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LETTER

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Klein tunneling phenomenon with pair creation process

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Abstract – In this paper, we study the Klein tunneling phenomenon with electron-positron pair creation process. Pairs can be created from the vacuum by a supercritical single-well potential (for electrons). In the time region, the time-dependent growth pattern of the created pairs can be characterized by four distinct regimes which can be considered as four different statuses of the single well. We find that if positrons penetrate the single well by Klein tunneling in different statuses, the total number of the tunneling positrons will be different. If Klein tunneling begins at the initial stage of the first status *i.e.* when the sing well is empty, the tunneling process and the total number of tunneling positrons are similar to the traditional Klein tunneling case without considering the pair creation process. As the tunneling begins later, the total tunneling positron number increases. The number will finally settle to an asymptotic value when the tunneling begins later than the settling-down time t_s of the single well which has been defined in this paper.

Introduction. – Klein tunneling predicts that relativistic fermions can pass through supercritical repulsive potential barriers without the exponential damping expected in the quantum tunneling processes of nonrelativistic particles [1]. This phenomenon is a property of relativistic wave equations and arises from the existence of negative-energy solutions of the Dirac equation. Klein tunneling for relativistic fermions, however, has never been realized. Its experimental observation requires extremely high fields which are not currently available. But thanks to the recent rapid advancements of high-energy laser technology [2–4], the Klein tunneling phenomenon will paly an important role in investigating high-energy physics in the foreseeable future. Therefore many important works have been done to study Klein tunneling phenomena [5–10].

In another field of high-energy physics, the quantum electrodynamics (QED) predicts that an external energy associated with a supercritical field can be converted into matter in the form of electron-positron pairs. The premier relevant calculation was done by Schwinger, he calculated the rate of pair creation due to a constant electric field using a nonperturbative approach [11]. From his work the critical electric field for pair production can be estimated to be on the order of 10^{18} V/m. Since then the electron-positron pair creation has been widely studied [12–19].

Since both the Klein tunneling phenomenon and pair creation process exist in supercritical field, the study of the Klein tunneling phenomenon with pair creation process will be very significant. A relevant research has been done [17], but the work is focused on the pair creation process. The specific behavior of Klein tunneling with pairs creation process has not been studied concretely yet. Some fundamental research about Klein tunneling with pair creation process in a supercritical single-well potential has been done in this paper by using the computational quantum field theory method. The theory of the method and the buildup of the model are introduced in the second section, and in the third section the specific results of calculation about different conditions are given, in the final section we give the conclusions.

Theoretical approach and model. – First of all, we illustrate the theoretical approach. The time-dependent evolution of the Dirac field operator $\hat{\varphi}(x,t)$ in a potential $V(x,t)$ is governed by the Dirac equation:

$$i\frac{\partial\hat{\varphi}(x,t)}{\partial t} = [cp\boldsymbol{\sigma}_1 + c^2\boldsymbol{\sigma}_3 + \boldsymbol{I}V(x,t)]\hat{\varphi}(x,t). \quad (1)$$

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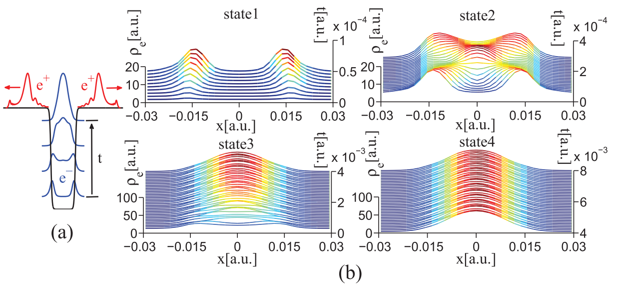

Fig. 1: (Colour online) (a) Sketch of the pair creation process in the supercritical single well. (b) Snapshots of the evolving electronic spatial distribution of the created electrons in the four statuses of the single well with the parameter: $V_0 = 2c^2 + 10^4$ a.u.; $D = 0.3\lambda_c$; $W_1 = 4.55\lambda_c$. Here $\lambda_c = 1/c = 7.3 \times 10^{-3}$ a.u. is the Compton wavelength.

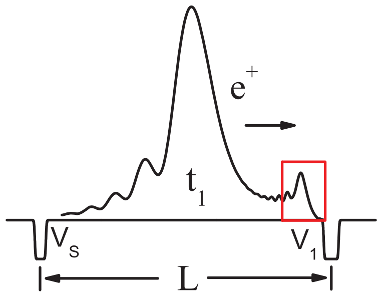

Fig. 2: (Colour online) Sketch of the double-well potential and the positron waveform created by the left well (positron wave in the red frame is the part that needs to be observed) at time t_1. Here L is the distance between the two wells.

