Theory of nonlinear Landau-Zener tunneling

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We present a comprehensive analysis of the nonlinear Landau-Zener tunneling. We find characteristic scaling or power laws for the critical behavior that occurs as the nonlinear parameter equals to the gap of avoided crossing energy levels. For the nonlinear parameter larger than the energy gap, a closed-form solution is derived for the nonlinear tunneling probability, which is shown to be a good approximation to the exact solution for a wide range of the parameters. Finally, we discuss the experimental realization of the nonlinear model and possible observation of the scaling or power laws using a Bose-Einstein condensate in an accelerating optical lattice.

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I. INTRODUCTION

It is common in the study of quantum systems to consider only a finite number of energy levels that are strongly coupled. The special case of two coupled levels is of enormous practical interest, and a vast amount of literature has been devoted to the dynamical properties of the two-level systems [1]. One of the interesting phenomena is the Landau-Zener tunneling between energy levels. As a basic physical process [2], it has found wide applications in various systems, such as current driven Josephson junctions [3], atoms in accelerating optical lattices [4], and field-driven superlattices [5].

A nonlinear two-level system, where the level energies depend on the occupation of the levels, may arise in a mean-field treatment of a many-body system where the particles predominantly occupy two energy levels. For example, such a model arises in the study of the motion of a small polaron [6], a Bose-Einstein condensate in a double-well potential [7–9] or in an optical lattice [10,11], or for a small capacitance Joseph junction where the charging energy may be important. In contrast to the linear case, the dynamical property of a nonlinear two-level model is far from being fully understood, and many novel features have been revealed recently [12,13], including the discovery of a nonzero Landau-Zener tunneling probability even in the adiabatic limit when the nonlinear parameter C exceeds a critical value V.

In this paper, we present an analytic study on the nonlinear Landau-Zener tunneling. For the behavior near the critical point $C=V$, we find that the adiabatic tunneling probability between the two energy levels rises as a 3/2 power law of the function $C/V-1$. Below the critical point, the tunneling probability as a function of sweeping rates α follows an exponential law as in the linear case but with the exponent modified due to the nonlinearity. The explicit expression of the modification factor is obtained analytically, and it is found to decrease monotonously with the nonlinear parameter and tends to zero at the critical point, indicating the breakdown of the exponential law. Indeed, our analysis shows that the exponential law breaks down at the critical point and turns into a 3/4 power law. Beyond the critical regime, i.e., $C>V$, we employ the stationary phase method and obtain a closed-form solution of the nonlinear tunneling probability. This solution is compared with the numerical solution by integrating the Schrödinger equation; they exhibit a good agreement for a wide range of parameters. At the end, we discuss the possible experimental observation of our results with Bose-Einstein condensates (BECs) in accelerating optical lattices.

Our paper is organized as follows. In Sec. II we introduce the nonlinear two-level model and its equivalent classical Josephson Hamiltonian. We discuss the connection between the two representations in the context of breakdown of adiabatic tunneling. In Sec. III, we investigate the tunneling dynamics of the nonlinear Landau-Zener model near the critical regime and reveal the scaling or power laws that characterize the critical behavior. In Sec. IV, we show the exponential law of tunneling probability is modified by the nonlinearity in the subcritical regime. In Sec. V, we discuss the tunneling dynamics in the regime beyond the critical point and derive the nonlinear tunneling probability using the stationary phase approximation. In Sec. VI, we discuss how our findings may be observed experimentally.

II. NONLINEAR LANDAU-ZENER MODEL

Our model consists of two levels as in the standard Landau-Zener model but with an additional energy difference depending on the population in the levels. It is described by the following Hamiltonian [12]:

$$H(\gamma)=\begin{pmatrix} \frac{\gamma}{2}+\frac{C}{2}(|b|^2-|a|^2) & \frac{V}{2} \\ \frac{V}{2} & -\frac{\gamma}{2}-\frac{C}{2}(|b|^2-|a|^2) \end{pmatrix}, \tag{1}$$

where a and b are the probability amplitudes. The Hamiltonian is characterized by three parameters: the coupling V between the two levels, the level bias γ, and the nonlinear parameter C describing the level energy dependence on the populations. The amplitudes a and b satisfy the Schrödinger equation,

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix}=H(\gamma)\begin{pmatrix} a \\ b \end{pmatrix}, \tag{2}$$

which conserves the total probability $|a|^2+|b|^2$ that is set to be 1.

