PAPER

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Spin Hall separation of ultracold atom–molecule mixed gases

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Abstract

We propose a theoretical scheme to separate a molecular cloud from atoms in analogy to the spin Hall effect and to completely transfer Feshbach molecules to the ground state by applying a spatially modulated laser field to an atom–molecule mixed gas. In particular, the laser–molecule interaction induces a synthetic $U(1)$ gauge potential for the dressed molecular dark state. Through numerical simulation, we demonstrate that such a gauge field leads to a spin Hall separation of atoms and molecules. In such a process, molecules can be transformed into the ground state completely.

Keywords: spin Hall effect, artificial gauge potential

(Some figures may appear in colour only in the online journal)

1. Introduction

Recently, the light-induced effective gauge potential has opened a new avenue for studying spin–orbit coupling, magnetic monopoles and so on in ultracold systems [1]. The spin Hall effect (SHE) [2] is one such interesting phenomenon. It has been widely studied in condensed matter physics for its connection to topological insulators [3] and potential impact on information technology [4]. Experimentally, it has been observed in semiconductors [5] and semiconductor quantum wells [6]. The extremely clean environment and remarkable controllability of an ultracold atomic system provide a unique opportunity to study spin Hall physics [7–9]. Most recently, the SHE in a quantum gas has been observed experimentally [10].

In previous works [7], two dressed states were regarded as synthetic spin states. Their dynamics exhibits a typical spin Hall effect. However, their rotation in the full Hilbert space spanned by bare states had been ignored. It is a typical stimulated Raman adiabatic passage (STIRAP) [11]. The two-photon STIRAP is widely used in the preparation of ground-state molecules [12–16]. Weakly bound Feshbach molecules are coherently transformed into the rovibrational ground state via a two-photon STIRAP. In the Feshbach resonance [17], the generated molecules and the unconverted residual atoms are usually mixed in space. One needs to apply an additional gradient magnetic field to realize the Stern–Gerlach separation of the two species [18]. In the SHE, particles will move spin-dependently and experience a STIRAP; it is natural to ask whether it is possible to apply the SHE in the preparation of ground-state molecules.

In this paper, we show that $U(1)$ synthetic gauge potentials emerge in an atom–molecule mixed gas. Considering atoms and molecules in dark states as synthetic spins, we manifest the SHE in an atom–molecule mixture via numerical simulation with a reduced Hamiltonian. The atoms and molecules are separated spatially. The synthetic spin-down particles (molecules) experience a Raman passage. Feshbach molecules are transformed to ground-state molecules efficiently. Therefore we propose a scheme to prepare ground-state molecules from an atom–molecule mixture after Feshbach resonance. We justify our scheme via numerical simulation with a full Hamiltonian in the experimental parameter region.

