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Fulde-Ferrell-Like Molecular States in Spin-Orbit Coupled Ultracold Fermi Gases*

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Abstract *We study the molecular state in three-component Fermi gases with a single impurity of ^{6}Li immersing in a no-interacting Fermi sea of ^{40}K in the presence of an equal weight combination of Rashba-type and Dresselhaus-type spin-orbit coupling. In the region where the Fermi sea has two disjointed Fermi surfaces, we find that there are two Fulde–Ferrell-like molecular states with dominating contributions from the lower helicity branch. Decreasing the scattering length or the spin-orbit coupled Fermi energy, we find the Fulde–Ferrell-like molecular state with small center-of-mass momentum is always energy favored and the other one will suddenly disappear.*

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1 Introduction

The problem of a single impurity in the presence of a Fermi sea has arisen widespread interest on theoretical and practical sides.[1−8] Indeed, it is related to other fields of physics ranging from superconductivity, astrophysics, to high-energy physics, where similar situations arise.[9] For weak interspecies attraction, the ground state can be well described by a state with an impurity atom dressed by a single particle-hole excitation of the Fermi sea named as a "polaron",[1−2] while for strong attraction, a state based on a molecular picture gives a lower energy.[3−4]

Recently, the light-induced effective gauge potential[10−15] has opened a new avenue for studying the single-impurity problem in spin-orbit coupled Fermi gases.[7−8] With the state of the art experimental techniques, one can induce an equal weight combination of Rashba-type and Dresselhaus-type spin-orbit coupling (SOC) in the no-interacting Fermi gas of ^{40}K.[13] When a single impurity of ^{6}Li interacts with the spin up component of such spin-orbit coupled Fermi gas, there are two meta-stable states with a nonzero center-of-mass (c.m.) momentum named Fulde–Ferrell-like (FF-like) molecular states with dominating contributions from the lower and upper helicity branches respectively.[8] With the variation of the scattering length or the SOC strength, the ground state may experience a transition from one FF-like molecular state to another.[8]

In the same system, we study the molecular state in the region where the Fermi sea has two disjointed Fermi surfaces. We find two FF-like molecular states with dominating contributions from the lower helicity branch, which is very different from the case in Ref. [8]. In experiment, the scattering length can be tuned via a Feshbach resonance.[16] Therefore, we explore the two molecular states with the increasing of the inverse of the scattering length and find the FF-like molecular state with large c.m. momentum will suddenly disappear. We also find that the FF-like molecular state with a small c.m. momentum is always energy favored. The contribution of the upper helicity branch to the two FF-like molecular states will increase with the increasing of the inverse of the scattering length. By tuning the spin-orbit coupled Fermi energy by changing the particle number of ^{40}K, we find a similar phenomenon. The contribution of the upper helicity branch to the FF-like molecular state with small (large) c.m. momentum will decrease (increase) with the increasing of Fermi energy. Therefore, one will expect when the Fermi energy is big enough our case will smoothly transfer to the case where the two FF-like molecular states are with dominating contributions from the two helicity branches respectively. The relative energy gap between the two FF-like molecular states will diminish with the decreasing of the scattering length or the spin-orbit coupled Fermi energy.

2 Model

We explore the molecular state of three-component Fermi gases with a single purity of ^{6}Li and a no-interacting spin-orbit coupled Fermi sea of ^{40}K in three dimension. Here we consider an equal weight combination of Rashba-type and Dresselhaus-type SOC which had been generated experimentally[13] and an attractive s-wave contact