We want to examine the nonlinear Landau-Zener tunneling, i.e., how the system evolves when the level bias γ changes with time as $\gamma=\alpha t$. We call α sweeping rate. In this section, we focus on the adiabatic limit, that is, the sweeping rate α tends to zero.

As in the linear model, it is useful to find the adiabatic levels $\epsilon(\gamma)$ by diagonalizing the Hamiltonian (1). It is readily found that there are two eigenvalues when $C<V$ while there can be four eigenvalues when $C>V$, as demonstrated in Fig. 1. At $C/V=2$ [Fig. 1(b)], as the result of four eigenvalues, a loop appears at the tip of the lower level in the regime $-\gamma_c\leqslant\gamma\leqslant\gamma_c$, where

$$\gamma_c=(C^{2/3}-V^{2/3})^{3/2}. \tag{3}$$

The corresponding eigenstates are not orthogonal to each other for finite γ, but become so in the limits of $\gamma\to\pm\infty$, where $\epsilon\to\pm|\gamma|/2$. For instance, at the lower level, we have $(a,b)\to(1,0)$ at $\gamma\to-\infty$ and $(a,b)\to(0,1)$ at $\gamma\to+\infty$.

The direct consequence of the loop structure in Fig. 1(b), as first discussed in Ref. [12], is that as a quantum state moves along the lower lever to the singular point T, there is no way to go further except to jump to the upper and lower levels. As a result, the nonlinear Landau-Zener tunneling is not zero even in the adiabatic limit $\alpha\to 0$. The underlying mechanism of this interesting phenomenon is revealed with

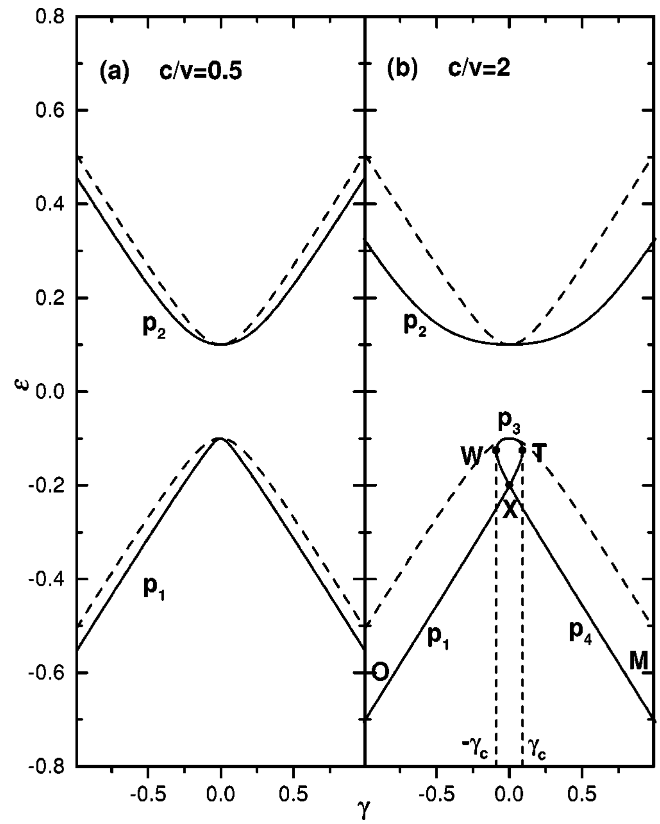

FIG. 1. Adiabatic energy levels (solid lines) for two typical nonlinear cases: (a) $C=0.1$, $V=0.2$; (b) $C=0.4$, $V=0.2$. The dashed lines are for the linear case $(C=0)$. The corresponding eigenstates are the fixed points P_i $(i=1,\ldots,4)$ of the H_e system (4) as shown in (b): $OXT\to P_1$, $MXW\to P_4$, $WT\to P_3$. Only P_3 is an unstable saddle point, others are stable elliptic points.

an equivalent classical Hamiltonian, where the nonzero adiabatic tunneling probability is viewed as the result of collision between fixed points.