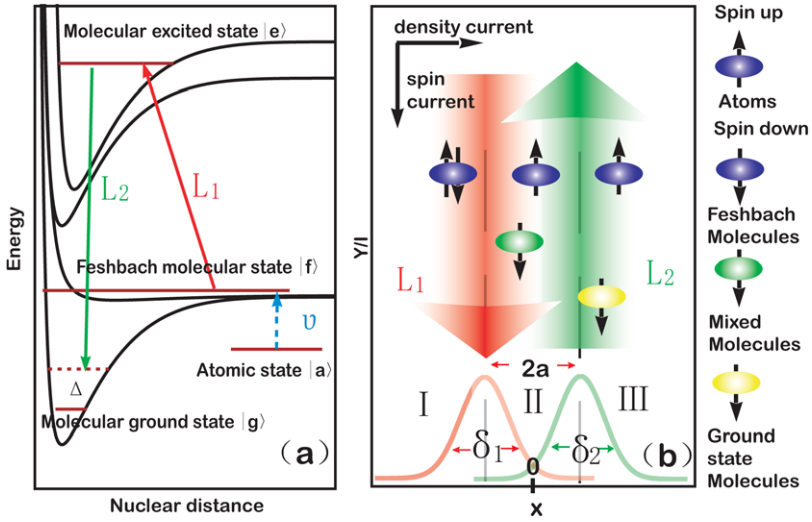

Figure 1. Schematic representation for generation of effective gauge fields and the spin Hall effect. (a) Energy levels of atom–molecule conversion. Laser beams L_1 and L_2 characterized by the Rabi frequencies Ω_1 and Ω_2 couple three molecular energy levels $|f\rangle, |e\rangle$ and $|g\rangle$ with detuning Δ. Feshbach resonance ν forms one Feshbach molecule from two atoms. (b) Configuration of the Raman laser beams and sketch of the spin Hall process. Laser beams L_1 and L_2 are two counterpropagating and overlapping Gaussian laser beams with shifted spatial profiles. Their waists are δ_1 and δ_2 and their center shift is $2a$. In the spin Hall process particles move spin-dependently and Feshbach molecules transform smoothly into ground state molecules.

2. Theoretical scheme

2.1. The total Hamitonian

In an atom–molecule mixed gas, we apply two laser beams L_1 and L_2. They couple the intermediate excited molecular state $|e\rangle$ to Feshbach molecular state $|f\rangle$ and molecular ground state $|g\rangle$ respectively, as shown in figure 1(a). The Hamiltonian of a mixed gas of free ultracold atoms and molecules in an external potential $V_{\rm ext}$ can be written in the second quantized form as follows:

$$\hat{H}_0 = \int \mathrm{d}^2 r \frac{\hbar^2}{2m}[-\hat{\Psi}^\dagger \nabla^2 \hat{\Psi} + \Psi^\dagger V_{\rm ext} \Psi], \tag{1}$$

where $\hat{\Psi}(\mathbf{r}) = (\hat{\psi}_a(\mathbf{r}), \hat{\psi}_f(\mathbf{r}), \hat{\psi}_e(\mathbf{r}), \hat{\psi}_g(\mathbf{r}))^{\rm T}$ in which $\hat{\psi}_a$, $\hat{\psi}_f$, $\hat{\psi}_e$ and $\hat{\psi}_g$ are annihilation operators of an atom, Feshbach molecule, excited molecule and ground-state molecule. Under the rotating wave approximation, the laser–molecule interaction reads

$$\hat{H}_{lm} = \int \mathrm{d}^2 r\, \hat{\Psi}^\dagger(\mathbf{r}) M \hat{\Psi}(\mathbf{r}) \tag{2}$$

with

$$M = \hbar \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_1^* & 0 \\ 0 & \Omega_1 & 0 & \Omega_2 \\ 0 & 0 & \Omega_2^* & \Delta \end{pmatrix}, \tag{3}$$

where Δ represents the two-photon detuning. $\Omega_1 = |\Omega_1| \mathrm{e}^{\mathrm{i}\mathbf{k_1}\cdot\mathbf{r}}$ and $\Omega_2 = |\Omega_2| \mathrm{e}^{\mathrm{i}\mathbf{k_2}\cdot\mathbf{r}}$ are the space-dependent Rabi frequencies of L_1 and L_2 respectively. The total Hamiltonian is $\hat{H} = \hat{H}_0 + \hat{H}_{lm}$.

2.2. Emergence of a U(1) gauge potential in the reduced Hamiltonian

In the resonant situation ($\Delta = 0$), the Hamiltonian (3) can be diagonalized by the matrix

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\beta & 0 & -\cos\beta \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \\ 0 & \frac{\sqrt{2}}{2}\cos\beta & \frac{\sqrt{2}}{2}\mathrm{e}^{-\mathrm{i}\mathbf{k}_1\cdot\mathbf{r}} & \frac{\sqrt{2}}{2}\sin\beta \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \\ 0 & \frac{\sqrt{2}}{2}\cos\beta & -\frac{\sqrt{2}}{2}\mathrm{e}^{-\mathrm{i}\mathbf{k}_1\cdot\mathbf{r}} & \frac{\sqrt{2}}{2}\sin\beta \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \end{pmatrix}, \tag{4}$$

