot{\sigma}_+ \rangle_\tau = i\frac{A}{4}\langle \sigma_z \rangle_\tau - \gamma\langle \sigma_- \rangle_\tau(2n(T) + 1), \quad (A4)$$

$$\langle \dot{\sigma}_- \rangle_\tau = -i\frac{A}{4}\langle \sigma_z \rangle_\tau - \gamma\langle \sigma_+ \rangle_\tau(2n(T) + 1). \quad (A5)$$

For the initial Gibbs state with $\alpha = \text{Tr}[\sigma_z \rho(0)]$, one can solve the above equations analytically

$$\begin{aligned} \langle \sigma_z \rangle_\tau = & -\frac{2\gamma^2 \chi}{2\gamma^2 \chi^2 + A^2} \left\{ 1 - e^{-3\tau\gamma\chi/2} \frac{1}{2\chi\gamma^2} \left\{ [(2\chi\gamma^2(1 + \alpha\chi) + \alpha A^2) \cos(\Omega\tau) \right. \right. \\ & \left. \left. - \frac{\gamma[2\gamma^2\chi^2(1 + \alpha\chi) + A^2(4 + \alpha\chi)]}{2\Omega} \sin(\Omega\tau) \right\} \right\}, \end{aligned} \quad (A6)$$

where the oscillation Rabi frequency is $\Omega = \sqrt{A^2 - \frac{\gamma^2 \chi^2}{4}}$ with $\chi = [1 + 2n(T)]$.

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