With $a=|a|e^{i\theta_a}$ and $b=|b|e^{i\theta_b}$, we introduce the population difference $s=|b|^2-|a|^2$ and the relative phase $\theta=\theta_b-\theta_a$. In terms of s and θ, the nonlinear two-level system is cast into a classical Hamiltonian system [8,13],

$$H_e(s,\theta,\gamma)=\frac{C}{2}s^2+\gamma s-V\sqrt{1-s^2}\cos\theta, \tag{4}$$

which has the form of a Josephson Hamiltonian. The fixed points of the classical Hamiltonian correspond to the eigenstates of the nonlinear two-level system, and are given by the following equations:

$$\theta^*=0,\pi, \quad \gamma+Cs^*+\frac{Vs^*}{\sqrt{1-s^{*2}}}\cos\theta^*=0. \tag{5}$$

The number of the fixed points depends on the nonlinear parameter C. For weak nonlinearity, $C/V<1$, there exist only two fixed points (P_1 and P_2 in Fig. 2), corresponding to the maximum and minimum of the classical Hamiltonian. They are elliptic points, each being surrounded by closed (elliptic) orbits. The fixed points are located on the lines of

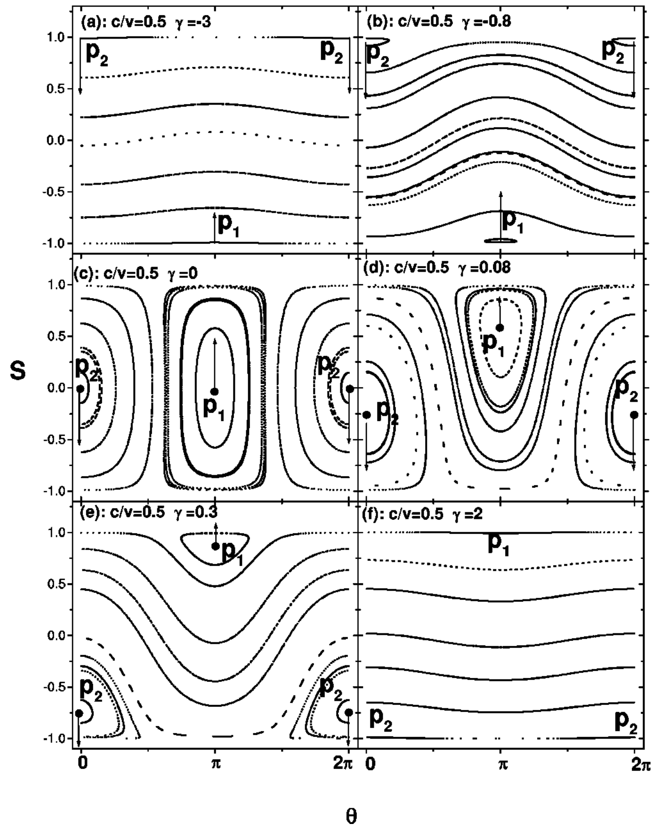

FIG. 2. Evolution of the phase-space motions of the Hamiltonian system H_e at $C/V=0.5$ as γ changes adiabatically. The arrows indicate the shifting direction of the fixed points P_i as γ increases. The closed curves are the periodic trajectories. In this case, no collision between fixed points occurs, implying zero adiabatic tunneling probability.

$\theta^*=\pi$ and 0, meaning that the two corresponding eigenstates of the two-level system have relative phase of π. As the level bias changes from $\gamma=-\infty$ to $+\infty$, P_1 moves smoothly along the line $\theta^*=\pi$ from the bottom ($s=-1$) to the top ($s=+1$), corresponding to the lower energy level in Fig. 1(a); the other point P_2 moves from the top to the bottom, corresponding to the upper level.

For stronger nonlinearity, $C/V>1$, two more fixed points appear in the window $-\gamma_c<\gamma<\gamma_c$. As shown in Figs. 3(c)–3(e), both of the new fixed points lie on the line $\theta^*=\pi$, one being elliptic (P_4) and the other being hyperbolic (P_3) as a saddle point of the classical Hamiltonian. One of the original fixed point P_2 still moves smoothly with γ, corresponding to the upper adiabatic level in Fig. 1(b). The other P_1 moves smoothly up to $\gamma=\gamma_c$, where it collides with P_3, corresponding to the branch OXT of the lower level in Fig. 1(b). The new elliptic point P_4, created at $\gamma=-\gamma_c$ together with P_3, moves up to the top, corresponding to the branch WXM of the lower level. The hyperbolic point P_3, moves down away from its partner P_4 after creation and is annihilated with P_1 at $\gamma=\gamma_c$, corresponding to the top branch WT of the lower level. The collision between P_1 and P_3 leads to nonzero adiabatic tunneling from the lower level to the upper level, which is determined by the eventual fate of the fixed point P_1.