where $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ and $\tan\beta = |\Omega_2|/|\Omega_1|$. The dressed states are

$$|\chi_1\rangle = |a\rangle, \tag{5}$$

$$|\chi_2\rangle = \sin\beta|f\rangle - \cos\beta \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}|g\rangle, \tag{6}$$

$$|\chi_3\rangle = \frac{\sqrt{2}}{2}(\cos\beta|f\rangle + \mathrm{e}^{-\mathrm{i}\mathbf{k}_1\cdot\mathbf{r}}|e\rangle + \sin\beta \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}|g\rangle), \tag{7}$$

and

$$|\chi_4\rangle = \frac{\sqrt{2}}{2}(\cos\beta|f\rangle - \mathrm{e}^{-\mathrm{i}\mathbf{k}_1\cdot\mathbf{r}}|e\rangle + \sin\beta \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}|g\rangle), \tag{8}$$

with energy eigenvalues $E_i = (0, 0, \hbar\Omega, -\hbar\Omega)$ and $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$. Correspondingly, we obtain the vector field operator $\hat{\Phi} = (\hat{\phi}_1, \hat{\phi}_2, \hat{\phi}_3, \hat{\phi}_4)^{\rm T} = U(\hat{\psi}_a, \hat{\psi}_f, \hat{\psi}_e, \hat{\psi}_g)^{\rm T}$. The Hamiltonian can be rewritten as

$$\hat{H} = \int \mathrm{d}^2 r\, \hat{\Phi}^\dagger \left(\frac{1}{2m}(-\mathrm{i}\hbar\nabla - \mathbf{A})^2 + V'\right)\hat{\Phi}, \tag{9}$$

where $\mathbf{A}' = \mathrm{i}\hbar U\nabla U^\dagger$ and $V'(\mathbf{r}) = UV_{\mathrm{ext}}U^\dagger + UMU^\dagger + \frac{\hbar^2}{2m}[(U\nabla U^\dagger)^2 + \nabla U\cdot\nabla U^\dagger]$. We straightforwardly calculate these matrices:

$$\mathbf{A}' = \hbar\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\cos^2\beta\mathbf{k} & \frac{\mathbf{k}\cos\beta\sin\beta - \mathrm{i}\nabla\beta}{\sqrt{2}} & \frac{\mathbf{k}\cos\beta\sin\beta - \mathrm{i}\nabla\beta}{\sqrt{2}} \\ 0 & \frac{\mathbf{k}\cos\beta\sin\beta + \mathrm{i}\nabla\beta}{\sqrt{2}} & -\frac{\mathbf{k}_1 + \sin^2\beta\mathbf{k}}{2} & \frac{\mathbf{k}_1 - \sin^2\beta\mathbf{k}}{2} \\ 0 & \frac{\mathbf{k}\cos\beta\sin\beta + \mathrm{i}\nabla\beta}{\sqrt{2}} & \frac{\mathbf{k}_1 - \sin^2\beta\mathbf{k}}{2} & -\frac{\mathbf{k}_1 + \sin^2\beta\mathbf{k}}{2} \end{pmatrix} \tag{10}$$

and

$$V' = \begin{pmatrix} V_{\mathrm{ext}} & 0 & 0 & 0 \\ 0 & V_{\mathrm{ext}} & 0 & 0 \\ 0 & 0 & V_{\mathrm{ext}} + \hbar\Omega & 0 \\ 0 & 0 & 0 & V_{\mathrm{ext}} - \hbar\Omega \end{pmatrix}. \tag{11}$$

When $\Delta \neq 0$, we have to add an additional term in equation (11), which is

$$\Delta' = \hbar\Delta\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2\beta & -\frac{\cos\beta\sin\beta}{\sqrt{2}} & -\frac{\cos\beta\sin\beta}{\sqrt{2}} \\ 0 & -\frac{\cos\beta\sin\beta}{\sqrt{2}} & \frac{\sin^2\beta}{2} & \frac{\sin^2\beta}{2} \\ 0 & -\frac{\cos\beta\sin\beta}{\sqrt{2}} & \frac{\sin^2\beta}{2} & \frac{\sin^2\beta}{2} \end{pmatrix}. \tag{12}$$