We use atomic units and only consider the one-dimensional space. The Dirac field operator can be expanded in the antielectron creation operators $\hat{d}_n^+(t)$ and the electron annihilation operators $\hat{b}_p(t)$:

$$\hat{\varphi}(x,t) = \Sigma_p \hat{b}_p(t) W_p(x) + \Sigma_n \hat{d}_n^+(t) W_n(x), \qquad (2)$$

where $W_p(x)$ and $W_n(x)$ are plane-wave solutions for the single-particle Dirac equation with positive and negative energies. Using Bogoliubov transformation we can obtain

$$\begin{aligned} &\Sigma_p \hat{b}_p(t) W_p(x) + \Sigma_n \hat{d}_n^+(t) W_n(x) = \\ &\Sigma_p \hat{b}_p(t=0) W_p(x,t) + \Sigma_n \hat{d}_n^+(t=0) W_n(x,t), \end{aligned} \qquad (3)$$

then the time-dependent evolution of creation and annihilation operators can be written as

$$\begin{aligned} \hat{b}_p(t) &= \Sigma_{p'} \hat{b}_{p'}(t=0)\langle p|U(t)|p'\rangle \\ &\quad + \Sigma_{n'} \hat{d}_{n'}^+(t=0)\langle p|U(t)|n'\rangle, \\ \hat{d}_n^+(t) &= \Sigma_{p'} \hat{b}_{p'}(t=0)\langle n|U(t)|p'\rangle \\ &\quad + \Sigma_{n'} \hat{d}_{n'}^+(t{=}0)\langle n|U(t)|n'\rangle, \end{aligned} \qquad (4)$$

where the coefficients are the matrix elements of the time-evolution operator $U(t) = \hat{T}\exp\{-i\int [cp\boldsymbol{\sigma}_1 + c^2\boldsymbol{\sigma}_3 + \boldsymbol{I}V(x,t)]\mathrm{d}t\}$ between the plane-wave states. The split-operator algorithm technique has been used to accomplish the time evolution of each state [20].

Consider a supercritical single-well potential $V_1(x,t) = \theta(t)V_0\{\tanh[(x - W_1/2)/D] - \tanh[(x + W_1/2)/D]\}$ for electrons, which is a barrier potential for positrons. W_1 is the width of the total well, and D is the width of the well edge. $\theta(t)$ is the Heaviside unit step function, which means that the single well is opened at $t = 0$. The pairs will be created from the vacuum since the opening of the well, the created positron wave will be emitted outward while the created electron wave is trapped in the well. Because of the restriction of the well edges, the well will ultimately be filled with created electrons (fig. 1(a)).

The process of the well being filled up is characterized by four distinct regimes in the time region, which can be considered as four different statuses of the single well [21]. In the first status ($t \ll 1/c^2$), the total number of created electrons $N(t) = \Sigma_p \langle 0|\hat{b}_p^+(t)\hat{b}_p(t)|0\rangle$ increases quadratically, where $|0\rangle$ is the vacuum state. The second status ($1/c^2 < t < 1/c^2 + W_1/c$) is characterized by a linear growth, $N(t) = 2\beta t$. The value of the rate β depends on the edges of the well, β can be obtained from the single-particle framework by integrating the edge transmission coefficient of the incoming electrons with different energies. In the third status ($t > 1/c^2 + W_1/c$), because the well edges begin to restrict the propagation of the created electron wave, the growth rate of the created electron number reduces. Ultimately the growth rate reduces to zero in the fourth status ($t \gg 1/c^2 + W_1/c$), and the number of the created electrons settles to an asymptotic value. The whole process of the well being filled up can be exhibited intuitively by the evolving spatial distribution of the created electrons $\rho_e(x,t) = \Sigma_n|\Sigma_p\langle p|U(t)|n\rangle W_p(x)|^2$ (fig. 1(b)).

Our thought is to explore the effect of the different well statuses on the Klein tunneling phenomenon. To build the model for the study, we use a double-well potential $V_2(x,t) = V_s(x,t) + V_1(x,t)$ (fig. 2). The left well $V_s(x,t) = \theta(t+t_0)V_0\{\tanh[(x+L-W_s/2)/D] - \tanh[(x+L+W_s/2)/D]\}$ can be considered as the emission source of positrons, and half of the emitted positron wave will spread to the right and penetrate the right well $V_1(x,t)$ by Klein tunneling, while the right well produces electrons and positrons (fig. 2). As a result, we can observe the Klein tunneling phenomenon with pair creation process in the right well. The distance between the two wells L is long enough to consider each well as an independent one. Here we only observe part of the created positron wave to make the tunneling phenomenon more distinct (fig. 2). Note that, the following big wave crest is too far to affect the tunneling process of the observed positron wave. We keep the left well and the distance between the two wells invariable to make sure the waveform of the observed positron wave keeps invariant. Moreover, since the created positron wave only intervenes with its own reflected wave, the positron wave created by the right well cannot affect the waveform of the observed wave. The