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interaction occuring among ^{6}Li and the spin up component of ^{40}K. The Hamiltonian of such Fermi mixture is[8] $H = H_0 + H_{\rm int}$ with the single particle Hamiltonian

$$H_0 = \sum_{\boldsymbol{k}} \varepsilon^b_{\boldsymbol{k}} b^\dagger_{\boldsymbol{k}} b_{\boldsymbol{k}} + \sum_{\boldsymbol{k},\sigma} \varepsilon^a_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\sigma} a_{\boldsymbol{k},\sigma} + \sum_{\boldsymbol{k}} (h a^\dagger_{\boldsymbol{k},\uparrow} a_{\boldsymbol{k},\downarrow} + \alpha k_x a^\dagger_{\boldsymbol{k},\uparrow} a_{\boldsymbol{k},\uparrow} + \text{h.c.}) , \quad (1)$$

and the attractive s-wave contact interaction

$$H_{\rm int} = \frac{U}{V} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} a^\dagger_{\boldsymbol{q}/2+\boldsymbol{k},\uparrow} b^\dagger_{\boldsymbol{q}/2-\boldsymbol{k}} b_{\boldsymbol{q}/2-\boldsymbol{k}'} a_{\boldsymbol{q}/2+\boldsymbol{k}',\uparrow} . \quad (2)$$

Here $a_{\boldsymbol{k},\sigma}$ ($\sigma = \uparrow, \downarrow$) denotes the annihilation operator of ^{40}K in a spin component and $b_{\boldsymbol{k}}$ is the annihilation operator of ^{6}Li. The kinetic energies of them are $\varepsilon^a_{\boldsymbol{k}} = \hbar^2 k^2/(2m_a)$ and $\varepsilon^b_{\boldsymbol{k}} = \hbar^2 k^2/(2m_b)$, where m_a and m_b are masses of ^{40}K and ^{6}Li. The spin-orbit coupling strength and the Zeeman field along the z direction are parameterized by α and h respectively.

In three dimension, the bare interaction rate U is renormalized as

$$\frac{1}{U} = \frac{1}{U_s} - \frac{1}{V} \sum_{\boldsymbol{k}} \frac{1}{(1+\eta)\varepsilon^a_{\boldsymbol{k}}} , \quad (3)$$

where $1/U_s = m_a/2\pi\hbar^2 a_s(1+\eta)$ with a_s being the s-wave scattering length, V is the quantization volume, and $\eta = m_a/m_b$ is the mass ratio. For the ^{40}K-^{40}K-^{6}Li mixture, the mass ratio is $\eta = 40/6$. Here we have ignored the modification of the renormalization relation by SOC.[17−21]

Without SOC, the ground state of a single impurity Fermi gas in the strong interaction region can be described by a molecular state with the binding of two fermion on the top of the Fermi sea. In the appearance of SOC, the single-particle dispersion of ^{40}K divides into two separated branches $\epsilon^a_{\boldsymbol{k},\pm} = \varepsilon^a_{\boldsymbol{k}} \pm \sqrt{h^2 + \alpha^2 k_x^2}$ where "+" and "−" label the upper and the lower helicity branches respectively. The eigen-state of each helicity branch is created

$$a^\dagger_{\boldsymbol{k},\pm} = \cos\theta^\pm_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\uparrow} + \sin\theta^\pm_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\downarrow}$$

with $\cos\theta^\pm_{\boldsymbol{k}} = \pm\beta^\pm_{\boldsymbol{k}}$, $\sin\theta^\pm_{\boldsymbol{k}} = \beta^\mp_{\boldsymbol{k}}$, and

$$\beta^\pm_{\boldsymbol{k}} = [\sqrt{h^2 + \alpha^2 k_x^2} \pm \alpha]^{1/2} / \sqrt{2} [h^2 + \alpha^2 k_x^2]^{1/4} .$$

The appearance of SOC also leads to two momentum-dependent effective interactions between ^{6}Li and ^{40}K in the two helicity branches for the momentum dependence of $\theta^\pm_{\boldsymbol{k}}$. In the following, we will show these modulations of system and the appearance of the Fermi sea give rise to rich exotic phenomena.