Furthermore, we assume the adiabatic condition [19]

$$\Omega \gg \frac{\hbar}{2m}\left|\frac{\mathbf{k}\cos\beta\sin\beta - \mathrm{i}\nabla\beta}{\sqrt{2}}\right|^2 \tag{13}$$

and

$$\Omega \gg \Delta. \tag{14}$$

They ensure that the off-diagonal elements of the matrices $\mathbf{A}'$, V'_{ext} and Δ' are much smaller than the energy differences $\hbar\Omega$. Particles in the subspace $\{|\chi_1\rangle, |\chi_2\rangle\}$ will not decay into the dressed states $|\chi_3\rangle$ and $|\chi_4\rangle$. The reduced Hamiltonian is

$$\hat{H}_{\mathrm{reduced}} = \int \mathrm{d}^2r\ \hat{\phi}_1^\dagger\left(-\frac{\hbar^2}{2m} + V_{\mathrm{ext}}\right)\hat{\phi}_1 + \int \mathrm{d}^2r\ \hat{\phi}_2^\dagger\left(\frac{1}{2m}(-\mathrm{i}\hbar\nabla - \mathbf{A})^2 + V_{\mathrm{ext}}\right)\hat{\phi}_2, \tag{15}$$

where the $U(1)$ adiabatic gauge potential $\mathbf{A} = \mathbf{A}'_{22} = -\hbar\cos^2\beta\mathbf{k}$.

2.3. Spin-dependent Hamiltonian

The Hamiltonian (15) can be written as two separated spin-dependent Hamiltonians

$$\hat{H}_\sigma = \int \mathrm{d}^2r\ \hat{\phi}_\sigma(\mathbf{r})\left(\frac{1}{2m}(\mathbf{P} - \mathbf{A}_\sigma)^2 + V_{\mathrm{ext}}\right)\hat{\phi}_\sigma(\mathbf{r}) \tag{16}$$

with $\sigma = \uparrow, \downarrow$, $\hat{\phi}_\uparrow = \hat{\phi}_1$ and $\hat{\phi}_\downarrow = \hat{\phi}_2$. The spin-dependent gauge fields are

$$\mathbf{A}_\sigma = -\eta_\sigma\hbar\cos^2\beta\mathbf{k} \tag{17}$$

and

$$\mathbf{B}_\sigma = \eta_\sigma\hbar\sin 2\beta\nabla\beta\times\mathbf{k}, \tag{18}$$

where $\eta_\uparrow = 0$ and $\eta_\downarrow = 1$. When the laser field is spatially modulated, we can get a *nonzero* magnetic field.

We consider two counterpropagating and overlapping Gaussian laser beams with shifted centers [10]. The corresponding Rabi frequencies Ω_j take the form $\Omega_j = \Omega_0\exp[-(x - x_j)^2/\delta_i^2]\exp(-\mathrm{i}k_j y)$ (with $j = 1, 2$, $\delta_1 = \delta_2 = \delta_0$ and $x_1 = -x_2 = a$) as shown in figure 1(b). The synthetic spin-dependent gauge potentials and magnetic fields are

$$\mathbf{A}_\sigma = \frac{\eta_\sigma\hbar k}{1 + \exp(-x/d)}\mathbf{e}_y \tag{19}$$

and

$$\mathbf{B}_\sigma = \frac{-\eta_\sigma\hbar k}{4d\cosh^2(x/2d)}\mathbf{e}_z, \tag{20}$$

where $d = \delta_0^2/8a$ and $k = k_1 - k_2$.