III. CRITICAL BEHAVIOR NEAR $C=V$

A. Adiabatic tunneling

For adiabatic change of the level bias γ, a closed orbit in the classical dynamics remains closed and the action

$$I=\frac{1}{2\pi}\oint s\,d\theta \tag{6}$$

stays invariant in time according to the classical adiabatic theorem [14]. The change of γ is adiabatic as long as the relative change of γ in a period of the orbit is small. The action equals the phase-space area enclosed by the closed orbit, and is therefore zero for a fixed point. Since the closed orbits surrounding an elliptic fixed point all have finite periods T, they should evolve with the area of each fixed in time. We thus expect an elliptic fixed point to remain as a fixed point during the adiabatic change of the level bias γ. For the case of $C/V<1$, the two fixed points (both elliptic) evolve adiabatically throughout the entire sweeping of γ, implying the absence of transition between the eigenstates in the adiabatic limit. This is still true for the fixed point P_2 in the case $C/V>1$, meaning a state starting from the upper level will remain in the upper level.

The adiabaticity is broken, however, when P_1 collides with the hyperbolic fixed point P_3 to form a homoclinic orbit where the period T diverges. Nevertheless, the classical ''particle'' will remain on this orbit, because the orbit is surrounded from both outside and inside by closed orbits of finite periods, which form barriers to prevent the particle from escaping. After this collision, the homoclinic orbit turns into an ordinary closed orbit of finite period, and evolves adiabatically for $\gamma>\gamma_c$ according to the rule of constant action, which is now nonzero. This orbit eventually evolves into a straight line of constant s.

With these observations, we can obtain the tunneling probability in the adiabatic limit,

$$\Gamma_{ad}=\frac{1}{2}I(s_c)=\frac{1}{4\pi}\oint s(\theta;E_c)\,d\theta, \tag{7}$$

where

$$s_c=-\sqrt{1-(V/C)^{2/3}} \tag{8}$$

and

$$E_c=\frac{C}{2}s_c^2+\gamma_c s_c-V\sqrt{1-s_c^2}. \tag{9}$$

The above analysis is consistent with the nonlinear hysteresis phenomenon presented in Ref. [13], where a similar formula for adiabatic tunneling probability was obtained.

The adiabatic tunneling probability can be evaluated analytically in the critical region of $\delta=C/V-1\to 0$. The singular point of the level bias is found to leading order as

