LETTER

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A Letters Journal Exploring the Frontiers of Physics

December 2016

EPL, **116** (2016) 60014 www.epljournal.org
doi: 10.1209/0295-5075/116/60014

High-fidelity superadiabatic population transfer of a two-level system with a linearly chirped Gaussian pulse

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received 11 December 2016; accepted in final form 23 January 2017
published online 16 February 2017

PACS 03.65.Xp – Tunneling, traversal time, quantum Zeno dynamics
PACS 32.80.Qk – Coherent control of atomic interactions with photons
PACS 42.50.Dv – Quantum state engineering and measurements

Abstract – We investigate high-fidelity superadiabatic quantum driving in a chirped Gaussian two-level model with a Gaussian temporal envelope and a linear detuning. We show that the nonadiabatic losses can be canceled to any desired order by constructing and adjusting an auxiliary Hamiltonian (counter-diabatic field) and a symmetry in the fidelity arises on the counter-diabatic field ratio. A high-fidelity, robust, and accelerated (in a shorter time) transitionless superadiabatic population transfer is achieved that ensures a perfect following of the instantaneous adiabatic ground state even in the nonadiabatic regime. The features make the superadiabatic protocol a potentially important tool for quantum information.

Introduction. – Quantum adiabatic processes are a powerful strategy to implement quantum state control which aims at manipulating a quantum system to attain a prescribed target state in a controlled and optimal way [1–3]. During adiabatic evolution, the system follows an eigenstate of the Hamiltonian, *i.e.*, if the system is prepared in an eigenstate of the Hamiltonian at an initial time, it will evolve to the corresponding instantaneous eigenstate at later times. Based on the adiabatic dynamics, many different processes, such as controlling chemical reactions, laser cooling, nuclear magnetic resonance, quantum information, and fast population transfer in quantum optics, were realized both theoretically and experimentally in the recent past [4–7]. However, often such an adiabatic process may be too slow to satisfy the adiabatic criteria, and in nearly all adiabatic techniques the population transfer is incomplete, with the fidelity close to, but less than, 1 [8]. Therefore various protocols have been devised to speed up the process and to enhance the fidelity of quantum manipulation processes [9–14].

Among the various popular methods, the superadiabatic (also known as transitionless or counterdiabatic) quantum driving [12,13] and shortcut to adiabaticity [14] are two valuable tools to speed up the adiabatic quantum behavior. The former suppresses the nonadiabatic transitions between energy eigenstates and ensures a perfect adiabatic following by constructing an auxiliary field (Hamiltonian). The latter puts forward another reverse engineering approach using the Lewis-Riesenfeld (LR) invariant to carry the eigenstates of a Hamiltonian from a specified initial to a final configuration, then to design the transient Hamiltonian from the LR invariant. Although different in form, those driving methods are shown to be essentially equivalent to each other by properly adjusting the reference Hamiltonian [14]. Recently, these protocols have been extended to many quantum systems [15–22]. Experiments with superadiabatic protocols have been demonstrated for a Bose-Einstein condensate loaded into an accelerated optical lattice [23,24], the electron spin of a single nitrogen-vacancy center in diamond [25], a large single-photon detuning system with a cold atomic ensemble [26], and a continuous variable system for adiabatic transport of a trapped ion [27].

Two-level systems are a fundamental ingredient and play an important role in quantum mechanics. Although rarely existing in nature in their pure form, they often serve as models in many areas of physics and are successful in describing a large variety of physical phenomena. Many