To observe the spin Hall effect, we need an effective electric field $\mathbf{E}$. It drives particles in one direction, and a spin current should be observed in the vertical direction. The external potential $V_{\mathrm{ext}} = xE$ plays the role of the effective field. Replacing the field operator $\hat{\phi}_\sigma(\mathbf{r})$ by the complex number $\phi_\sigma(\mathbf{r})$, we can obtain the mean-field spin-dependent Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\phi_\sigma(\mathbf{r}) = -\frac{\hbar^2}{2m}(\nabla - \mathrm{i}\mathbf{A}_\sigma)^2\phi_\sigma(\mathbf{r}) + xE\phi_\sigma(\mathbf{r}). \tag{21}$$

3. Spin Hall effect and spatial Raman passage

3.1. Numerical results

We make the following scalings of variables: $\tilde{l} = l/l_0$, $\tilde{t} = t/t_0$, $\tilde{E} = E/E_0$, $\tilde{k} = k/k_0$. Here $l_0 = 4\ \mu$m, $t_0 = ml_0^2/\hbar \simeq 30$ ms, $E_0 = \hbar^2/(ml_0^3)$ and $k_0 = 1/l_0$. The dimensionless form of the Schrödinger equation (16) is

$$\mathrm{i}\frac{\partial\psi_\sigma}{\partial t} = -\frac{1}{2}(\nabla - \mathrm{i}\mathbf{A}_\sigma)^2\psi_\sigma + xE\psi_\sigma \tag{22}$$

where we replace symbols $\tilde{o}$ with o, etc. To illuminate our scheme, We give the parameters $\delta_1 = \delta_2 = \delta_0 = 8$, $a = 8$, $k = 10$ and set the initial states to be $\phi_\uparrow(\mathbf{r}) = \phi_\downarrow(\mathbf{r}) = \frac{1}{\sqrt{2\pi}\delta_r}\exp(-|\mathbf{r} - \mathbf{r}_0|^2/2\delta_r^2)$ with $\delta_r = 2$ and $r_0 = (-15, 0)$. Using the algorithm of the lattice gauge difference scheme [21] to solve equation (22) numerically, we show the density profiles in figure 2. This displays a prominent spin Hall effect—the generation of a spin current transverse to an applied electric field [7]. In our scheme, the electric field is in the x direction and a spin current is generated in the y direction. Specifically the synthetic magnetic field $\mathbf{B}_\uparrow$ is equal to *zero*. The spin-up (atomic) cloud moves forward in the x direction as the result of the electric field $\mathbf{E}$. The spin-down particles (molecules) feel a *nonzero* magnetic field $\mathbf{B}_\downarrow$ in the z direction.

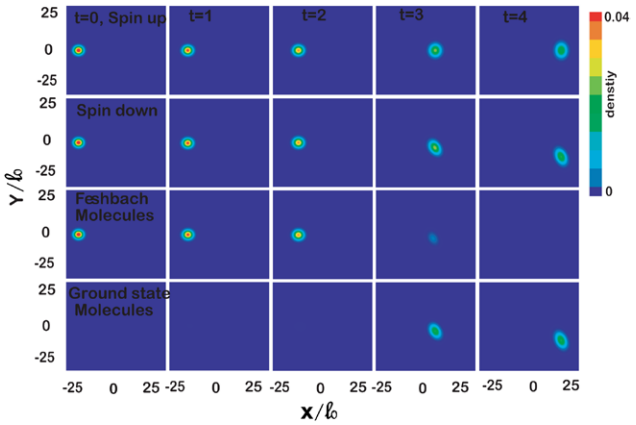

Figure 2. The density profiles of spin-up particles, spin-down particles, Feshbach molecules and ground-state molecules evolve under the electric field $E = -5$ for evolution times $t = 0, 1, 2, 3$ in the synthetic gauge field equation (16) where $\sigma_0 = 8$, $a = 8$ and $k = -10$. The initial states are $\phi_\uparrow(\mathbf{r}) = \phi_\downarrow(\mathbf{r}) = \frac{1}{\sqrt{2\pi}\delta_r}\exp(-|\mathbf{r}-\mathbf{r}_0|^2/2\delta_r^2)$, where $\delta_r = 2$ and $\vec{r}_0 = (-15, 0)$.