$$\gamma_c\simeq V(\tfrac{2}{3}\delta)^{3/2}. \tag{10}$$

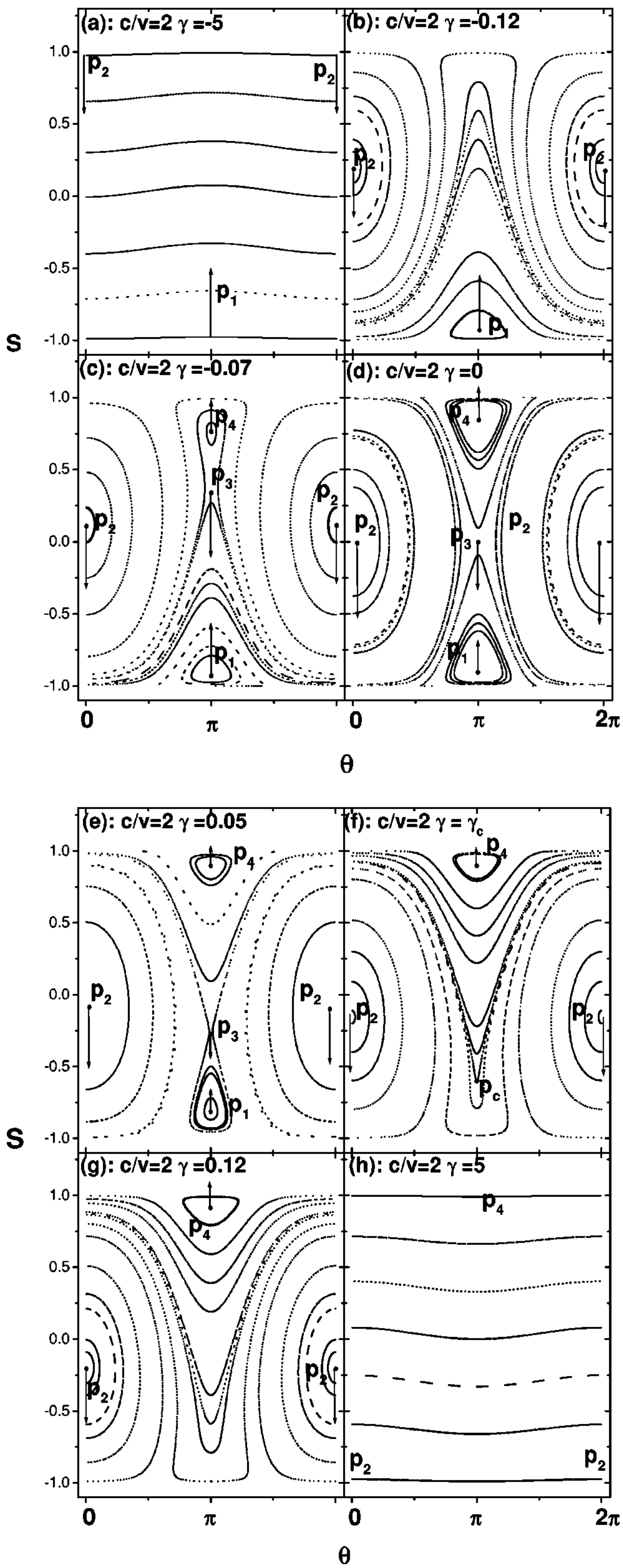

FIG. 3. Evolution of the phase-space motions of the Hamilton H_e system at $C/V=2$ as γ changes adiabatically. The arrows refer to the moving directions of the fixed points as γ increases. In this case, the fixed points P_2 and P_3 collide at the singular point γ_c and form a homoclinic orbit with nonzero action. This jump of the action leads to nonzero adiabatic tunneling probability.

The homoclinic orbit is confined near the critical point, with its top at

$$s_t \simeq s_c + \sqrt{6}\delta. \tag{11}$$

We expand the classical Hamiltonian to leading orders of $s - s_c$ and $\theta - \pi$, and find

$$\theta - \pi \simeq \sqrt{\frac{2\gamma_c(s-s_c)}{V}} + \frac{1}{2}\sqrt{\frac{2\gamma_c}{V}}(s-s_c)^{3/2}. \tag{12}$$

From the area of this orbit the adiabatic tunneling probability for this limiting case is found to be

$$\Gamma_{ad} = \frac{1}{2\pi}\int_{s_c}^{s_t}(\theta-\pi)ds = \frac{4}{3\pi}\delta^{3/2}. \tag{13}$$

Clearly, both Γ_{ad} and its first-order derivative are continuous at the critical point. However, its second-order derivative turns to be discontinuous.

B. Nonadiabatic tunneling

In the linear case $C=0$, there is an exact formula that prescribes an exponential dependence of tunneling probability on the sweeping rate [2],

$$\Gamma_{lz} = \exp\left(-\frac{\pi V^2}{2\alpha}\right). \tag{14}$$

It is interesting to know how this exponential law is changed due to the nonlinearity. We first focus on the near adiabatic case (i.e., $\alpha \neq 0$ and $\alpha \ll 1$).

For this purpose, we need to investigate the evolution of the fixed point P_1 as well as the nearby periodic orbits by introducing the angle variable ϕ, the canonical conjugate of the action variable I. As in the adiabatic case considered above, the transition probability is still given by the increment of the action, i.e., $\Gamma = \frac{1}{2}\Delta I$. According to the standard theory on the nonadiabatic correction [14], we have

$$\Delta I = \int_{-\infty}^{+\infty} R(I,\phi)\frac{d\gamma}{dt}\frac{d\phi}{\dot{\phi}}, \tag{15}$$

where $R(I,\phi)$ is the periodic function of ϕ with zero average, and related to the generating function of the canonical transformation from variables (s,θ) to (I,ϕ). The concrete form of the function R is not important our following discussions.