To study the molecular state of such impurity Fermi gas, we introduce the ansatz wave function[8]

$$|M_{\boldsymbol{Q}}\rangle = \sum_{\lambda=\pm} \sum_{\epsilon^a_{\boldsymbol{k},\lambda} > E_h} \Phi^{\boldsymbol{Q}}_\lambda(\boldsymbol{k}) b^\dagger_{\boldsymbol{Q}-\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\lambda} |\text{FS}\rangle , \quad (4)$$

where $\Phi^{\boldsymbol{Q}}_\lambda(\boldsymbol{k})$ is the variational coefficient, $\boldsymbol{Q}$ is the c.m. momentum of the molecular state, and $|\text{FS}\rangle$ denotes the spin-orbit coupled Fermi sea with a spin-orbit coupled Fermi energy E_h. The two-body energy (molecular energy) $E_{\boldsymbol{Q}}$ can be achieved by solving the Schrödinger equation $H|M_{\boldsymbol{Q}}\rangle = E_{\boldsymbol{Q}}|M_{\boldsymbol{Q}}\rangle$ in the following process.

Firstly, rearranging the Schrödinger equation in the form $H_{\rm int}|M_{\boldsymbol{Q}}\rangle = (E_{\boldsymbol{Q}} - H_0)|M_{\boldsymbol{Q}}\rangle$ and multiplying $\langle \boldsymbol{k}, \lambda; \boldsymbol{Q} - \boldsymbol{k}|$ on each side, we arrive

$$\Phi^{\boldsymbol{Q}}_\lambda(\boldsymbol{k}) = G \frac{\beta^\lambda_{\boldsymbol{k}}}{E_{\boldsymbol{Q}} - \varepsilon^b_{\boldsymbol{Q}-\boldsymbol{k}} - \epsilon^a_{\boldsymbol{k},\lambda}} , \quad (5)$$

where

$$G = \frac{U}{V} \sum_{\lambda'=\pm} \sum_{\epsilon^a_{\boldsymbol{k}',\lambda'} > E_h} (\beta^{\lambda'}_{\boldsymbol{k}'})^* \Phi^{\boldsymbol{Q}}_{\lambda'}(\boldsymbol{k}')$$

and $|\boldsymbol{k}, \lambda; \boldsymbol{Q} - \boldsymbol{k}\rangle = b^\dagger_{\boldsymbol{Q}-\boldsymbol{k}} a^\dagger_{\boldsymbol{k},\lambda} |\text{FS}\rangle$. Then, multiplying $(\beta^\lambda_{\boldsymbol{k}})^*$ on each side and summing over all the available states, we arrive the self-consistent equation for $E_{\boldsymbol{Q}}$

$$\frac{1}{U} = \frac{1}{V} \sum_{\lambda=\pm} \sum_{\epsilon^a_{\boldsymbol{k},\lambda} > E_h} \frac{|\beta^\lambda_{\boldsymbol{k}}|^2}{E_{\boldsymbol{Q}} - \varepsilon^b_{\boldsymbol{Q}-\boldsymbol{k}} - \epsilon^a_{\boldsymbol{k},\lambda}} . \quad (6)$$

For calculation, we take the unit of energy $E_0 = 2\alpha^2 m_a/\hbar^2$ and the unit of momentum $k_0 = 2\alpha m_a/\hbar^2$.

3 Disappearing of One Fulde–Ferrell-like Molecular

In Ref. [8], the existence of the spin-orbit coupled Fermi sea can lead to two FF-like molecular states with dominating contributions from the lower and upper helicity branches. In our calculation, we consider the parameter region where the Fermi sea has two disjointed Fermi surfaces. This parameter region is depicted as $h < m_a\alpha^2$ and $E_h < -h$. Protected by the rotation symmetry around the x axis, c.m. momenta of ground states will always lie in the x direction. Thus we study the cases with c.m. momenta $\boldsymbol{Q} = Q_x e_x$.