The spin-down (molecular) cloud moves in the $-y$ direction as the result of the Lorentz force $\mathbf{v}\times\mathbf{B}_\downarrow$. In figure 2 one can see spin-dependent shearing of the density profile [10]. The atomic clouds are unsheared, while the molecular clouds are sheared. This spin-dependent shearing of the density profile is caused by spin-dependent 'electric' forces $-\partial\mathbf{A}_\sigma/\partial t$ [22].

Using equation (6), we give the density profiles of Feshbach molecules and ground-state molecules in figure 2 as well. It is clear that Feshbach molecules are transferred to the ground state smoothly. This is a typical STIRAP [11]. The mixed angle β of the spin-down state is modulated in the x direction. We distinguish three intervals as shown in figure 4(b). From I to III, β changes smoothly from *zero* to $\pi/2$. When the spin-down cloud moves from region I to region III, $|\phi_\downarrow\rangle$ rotates from a direction parallel to $|f\rangle$ to a direction parallel to $|g\rangle$ in the plane perpendicular to $|e\rangle$. Therefore Feshbach molecules are transformed to ground-state molecules smoothly. In experiments [12–16] the STIRAP is caused by the temporal modulation of the laser field. In our case the STIRAP is caused by the spatial modulation of the laser field. Thus we call it spatial Raman passage.

3.2. Physical interpretation

The two-photon STIRAP is essential for both the spin Hall effect and spatial Raman passage. It causes the transition from Feshbach molecules to ground-state molecules. A spin-down particle is initially located at time t_i at x_i in region I. At a later time t_f, it arrives at the point x_f in region III. Its internal state rotates adiabatically from $|f\rangle$ to $|g\rangle$. This is caused by absorption of a photon from laser L_1, driving the $|f\rangle \to |e\rangle$ transition, and the stimulated emission of a photon from laser L_2, driving the $|e\rangle \to |g\rangle$ transition. In this process, the spin-down particle gains momentum $\hbar\mathbf{k}$ from the laser field. We study the cumulative effect of the Lorentz force, i.e. the change of the momentum along y,

$$\Delta\mathbf{p}_y = \int_{t_i}^{t_f}\mathbf{v}_x\times\mathbf{B}_\downarrow\,\mathrm{d}t = \int_{x_i}^{x_f}\mathbf{e}_x\times\mathbf{B}_\downarrow\,\mathrm{d}x$$
$$= \frac{\hbar\mathbf{k}}{2}[\tanh(x_1/2d) - \tanh(x_2/2d)] \simeq \hbar\mathbf{k}, \qquad (23)$$

where $\tanh(x_1/2d) = -\tanh(x_2/2d) \simeq 1$. Therefore the origin of the Lorentz force is closely related to the two-photon STIRAP. More precisely, the Lorentz force is associated with the vector potential that results from the perturbation of the internal molecular state due to the slow molecular motion [23].

4. Preparation of ground-state molecules

We find that Feshbach molecules can be separated from atoms and transformed into the ground state in the spin Hall process. Therefore we propose a theoretical scheme to get pure ground-state molecules from the mixed gas of atoms and Feshbach molecules after Feshbach resonance. We apply two laser beams L_1 and L_2 to couple an excited state with a Feshbach molecular state and a molecular ground state as shown in figure 1(a). The configuration of laser beams is sketched in figure 1(b) similar to [20]. The mixed gas cloud of atoms and Feshbach molecules is located in region I. Due to the effective electric field the numbers of atoms and molecules fall off in the x direction. After a while in region III, molecules are separated from the atomic cloud and transferred to the ground state.