To evaluate the above integral, we need to express $\dot{\phi}$ as a function of ϕ itself. In the near-adiabatic limit, the change of the angle variable is equal to frequency of the fixed point P_1, i.e., $\dot{\phi} = \omega^*$. The frequency can be calculated by linearizing the equations of motion (4) near the fixed point (5),

$$\omega^* = V\left(\frac{1}{1-(s^*)^2} - \frac{C}{V}\sqrt{1-(s^*)^2}\right)^{12}. \tag{16}$$

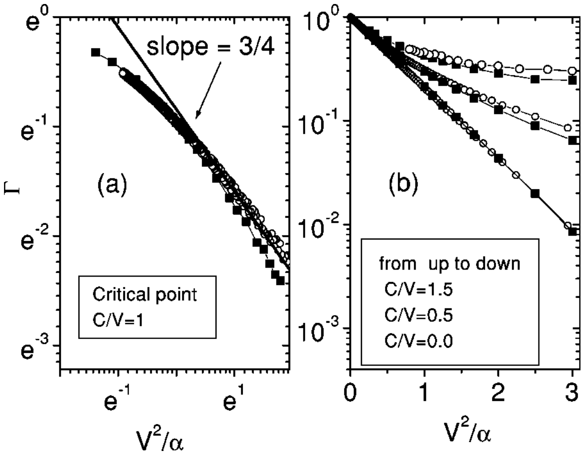

FIG. 4. Dependence of the tunneling probability on the scaled sweeping rate V^2/α (a) for $C/V=1$, and (b) other values of C/V. In (b) we see a clear breakdown of the exponential law for $C/V>1$. The open circles are obtained with the integration of Eq. (2); the solid squares are the numerical results of a Bose-Einstein condensate in an accelerating optical lattice, where α is the acceleration (see Sec.VI for detailed discussions).

On the other hand, by substituting $\theta^*=\pi$ into Eq. (5) and differentiating it with respect to time, we have

$$\frac{dt}{ds^*}=\frac{V}{\alpha}\left(\frac{1}{[1-(s^*)^2]^{3/2}}-C/V\right). \tag{17}$$

Combining these equations, we can relate s^* to ϕ and thus express $\dot{\phi}$ as a function of ϕ itself.

The principal contribution to the integral comes from the neighborhood of the singularities of the integrand, which are the zeros of the frequency $\dot{\phi}=\omega^*(\gamma)$. These zero points are easily found from Eq. (16) as

$$s_0^*=[1-(V/C)^{2/3}]^{1/2}. \tag{18}$$

The integral (15) is exponentially small if there are no real singularities, and becomes a power law in the sweeping rate if there is a singularity on the real axis.

We consider the case of critical nonlinearly, $C/V=1$, for which the singular point occurs at $s^*=0$. Near this point, we find from Eq. (17) that $\omega^*\simeq\sqrt{3/2}Vs^*$ and $\phi\simeq(1/4)(3/2)^{3/2}(V^2/\alpha)(S^*)^4$. Then, we have an approximate relation $\omega^*\sim\alpha^{1/4}\phi^{1/4}$ near the singularity. Substituting these expressions back to Eq. (15), and utilizing the fact that $\partial R/\partial\phi$ is independent of α, we find a power-law behavior for the tunneling probability

$$\Gamma\sim\alpha^{3/4}. \tag{19}$$

This power law, indicating a drastic change of tunneling behavior beyond the critical regime $C=V$, has been verified by our numerical calculations [Fig. 4(a)].

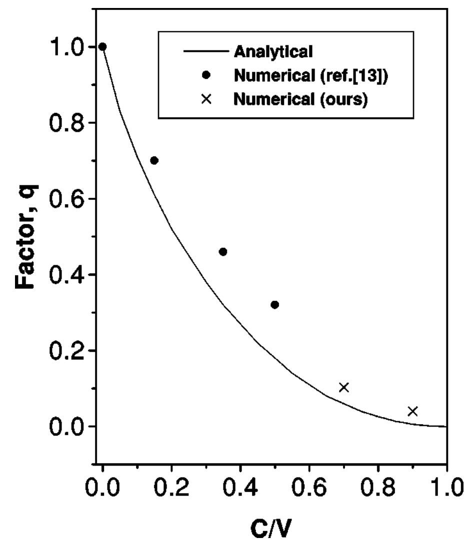

FIG. 5. Dependence of the factor q on C/V.