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problems involving multiple states and complicated linkage patterns can often be understood only by reduction to effective two-level problems. The quest for optimal control of the two-level systems has a long history, and protocols denoted as Landau-Zener, Rosen-Zener, Demkov-Kunike, Roland-Cerf, and composite pulses have been studied for many years [28–31]. Within the framework of adiabatic quantum manipulation, the high-fidelity superadiabatic quantum driving has been achieved in generalized Landau-Zener, Allen-Eberly, and tangent models [23,24]. A linearly chirped Gaussian pulse with a Gaussian temporal envelope and a linear detuning is also a typical protocol for two-level quantum systems and has been widely used in atomic, molecular, optical and plasma physics [32–35]. Experimentally, the model has been applied to tune bandwidth frequency, to simulate the process of laser triggered lightning in atmosphere, to measure the phase structure of soliton molecules, and to realize the strong-field ultrafast coherent control [32–34]. A very accurate analytic approximation to the transition probability has been derived by using the Dykhne-Davis-Pechukas approach [36] and a population transfer has been studied [34,35,37]. The results show that a complete adiabatic following can be realized under adiabatic condition. However, the adiabatic transition process require a long evolution time to satisfy the complex adiabatic condition and the fidelity of transition probability is not high [36,37].

In this paper, we study the superadiabatic population transfer in a two-level system with a linearly chirped Gaussian pulse. The superadiabatic protocol shows how the efficiency of the transfer can be improved by adding a suitably chosen counter-diabatic term, which aims to nullify the nonadiabatic coupling and to speed up the adiabatic dynamics. The high-fidelity requirement is achieved and the system follows the instantaneous adiabatic ground state near perfectly for all time. In the following section we present the two-level model and briefly introduce the superadiabatic quantum driving protocol. The third section discusses how to apply the protocol to the proposed chirped Gaussian model and study the stability of the approximate counter-diabatic field control protocol to variation of the counter-diabatic field intensity. The last section gives the conclusions.

Model and superadiabatic protocol. – The two-level system driven by an external coherent field is described by the dimensionless Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} = H(t)\begin{pmatrix} a \\ b \end{pmatrix}, \tag{1}$$

with the Hamiltonian given by

$$H(t) = \gamma(t)\hat{\sigma}_z + v(t)\hat{\sigma}_x, \tag{2}$$

where a and b are the probability amplitudes of diabatic states $|0\rangle$ and $|1\rangle$. The total probability $|a|^2 + |b|^2$ is conserved and set to be 1. $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are Pauli matrices, and $\gamma(t)$ and $v(t)$ are the energy bias and coupling strength between two diabatic levels, respectively.

The above system has instantaneous adiabatic eigenstates $|\psi_\pm(t)\rangle$,

$$H(t)|\psi_\pm(t)\rangle = \varepsilon_\pm(t)|\psi_\pm(t)\rangle, \tag{3}$$

where the eigenvalues $\varepsilon_\pm(t) = \pm\sqrt{\gamma^2 + v^2}$, and the subscripts $-$ and $+$ stand for the ground state and the excited state, respectively. Their difference $\varepsilon(t) = \varepsilon_+(t) - \varepsilon_-(t) = 2\sqrt{\gamma^2 + v^2}$ defines the energy splitting. The Hamiltonian can be diagonalized using the unitary transformation to a new basis (A, B), which is the adiabatic basis, given by

$$\begin{pmatrix} A \\ B \end{pmatrix} = U_0^{-1}(t)\begin{pmatrix} a \\ b \end{pmatrix}, \tag{4}$$

where U_0 is the rotation matrix and can be taken as

$$U_0 = \begin{pmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}. \tag{5}$$

Here the mixing angle $\theta = \frac{1}{2}\arctan(v(t)/\gamma(t))$. The Hamiltonian of the system in adiabatic basis is

$$H'(t) = U_0^{-1}H(t)U_0 - iU_0^{-1}\dot{U}_0, \tag{6}$$

where the overdot represents the derivative with respect to time t. The first term is the diagonal part, while the second term is the nondiagonal part regarded as a nonadiabatic correction. The Schrödinger equation in the adiabatic basis reads

$$i\frac{\partial}{\partial t}\begin{pmatrix} A \\ B \end{pmatrix} = H'(t)\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \varepsilon_- & -i\dot{\theta} \\ i\dot{\theta} & \varepsilon_+ \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}. \tag{7}$$