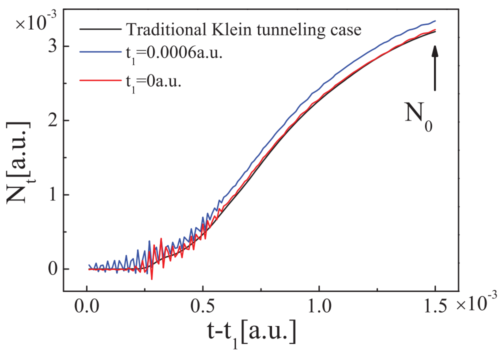

Fig. 3: (Colour online) Evolving number of the tunneling positrons with the parameter: $V_0 = 2c^2 + 10^4$ a.u.; $D = 0.3\lambda_c$; $W_s = 4.55\lambda_c$; $W_1 = 18.78\lambda_c$; $L = W_1/2 + 8.43 \times 10^{-3}$ a.u. N_0 is the total number of the tunneling positrons.

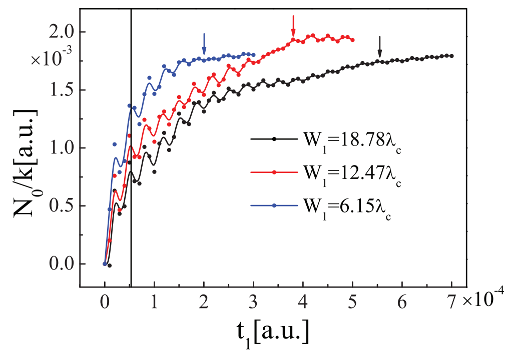

Fig. 4: (Colour online) The functional relationship between the total number of the tunneling positrons N_0 and the parameter t_1 from three different right wells: $W_1 = 6.15\lambda_c, 12.47\lambda_c, 18.78\lambda_c$, with the corresponding right well transmission coefficient: $k = 9.17 \times 10^{-2}, 6.91 \times 10^{-2}, 6.69 \times 10^{-2}$. Here the vertical black line is the separatrix between the first status and the second status of the right wells, and the arrows point out the middle time point of the second status of the right wells. The points in this graph are calculation results and the curves are fitted by B-splines. ($V_0 = 2c^2 + 10^4$ a.u.; $D = 0.3\lambda_c$; $W_s = 4.55\lambda_c$; $L = W_1/2 + 8.43 \times 10^{-3}$ a.u.)

Coulomb interaction is ignored because of the time scale in our simulation.

We can see that the right well is exactly the mentioned single well, so we open the right well at time $t = 0$ to fit the mentioned pair creation process in the single well. The left well is opened at $t = -t_0$, then at $t = L/p - t_0 = t_1$ the observed positron wave will arrive the left border of the right well and begin to tunnel though the right well (fig. 2), here p is the propagation velocity of the observed positron wave. We call t_1 the beginning time of the tunneling for the right well, t_1 determines the status of the right well when the tunneling begins. By changing t_1, we can keep the right well in different statuses when the tunneling begins.

Klein tunneling phenomenon with pair creation process. – After these remarks about method and model, let us begin to analyze the Klein tunneling phenomenon with pair creation process. First, we concern about the time-evolved tunneling positron number $N_t(t)$. Considering the mentioned double-well potential model, the time-evolved tunneling positron number $N_t(t)$ with different beginning times can be obtained by calculating the positron number on the right side of the right well and then minus the component created by the right well itself during the tunneling (fig. 3). To avoid the effect of the positron wavefront, we use the probability current density of positrons to calculate the needed positron number:

$$J = \int -2\mathrm{Im}\Sigma_{n'}\{[\Sigma_n \langle n|U(t)|n'\rangle^* W_n^+(x)] \times [\Sigma_n \langle n|HU(t)|n'\rangle W_n(x)]\}\mathrm{d}x. \tag{5}$$

For comparison, the traditional Klein tunneling case without considering the pair creation process is also calculated (fig. 3). $N_t(t)$ in the traditional Klein tunneling case can be got by using the single left well to generate the single-positron wave functions $\varphi_p(x,t) = \Sigma_n \langle n|U(t)|p\rangle W_n(x)$, and then evolving them in the single-right-well potential according to the single-particle Dirac equation.