IV. TUNNELING IN THE SUBCRITICAL REGIME, $C<V$

We shift our attention to nonadiabatic tunneling for subcritical nonlinearity, $C<V$, where the zeros of the frequency ω^* are complex. The principal contribution to the integral (15) comes from the neighborhood of this point and the integral can be evaluated by deforming the contour of integration into the complex plane [14]. The tunneling probability is found to be exponential

$$\Gamma\sim\exp\left(-q\frac{\pi V^2}{2\alpha}\right), \tag{20}$$

where the factor in the exponent is given by

$$q=\frac{4}{\pi}\int_0^{\sqrt{(V/C)^{2/3}-1}}(1+x^2)^{1/4}\left(\frac{1}{(1+x^2)^{3/2}}-\frac{C}{V}\right)^{3/2}dx. \tag{21}$$

For the linear case $C=0$, the factor q is exactly unit, consistent with the standard Landau-Zener formula (14). For the nonlinear case, $C/V>0$, this factor becomes smaller than one, showing the enhancement effect on the nonadiabatic tunneling. As C/V goes up to 1, the critical point, this factor vanishes, signaling the breakdown of the exponential law. Near the critical point $C/V=1$, we have the approximate expression $q\simeq\frac{3}{4}\sqrt{2/3}(1-C/V)^2$, i.e., the factor converges to zero with a square power law.

With numerical integration of the nonlinear Schrödinger equation (2), we show in Fig. 4 the sweeping rate α dependence of the tunneling probability, where the slope of the curve tends to be zero for $C/V>1$ clearly indicating the breakdown of the exponential law. We read the factor q from the slope and compare it with our analytical results in Fig. 5, where we see a reasonable good agreement.

V. TUNNELING BEYOND THE CRITICAL REGIME, $C>V$

In this section, we will discuss the nonlinear Landau-Zener tunneling beyond the critical regime and derive the tunneling probability using the stationary phase approximation. We concentrate on the case of strong nonlinearity $C/V\gg 1$, where there is a near unity tunneling probability to the upper adiabatic level even in the adiabatic limit. This probability can only get larger when the sweeping rate is finite. We thus expect the amplitude b in the Schrödinger equation (2) remains small and $|a|\sim 1$ all the times, and a perturbation treatment of the problem becomes adequate.

We begin with the variable transformation,

$$a=a'\exp\left[-i\int_0^t\left(\frac{\gamma}{2}+\frac{C}{2}(|b|^2-|a|^2)\right)dt\right], \tag{22}$$

$$b=b'\exp\left[i\int_0^t\left(\frac{\gamma}{2}+\frac{C}{2}(|b|^2-|a|^2)\right)dt\right]. \tag{23}$$

As a result, the diagonal terms in Hamiltonian are transformed away, and we have

$$b'=\frac{V}{2i}\int_{-\infty}^t dt\,\exp\left(-i\int_0^t[\gamma+C(|b|^2-|a|^2)]dt\right). \tag{24}$$

We need to evaluate the above integral self-consistently. Because of the large C, the nonlinear term in the exponent generally gives a rapid phase oscillation, which makes the integral small. The dominant contribution comes from the stationary point t_0 of the phase around which we have

$$-\gamma+C(1-2|b|^2)=-\overline{\alpha}(t-t_0), \tag{25}$$

with

$$\overline{\alpha}=\alpha+2C\left[\frac{d}{dt}|b|^2\right]_{t_0}. \tag{26}$$

We thus have

$$|b|^2=\left(\frac{V}{2}\right)^2\left|\int_{-\infty}^t dt\,\exp\left(-\frac{i}{2}\overline{\alpha}(t-t_0)^2\right)\right|^2. \tag{27}$$

We can differentiate this expression and evaluate its result at time t_0, obtaining a few standard Fresnel integrals with the result $[(d/dt)|b|^2]_{t_0}=(V/2)^2\sqrt{\pi/\overline{\alpha}}$. Combining this with the relation (26), we come to a closed equation for $\overline{\alpha}$,

$$\overline{\alpha}=\alpha+2C\left(\frac{V}{2}\right)^2\sqrt{\frac{\pi}{\overline{\alpha}}}. \tag{28}$$