Adiabatic evolution takes place when the nonadiabatic coupling in the Hamiltonian is negligible compared to the eigenenergy splitting. Mathematically, the adiabatic evolution requires the off-diagonal elements of the Hamiltonian (7) to be negligible compared to the diagonal ones, *i.e.*, $|\dot{\theta}| \ll \varepsilon$, which expresses the adiabatic condition [12]. The efficiency of this transfer is limited by the adiabatic condition, which requires slow evolution. When the adiabatic condition cannot be fulfilled, a complete population transfer does not occur due to the effect of the nonadiabatic term in the Hamiltonian. To overcome this one constructs an auxiliary Hamilitonian H_{cd} (also called counter-diabatic field) that cancels the nonadiabatic part of the evolution under H alone [12,13]. It thus ensures a transitionless adiabatic following such that the system evolving under $H + H_{cd}$ always remains the instantaneous adiabatic ground state of H with probability 1, even for a finite duration of the protocol. In general H_{cd} can be given by $H_{cd} = i\dot{U}_0U_0^{-1}$. For a two-level system of the form (1) one finds that [12,13]

$$H_{cd}(t) = \frac{\partial\theta}{\partial t}\hat{\sigma}_y, \tag{8}$$

where $\hat{\sigma}_y$ is the Pauli matrix.

To study the stability of the efficiency of the population transfer driven by a variable counter-diabatic field, we represent the total driving Hamiltonian in the form

$$\begin{aligned} H_{tot}(t) &= H + (1+\lambda)H_{cd} \\ &= \gamma(t)\hat{\sigma}_z + v(t)\hat{\sigma}_x + (1+\lambda)\dot{\theta}\hat{\sigma}_y \\ &= \begin{pmatrix} \gamma(t) & v(t) - i(1+\lambda)\dot{\theta} \\ v(t) + i(1+\lambda)\dot{\theta} & -\gamma(t) \end{pmatrix}, \end{aligned} \quad (9)$$

where λ represents a counter-diabatic field ratio [12]. $\lambda = -1$ corresponds to driving the system with only the original Hamiltonian H and $\lambda = 0$ to driving the system with the original Hamiltonian added a counter-diabatic field. We can describe the Hamiltonian (9) as a combination of an effective coupling and a phase term

$$H_{tot}(t) = \begin{pmatrix} \gamma(t) & v_{eff}(t)\exp(-i\phi) \\ v_{eff}(t)\exp(i\phi) & -\gamma(t) \end{pmatrix}, \quad (10)$$

where $v_{eff}(t) = \sqrt{v^2(t) + ((1+\lambda)\dot{\theta})^2}$. To eliminate the phase dependence, we take the following transformation [17]:

$$U_1 = \begin{pmatrix} \exp(-i\phi/2) & 0 \\ 0 & \exp(i\phi/2) \end{pmatrix}, \quad (11)$$

which again provides a new set of bases, and now the resulting Hamiltonian becomes

$$H_{tot}(t) = \begin{pmatrix} \gamma_{eff}(t) & v_{eff}(t) \\ v_{eff}(t) & -\gamma_{eff}(t) \end{pmatrix}, \quad (12)$$

where $\gamma_{eff}(t) = \gamma(t) - \dot{\phi}/2$ with $\phi = \arctan((1+\lambda)\dot{\theta}/v(t))$. This means that the effect of the extra field can also be achieved through an appropriate transformation $\gamma \to \gamma_{eff}$ and $v \to v_{eff}$. Complete population transfer by adiabatic following has been realized, for instance by using the LZ model and the AE model [23,24].

High-fidelity superadiabatic quantum driving in chirped Gaussian model. – In this section we will apply the superadiabatic quantum driving protocol to the chirped Gaussian model, in which the coupling is a Gaussian temporal shape and the energy bias is a linear function of time [36],

$$v(t) = v_0 \exp\left[-\left(\frac{t}{T}\right)^2\right], \quad \gamma(t) = \alpha t, \quad (13)$$