Numerical solving Eq. (6) with the renormalization relation Eq. (3), we give dispersions of $E_{\boldsymbol{Q}}$ in terms of Q_x for different $(k_0 a_s)^{-1}$ in Fig. 1(a). Here the other parameters used for calculation are $h = 0.1E_0$ and $E_h = -0.2E_0$. We find that there are two local minima corresponding to two FF-like molecular states when $(k_0 a_s)^{-1}$ is small (e.g. Figs. 1(a1) and 1(a2)). When $(k_0 a_s)^{-1}$ is large, there are one local minimum corresponding to one FF-like molecular states (e.g. Fig. 1(a3) and Fig. 1(a4)). In order to see how one FF-like molecule disappears, we show the variation of c.m. momenta of the two FF-like molecular states in terms of $(k_0 a_s)^{-1}$ in Fig. 1(b1). In Fig. 1(b1), we distinguish two regions I and II named the double-molecule region and the single-molecule region corresponding to the number of FF-like molecular states. We find that on the boundary of the two regions the FF-like with large c.m. momentum suddenly disappears.

In Ref. [8], tuning the interaction strength will give rise to the competition between the two FF-like molecular states. However, whether the two FF-like molecular states in our case will present the competition is not clear. Therefore, we give function of the relative energy

difference $\Gamma = (E_b - E_a)/(|E_a + E_b|)$ of the two FF-like molecular states in terms of interacting strength $(k_0 a_s)^{-1}$ in Fig. 1(b2). Here, E_a and E_b are energies of the two FF-like molecular states $|Q_a\rangle$ and $|Q_b\rangle$ with c.m. momenta Q_a and Q_b ($Q_b > Q_a$). It is clear that the FF-like molecular state $|Q_a\rangle$ with small c.m. momentum is always energy favored and Γ decreases with the increasing of $(k_0 a_s)^{-1}$. Varying the zeeman field in the region where the Fermi sea has two disjointed Fermi surface, we find similar phenomena when $E_h = -0.2E_0$. We also find the critical scattering length for the disappearing of the FF-like molecular state almost unchanged with the varying of the zeeman field.

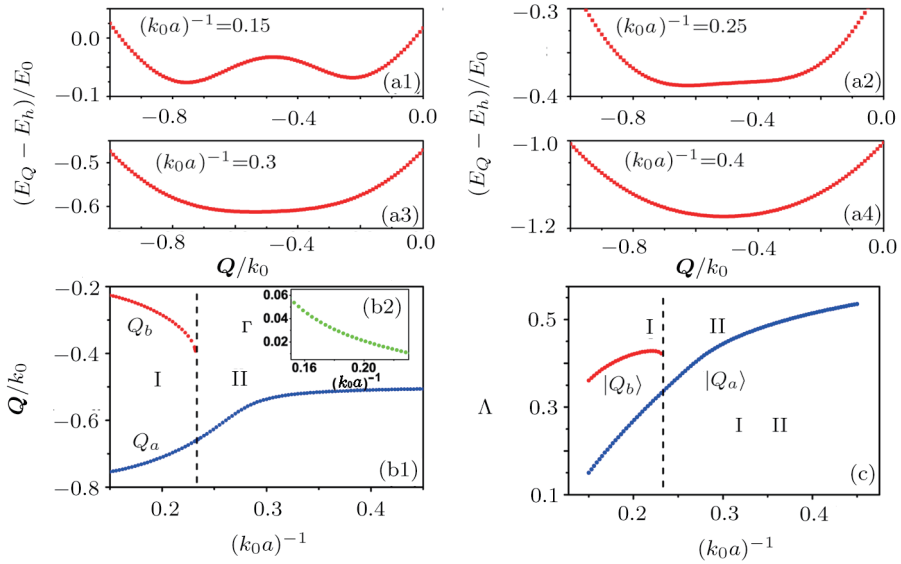

Fig. 1 (a) Molecular energy in terms of $\boldsymbol{Q}$ for different $(k_0 a_s)^{-1}$. (b1) The c.m. momenta of the two FF-like molecular states in terms of $(k_0 a_s)^{-1}$. (b2) The relative energy difference of the two FF-like molecular states in terms of $(k_0 a_s)^{-1}$. (c) The relative contribution of the two helicity branch to the FF-like molecular states in terms of $(k_0 a_s)^{-1}$. The zeeman field is $h = 0.1E_0$. The spin-orbit coupled Fermi energy is $E_h = -0.2E_0$.