The nonadiabatic transition probability Γ is given by

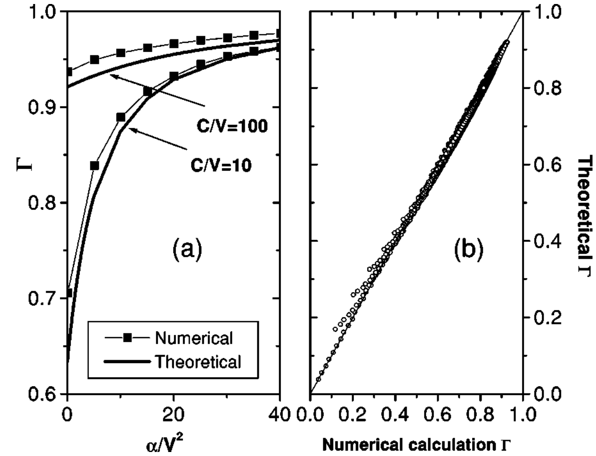

FIG. 6. Comparison between our analytic results and the numerical integration of the Schrödinger equation (2).

$$\Gamma=1-|b|^2_{+\infty}=1-\left(\frac{V}{2}\right)^2\left|\int_{-\infty}^{+\infty}dt\,\exp\left(-\frac{i}{2}\overline{\alpha}(t-t_0)^2\right)\right|^2$$

$$=1-\frac{\pi V^2}{2\overline{\alpha}}. \tag{29}$$

Then the above result yields a closed equation for the Γ,

$$\frac{1}{1-\Gamma}=\frac{1}{P}+\frac{\sqrt{2}}{\pi}\frac{C}{V}\sqrt{1-\Gamma}, \tag{30}$$

where $P=\pi V^2/2\alpha$. In the adiabatic limit, i.e., $1/P=0$, we find that $\Gamma=1-1.7(V/C)^{2/3}$; in the sudden limit, $1/P\to\infty$, we have $\Gamma=1-P$, which is exact. In Fig. 6(a), we compare the above analytical results with that from directly solving the Schrödinger equation (2) and see a good agreement.

The above deduction is made for the strong nonlinearity, however, its result can be extended to a wide range of parameters if we take the quantity P as the $1-\Gamma_{lz}$, the linear Landau-Zener tunneling. Then, the above equation indicates that the nonlinear tunneling probability is a function of the linear Landau-Zener tunneling and ratio between the nonlinear parameter and the energy gap. This relation has been confirmed by our numerical calculation. We have calculated the nonlinear tunneling probability using Eq. (30) with 2500 pairs of Γ_{lz} and C/V, randomly distributed in the range (0.05,0.95) and (1,20), respectively. These results are compared with the tunneling probabilities obtained by directly integrating the Schrödinger equation (2) in Fig. 6(b), where a very good agreement is shown.

VI. EXPERIMENTAL REALIZATION

One possible experimental study of this simple two-level nonlinear model is to use a BEC in an accelerating optical lattice [11–13]. As shown in Refs. [12,13], this BEC system can be reduced to a two-level model near the edge of the Brillouin zone. The nonlinear Landau-Zener tunneling is the transition between two lowest Bloch bands, as induced by the acceleration.

To check the validity of this experimental realization, we numerically solve the Gross-Pitaevskii equation,

$$i\frac{\partial\psi}{\partial t}=-\frac{1}{2}\left(\frac{\partial\psi}{\partial x}-i\alpha t\right)^2+V\cos(x)\psi+C|\psi|^2\psi, \quad (31)$$

which describes a BEC in an optical lattice with acceleration of α. In the above equation, the variables are scaled to be dimensionless as in Refs. [11,12]. We prepare a Gaussian wave packet that covers over 200 lattice sites, then slowly turn on the optical lattice and finally accelerate the lattice. The duration of the acceleration is two Bloch periods to ensure the well separation of the portion of the wave packet tunneled into the upper band and the rest remained in the lower band.

The results are shown in Fig. 4. The agreement with the two-level model is rather remarkable, especially the transition from the exponential law to power laws when the nonlinearity C gets over the critical value V. Some experiments with BECs in accelerating optical lattices have already been done along this direction [15].

In summary, we have investigated analytically the tunneling dynamics of the nonlinear Landau-Zener model and found many interesting phenomena as the power laws characterizing the critical behavior of the parameter dependence of tunneling probability. We have also checked the possibility of experimental study of this nonlinear tunneling with a BEC in an accelerating optical lattice.

ACKNOWLEDGMENTS

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