where v_0 is the coupling strength (or peak Rabi frequency) and T is the pulse duration. The parameter α stands for sweep rate (the chirp rate). We assume that the system is initially prepared in the adiabatic ground state $|\psi_-(t_{ini})\rangle$ at time $t = t_{ini}$. The final state at time $t = t_{fin}$ is the state $|\psi_{fin}\rangle$ after an evolution of duration $t_{fin} - t_{ini}$. Our aim is to realize the superadiabatic protocol that ensures a perfect following of the instantaneous adiabatic ground state $|\psi_-(t)\rangle$ for all time. The protocol can drive the system from the starting state $|\psi_-(t_{ini})\rangle$ to the final state $|\psi_{fin}\rangle$ in a speed-up way and with high fidelity, *i.e.*, the final state $|\psi_{fin}\rangle$ is as close as possible to the adiabatic ground state $|\psi_-(t_{fin})\rangle$, realizing a fidelity close to unity. Here the fidelity function F_{fin} is defined as follows:

$$F_{fin} = |\langle\psi_{fin}|\psi_-(t_{fin})\rangle|^2, \quad (14)$$

which can be used to characterize the protocol efficiency. For the chirped Gaussian model, we can obtain $\dot{\theta} = (\dot{v}(t)\gamma(t) - v(t)\dot{\gamma}(t))/2(v^2(t) + \gamma^2(t)) = -\alpha v(t)(t^2/T^2 + 0.5)/(v^2(t) + \gamma^2(t))$. The time dependences of the effective energy bias and coupling strength for different counter-diabatic field ratios are shown in fig. 1. The fidelity of the final state are plotted fig. 2 as a function of the coupling strength v_0 in different counter-diabatic field ratios and sweep rates. For simplicity, all the variables here should be understood as scaled dimensionless variables. Throughout, we use T to scale. Then, $T = 1$, v_0 and α are in units of $1/T$ and $1/T^2$, respectively. In all numerical simulations, the numerical time was performed from times -20 to 20.

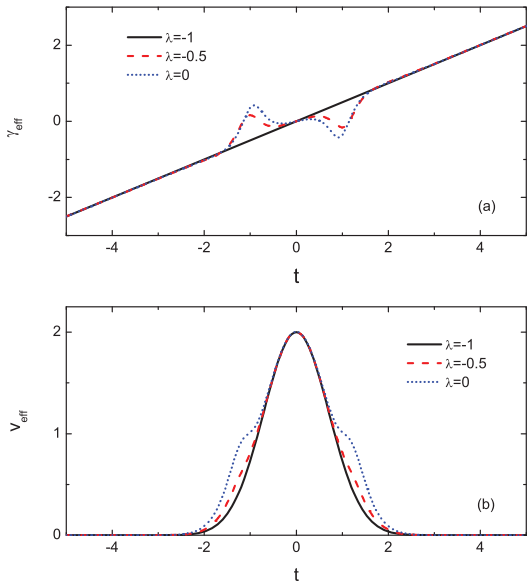

Fig. 1: (Color online) The time dependences of the effective energy bias and coupling strength of different counter-diabatic field ratios $\lambda = -1, -0.5, 0$ for (a) $\gamma_{eff}(t) - t$ and (b) $v_{eff}(t) - t$. Parameters: $v_0 = 2, \alpha = 0.5$, and $T = 1$.

We see that when the counter-diabatic field ratio $\lambda = -1$, corresponding to the original chirped Gaussian model without counter-diabatic field in the Hamiltonian of the system, for small v_0 there is a monotonic increase of F_{fin} with v_0. As v_0 increases, Rabi-type oscillations occur (except for very large α). With the emergence of a counter-diabatic field, the nonadiabatic losses can be reduced and the oscillation amplitude decreases, regardless of the sweep rate α. For $\lambda = -0.5$, the fidelity has been