Now, we have shown in the parameter region where the Fermi sea has two disjointed Fermi surfaces there are two FF-like molecular states. With the variation of the interacting strength, they represent very different phenomena from the case in Ref. [8]. In the following, we will show an important difference in our case. Exploring the wavefunction of the two FF-like molecular states, we find that they are both with dominating contributions from the lower helicity branch, while in Ref. [8] the two FF-like molecular states are with dominating contributions from the lower and the upper branches respectively. In order to demonstrate this, we introduce

$$\Lambda = \frac{\sum_{\boldsymbol{k}}' |\Phi_+^{\boldsymbol{Q}}(\boldsymbol{k})|^2}{\sum_{\boldsymbol{k}}' |\Phi_-^{\boldsymbol{Q}}(\boldsymbol{k})|^2}, \tag{7}$$

where $\sum_{\boldsymbol{k}}'$ excludes the states below the Fermi surface. It characterizes the relative contribution of the two helicity branch to the FF-like molecular states. In Fig. 1(c), we show the function of Λ in terms of $(k_0 a_s)^{-1}$ for the two FF-like molecular states. It is clear that $\Lambda < 1$ for the two FF-like molecular states, which demonstrates they are both with dominating contributions from the lower helicity branch. In our calculation, the upper helicity branch is well separated from the Fermi surface. From Eq. (5), one can find that the state with higher energy will have smaller distribution probability. Therefore our two FF-like molecular states are with dominating contributions from the lower helicity branch. While in Ref. [8], the Fermi surface contains both the lower and upper helicity branches. Therefore, the FF-like molecular states in Ref. [8] are with dominating contributions from the lower and upper helicity branches respectively.

Now, we have shown one FF-like molecular state will suddenly disappear with the increasing of the interacting strength. In the following, we will show this phenomenon happens when the spin-orbit coupled Fermi energy is tuned. Experimentally, the interacting strength and the spin-orbit coupled Fermi energy can be manipulated via sweeping the magnetic field near a Feshbach resonance and changing the particle number of ^{40}K respectively.

In Fig. 2(a), with the decreasing of the spin-orbit coupled Fermi energy E_h, the FF-like molecular state with large c.m. momentum will suddenly disappear and the other FF-like molecular state is always energy favored. Here the other parameters used for calculation are $h = 0.1E_0$ and $(k_0 a_s)^{-1} = 0.15$. From Fig. 2(b), we find the two FF-like molecular states are with dominating contributions from the lower helicity branch, i.e., $\Lambda < 1$. With the increasing of the spin-orbit coupled Fermi energy, Λ

of the FF-like molecular state with small c.m. momentum decreases, in contrast to Λ of the FF-like molecular state with large c.m. momentum which increases.