The above proposal is only demonstrated in principle based on adiabatic evolution with the reduced Hamiltonian. In order to obtain more convincing results, we study in the full Hilbert space by numerically solving the mean-field Schrödinger equation of the Hamiltonian $\hat{\mathcal{H}}$. Replacing $\hat{\psi}_\alpha(\mathbf{r})$ with the complex number $\psi_\alpha(\mathbf{r})$ ($\alpha = a, f, e, g$), we can obtain the mean-field Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r}) = -\frac{\hbar^2\nabla^2}{2m}\Psi(\mathbf{r}) + V_{\mathrm{ext}}\Psi(\mathbf{r}) + M\Psi(\mathbf{r}) \qquad (24)$$

where $\Psi = (\psi_a, \psi_f, \psi_e, \psi_g)^{\mathrm{T}}$ and $V_{\mathrm{ext}} = xE$.

We take ^{132}Cs as an example. In [15], molecules in a weakly bound Feshbach state are transferred to the ground state via a two-photon STIRAP. The wavelengths of the two laser beams are near 1126 nm and 1006 nm. Their peak Rabi frequencies are about $2\pi\times 3$ MHz and $2\pi\times 6$ MHz. We take the wavevectors of the two laser beams to be $k_1 \simeq -k_2 \simeq 10^{-6}$ m^{-1}. The peak Rabi frequencies are $\Omega_0 = 30$ MHz. Their waists are $\delta_1 = \delta_2 = \delta_0 = 45$ μm. Their center shift is $2a = 120$ μm. Double STIRAP efficiency gets its maximum value in the resonant case. Therefore we choose $\Delta = 0$. The adiabatic conditions (13) and (14) are satisfied in our laser scheme. The effective electric field can be conveniently generated through gravity on the neutral particles or the trap displacement technique [10]. Here we set the effective electric field to be $E \simeq 0.15$ m s^{-2}.

Without losing generality, we choose the initial state as $\Psi_0(\mathbf{r}) = (\psi_a(\mathbf{r}), \psi_f(\mathbf{r}), 0, 0)^{\mathrm{T}}$ where $\psi_a(\mathbf{r}) = \psi_f(\mathbf{r}) = \frac{1}{\sqrt{2\pi}\delta_r}\exp(-|\mathbf{r}-\mathbf{r}_0|^2/2\delta_r^2)$ is a Gaussian wave packet. In [18] Cs atoms

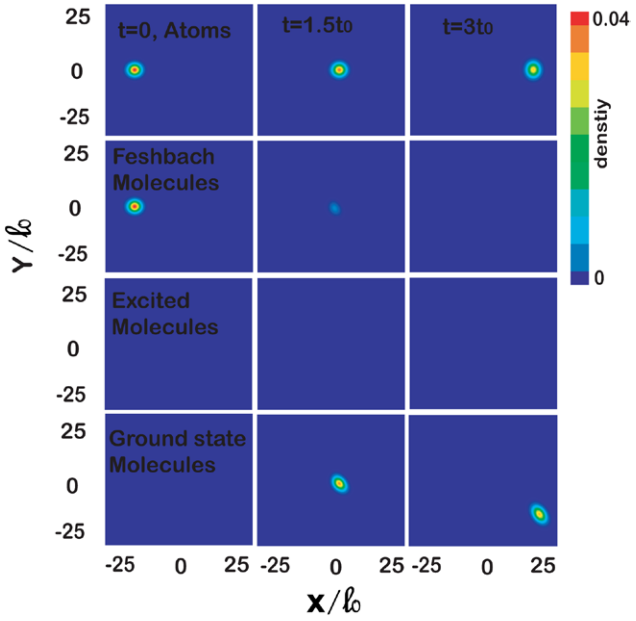

Figure 3. Full Hilbert space calculation. Evolutions of density profiles following the Schrödinger equation (24) in the laser field as shown in figure 1(b). The wavevectors of the two counterpropagating laser beams are $k_1 \simeq -k_2 \simeq 10^6\ \mathrm{m}^{-1}$. Their peak Rabi frequencies are both about 30 MHz. Their waists are about 45 μm and the center shift $2a$ is near 120 μm. The initial state is $\Psi_0(\mathbf{r}) = (\psi_a(\mathbf{r}), \psi_f(\mathbf{r}), 0, 0)^{\mathrm{T}}$ where $\psi_a(\mathbf{r})$ and $\psi_f(\mathbf{r})$ are both Gaussian wave packets located at $x = -60$ μm on the x axis with waist $\delta_r = 8$ μm.