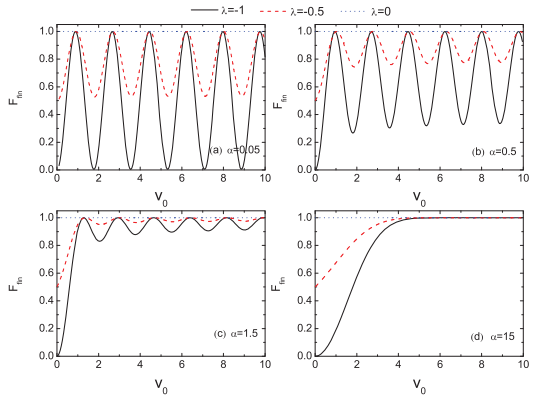

Fig. 2: (Color online) The fidelity of the final state as a function of the coupling strength v_0 in different counter-diabatic field ratios and sweep rates. (a) $\alpha = 0.05$, (b) $\alpha = 0.5$, (c) $\alpha = 1.5$, and (d) $\alpha = 15$.

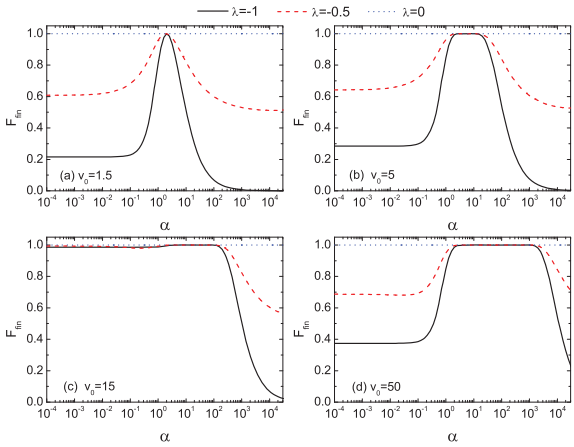

Fig. 3: (Color online) The fidelity of the final state as a function of the sweep rate α in different counter-diabatic field ratios and coupling strength. (a) $v_0 = 1.5$, (b) $v_0 = 5$, (c) $v_0 = 15$, and (d) $v_0 = 50$.

greatly improved. Until the counter-diabatic field ratio is taken as 0, the nonadiabatic oscillations completely disappear and the fidelity of the final state holds in 1. In fig. 3 the fidelity of the final state is plotted *vs.* the sweep rate α for different values of the coupling strength v_0 and counter-diabatic field ratio λ. The dependence of fidelity on the α is nonoscillatory and in the case of $\lambda = -1$ there are three distinctly different regimes. For small α, the value of F_{fin} depends strongly on v_0, as seen in the figure. For moderate α, the fidelity is nearly unity. For very large α, the fidelity decreases. If we take $\lambda = -0.5$, the fidelity dramatically increases. When $\lambda = 0$, meaning that the counter-diabatic field is fully implemented, the ultrahigh fidelity is achieved in all parameter regimes. The fidelity of the final state is further illustrated in fig. 4 *vs.* the counter-diabatic field ratio λ. The counter-diabatic field greatly enhances the transition probability and achieves a high fidelity. A symmetry in the fidelity arises and the maximum

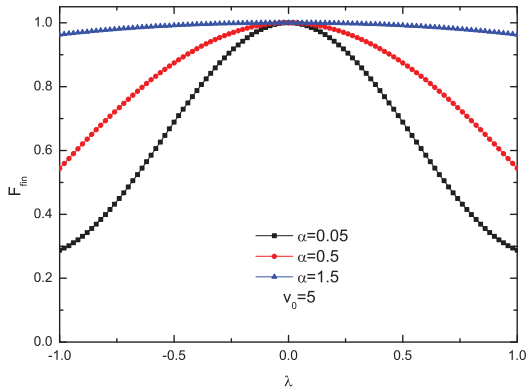

Fig. 4: (Color online) Comparison of the fidelity of the final state for various λ in the population transfer with $\alpha = 0.05, 0.5, 1.5$.