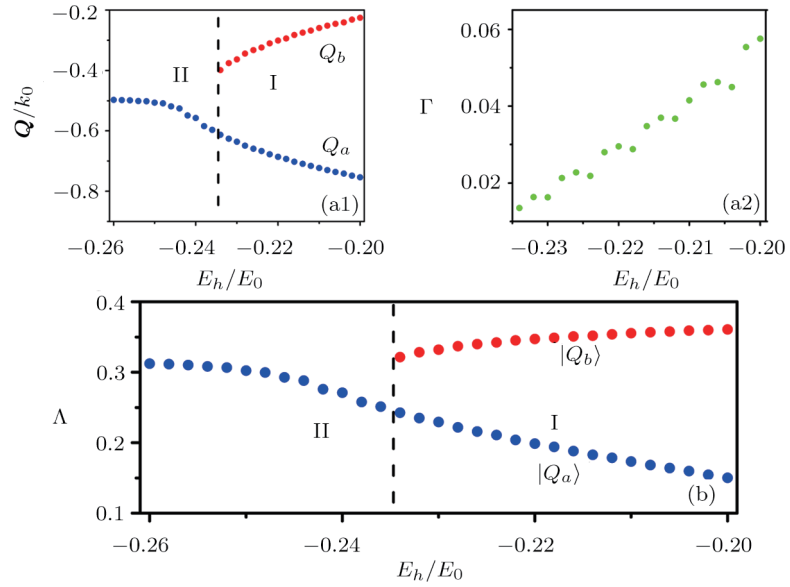

Fig. 2 (a1) The c.m. momenta of the two FF-like molecular states in terms of the spin-orbit coupled Fermi energy E_h. (a2) The relative energy difference of the two FF-like molecular states in terms of the spin-orbit coupled Fermi energy E_h. (b) The relative contribution of the two helicity branch to the FF-like molecular states in terms of the spin-orbit coupled Fermi energy E_h. The zeeman field is $h = 0.1E_0$. The interacting strength is $(k_0 a_s)^{-1} = 0.15$.

4 Summary

In summary, we have studied the molecular states of a three component Fermi mixture under spin-orbit coupling in the region where the two FF-like molecular states with dominating contributions from the lower helicity branch. We have explored the suddenly disappearing of one FF-like molecular state.

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References

[1] F. Chevy, Phys. Rev. A **74** (2006) 063628.
[2] R. Combescot, A. Recati, C. Lobo, and F. Chevy, Phys. Rev. Lett. **98** (2007) 180402.
[3] N. Prokofév and B. Svistunov, Phys. Rev. B **77** (2008) 020408.
[4] N. V. Prokofév and B. V. Svistunov, Phys. Rev. B **77** (2008) 125101.
[5] Sascha Zöllner, G. M. Bruun, and C. J. Pethick, Phys. Rev. A **83** (2011) 021603(R).
[6] Marco Koschorreck, Daniel Pertot, Enrico Vogt, Bernd Fröhlich, Michael Feld, and Michael Köhl, Nature (London) **485** (2012) 619.
[7] Wei Yi and Wei Zhang, Phys. Rev. Lett. **109** (2012) 140402.
[8] Lihong Zhou, Xiaoling Cui, and Wei Yi, Phys. Rev. Lett. **112** (2014) 195301.
[9] R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. **76** (2004) 263.
[10] Jean Dalibard, Fabrice Gerbier, Gediminas Juzeliūnas, and PatrikÖhberg, Rev. Mod. Phys. **83** (2011) 1523.
[11] Y. J. Lin, K. Jimenéz-García, and I. B. Spielman, Nature (London) **471** (2011) 83.
[12] Jin-Yi Zhang, Si-Cong Ji, Zhu Chen, *et al.*, Phys. Rev. Lett. **109** (2012) 115301.
[13] Pengjun Wang, Zeng-Qiang Yu, Zhengkun Fu, *et al.*, Phys. Rev. Lett. **109** (2012) 095301.
[14] Lawrence W. Cheuk, Ariel T. Sommer, *et al.*, Phys. Rev. Lett. **109** (2012) 095302.
[15] Lianghui Huang, Zengming Meng, Pengjun Wang, *et al.*, Nat. Phys. **10** (2016) 1038.
[16] Cheng Chin, Rudolf Grimm, Paul Julienne, and Eite Tiesinga, Rev. Mod. Phys. **82** (2010) 1225.
[17] Xiaoling Cui, Phys. Rev. A **85** (2012) 022705.
[18] Peng Zhang, Long Zhang, and Wei Zhang, Phys. Rev. A **86** (2012) 042707.
[19] Peng Zhang, Long Zhang, and Youjin Deng, Phys. Rev. A **86** (2012) 053608.
[20] Yuxiao Wu and Zhenhua Yu, Phys. Rev. A **87** (2013) 032703.
[21] Hao Duan, Li You, and Bo Gao, Phys. Rev. A **87** (2013) 052708.