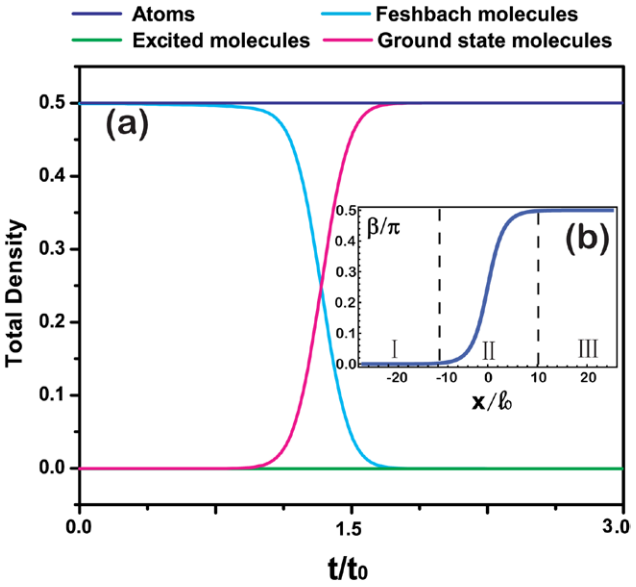

Figure 4. Spatial Raman passage. (a) The temporal evolution of the total densities of atoms, Feshbach molecules, excited molecules and ground-state molecules in figure 3. (b) The spatial modulation of the mixing angle, β, of the spin-down state $|\phi_\downarrow\rangle = \sin\beta|f\rangle - \cos\beta e^{i\mathbf{k}\cdot\mathbf{r}}|g\rangle$.

are trapped in an optical trap with a radial trap frequency ω of 18 Hz. The radial radius of the non-interacting particle cloud, $r = \sqrt{\hbar m/\omega}$, is about 5 μm. We choose the waist $\delta_r = 8$ μm. The initial wave packets are located at $\mathbf{r}_0 = -60$ $\mu\mathrm{m}\,\mathbf{e}_x$ where the spin-down state $|\phi_\downarrow\rangle \simeq |\phi_f\rangle$.

Solving equation (24) numerically, we show the density profiles at evolution times $t = 0$, $1.5t_0$ and $3t_0$ in figure 3. The molecular cloud is separated from the atomic cloud. In order to see the STIRAP transfer clearly, we give the temporal evolution of the total densities of each bare state in figure 4(a). Feshbach molecules are transformed into the ground state completely. During the evolution one can hardly see the emergence of excited molecules, which means that the state vector is always in the degenerate manifold $\{|\phi_\uparrow\rangle, |\phi_\downarrow\rangle\}$. Therefore we can understand the results with the spin Hall language in the same spirit as what has been described by equation (21). An atom and a molecule are two different spin particles, so they move spin-dependently as the result of the spin Hall effect. Molecules experience the spatial Raman passage in the spin Hall process so Feshbach molecules are transformed into the ground state smoothly.

Our reduced-space and full-space numerical results both show that in the spatially modulated Raman light field we can separate a molecular cloud from an atomic cloud and completely transfer Feshbach molecules to the ground state via a spin Hall process. Since the two-photon STIRAP plays the essential role in the spin Hall process, our scheme has the prospect of wide application in the preparation of ground-state molecules.

5. Conclusion

In summary, we have proposed a theoretical scheme to realize spin Hall separation in ultracold atom–molecule mixed gases. Through numerical solution of the spin-dependent Schrödinger equation in reduced space and the Schrödinger equation in full space, we demonstrate spatial separation of unconverted atoms and molecules, spin Hall shearing of density profiles, and spatial Raman passage in the spin Hall process. According to our results, our scheme may have an important application in the preparation of pure ground-state ultracold molecules.

Acknowledgments

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