From fig. 3, we can see that because of the effect of pair creation, the number of the transmission positrons keeps oscillatory growth at the earlier stage of the Klein tunneling process. After the oscillation, the tunneling positron number increases continually. Figure 3 also indicates that the tunneling process and the total number of the tunneling positrons N_0 at the condition of $t_1 = 0$ are quite similar to the traditional Klein tunneling case. This conclusion is very reasonable, because $t_1 = 0$ means that the right well is totally empty when the tunneling begins, the traditional Klein tunneling case has the same condition. Moreover, the most important conclusion we can get from fig. 3 is that the total number of the tunneling positrons N_0 is larger at the condition of $t_1 = 6 \times 10^{-4}$ a.u. The change of the total number of Klein tunneling particles may have major effects on many high-energy physical processes.

In the following study, a series of calculations has been done to explore the functional relationship between the total number of the tunneling positrons N_0 and the parameter t_1 (fig. 4). Cases of different right well widths have been considered since the right well width determines the time length of different right well statuses. To analyse the result more clearly, here we set $N_0(t_1 = 0)$ to zero and divide $N_0(t_1)$ by the right well transmission coefficient k for the observed positron wave. The transmission coefficient k can be calculated by dividing the total number of the tunneling positrons by the number of the incident positrons in the traditional Klein tunneling case.

From fig. 4, we can see that with the increasing of t_1, the total number $N_0(t_1)$ keeps a fluctuant growth and the growth pattern is nontrivial. If the tunneling begins in the first status of the right well, the growth rate is large. Then the growth rate reduces in a nontrivial way if the tunneling begins in the second status, and $N_0(t_1)$ finally settles to an asymptotic value at the middle time point of the second status of the right well. We can see that if

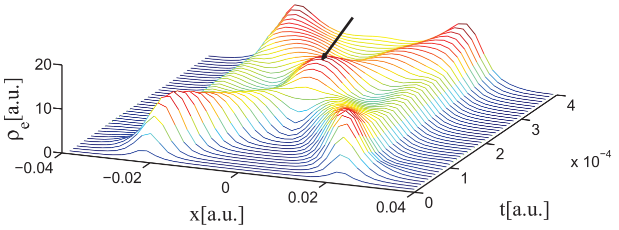

Fig. 5: (Colour online) Evolving spatial distribution of the created electrons in the first two statuses of the single well with the parameter: $V_0 = 2c^2 + 10^4$ a.u.; $D = 0.3\lambda_c$; $W_1 = 6.15\lambda_c$. The arrow points out the spatial distribution at the settling-down time t_s of the single well.

the tunneling begins after the middle of the second status, the total number $N_0(t_1)$ will fluctuate slightly around the asymptotic value. This phenomenon does not change in the last two statuses according to our calculation. According to this result, we define the middle time point of the second status as the settling-down time t_s of the right well for Klein tunneling.

To explore the implication of the settling-down time t_s, the evolving spatial distribution $\rho_e(x,t)$ of the created electrons in the first two statuses of the single well $V_1(x,t)$ must be analyzed in detail, since it determines the condition inside the well (fig. 5). From fig. 5, we can see that at the settling-down time t_s, the peaks of the electron wave meet in the middle of the well. After that, the created electron wave begins to be reflected by the opposite edge of the well, even though the growth pattern of the total number of created pairs $N(t)$ still keeps linear. As what we see, Klein tunneling will settle down after the created electron wave begins to be reflected by the well edges.

Another obvious phenomenon we can get from fig. 4 is the fluctuation of $N_0(t_1)$. It is obvious that the periods of the fluctuations are the same for different widths of the right well. The periods of the fluctuations are decided by the well edges which are all the same in our calculation ($D = 0.3\lambda_c$). According to the fact that we mentioned before, the well edges also determine the value of the linear growth rate β of the created pair number in the single well. In fact the period is approximatively equal to $2D/c$ numerically. The specific relationship between the period of the fluctuation and the structure of the well edges will be explored in the next work.

Conclusions. – In this paper, we explore the Klein tunneling phenomenon with pair creation process in the supercritical single well. We find that the beginning time t_1 of Klein tunneling can affect the the total number of the tunneling positrons $N_0(t_1)$. If Klein tunneling begins when the sing well is empty ($t_1 = 0$), the tunneling process and the total number of the tunneling positrons are quite similar to the traditional Klein tunneling. The total number of the tunneling positrons $N_0(t_1)$ keeps a fluctuant growth with the increasing of t_1, and the period of the fluctuation is approximatively equal to $2D/c$ numerically. The growth rate is large when the beginning time t_1 is in the first status of the single well, then the growth rate will reduce in a nontrivial way and $N_0(t_1)$ will finally settle to an asymptotic value in the earlier stage of the second status. The total tunneling number $N_0(t_1)$ will fluctuate slightly around the asymptotic value if the tunneling begins after the middle of the second status. In that case, we define the middle time point of the second status as the settling-down time t_s of the single well for Klein tunneling.

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