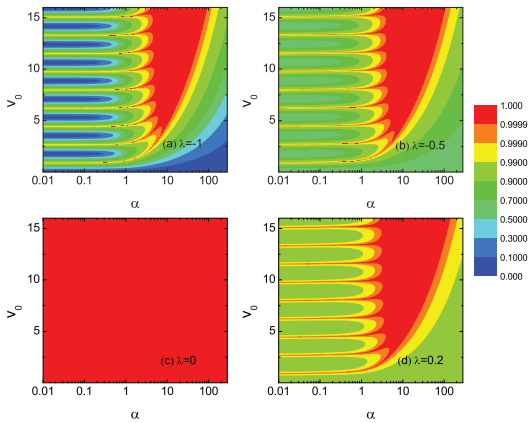

Fig. 5: (Color online) Contour plots of the fidelity of the final state as the function of the sweep rate α and the coupling strength v_0 for four different counter-diabatic field ratios λ.

values of fidelity correspond to the counter-diabatic field ratio $\lambda = 0$ (see footnote [1]).

To further investigate the effectiveness and robustness, we show the contour plots of fidelity final state as the function of both the sweep rate α and the coupling strength v_0 for four different counter-diabatic field ratios λ in fig. 5. The blue zones correspond to low fidelity whereas red areas indicate high fidelity. The superadiabatic driving protocol greatly enhances the robustness of the fidelity against variations of α and v_0 and achieves ultrahigh fidelity (the error below the 10^{-4} quantum computation benchmark) even for moderate parameter values under apparently unfavorable adiabatic conditions.

Moreover, we define a fidelity of system as

$$F(t) = |\langle\psi(t)|\psi_-(t)\rangle|^2, \tag{15}$$

to characterize the protocol efficiency during the adiabatic evolution process. Here $|\psi(t)\rangle$ is the actual state

[1]The Schrödinger equation with the total Hamiltonian (9) in the adiabatic basis reads $i\frac{\partial}{\partial t}\binom{A}{B} = H'(t)\binom{A}{B} = \begin{pmatrix}\varepsilon_- & -i\lambda\dot{\theta}\\ i\lambda\dot{\theta} & \varepsilon_+\end{pmatrix}\binom{A}{B}$. It is obvious that the system can perfectly adiabatically follow for $\lambda = 0$ and the transition dynamics are in complete symmetry for $\pm\lambda$.

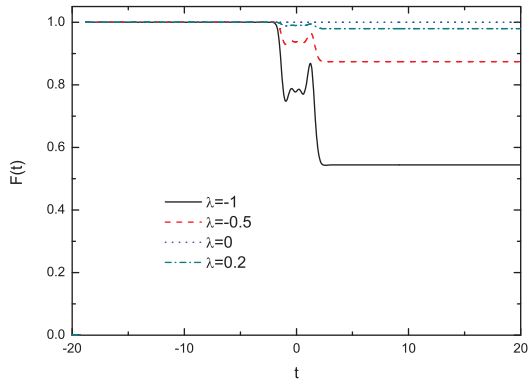

Fig. 6: (Color online) The time dependences of fidelity $F(t)$ for different counter-diabatic field ratios with $\alpha = 0.5, v_0 = 5$.

of the system. If the system can adiabatically evolve along the adiabatic ground state (adiabatic following), then the value of the fidelity should be 1. It is evident that $F(t) = F_{fin}$ at $t = t_{fin}$. The variation of the fidelity $F(t)$ with time for different λ with $\alpha = 0.5, v_0 = 5$ is shown in fig. 6. It is obvious that the superadiabatic protocol can be achieved for $\lambda = 0$, where the value of fidelity holds in 1 for all the time.

Conclusions. – In conclusion, we have investigated the high-fidelity superadiabatic quantum driving in a chirped Gaussian two-level system with a Gaussian temporal envelope and a linear detuning. The protocol allows one to suppress the nonadiabatic oscillations in the transition probability and to enhance the fidelity of the evolution process. The additional counter-diabatic field make it possible to readily implement the protocol ensuring a perfect adiabatic following in shorter time. We expect that the superadiabatic population transfer can be achieved in experiment. For example, in a Bose-Einstein condensate loaded into an accelerated optical lattice, the time dependence of linearly detuning can be achieved through a variation of the quasimomentum and the time dependence of the Gaussian coupling can be controlled through the power of the lattice laser beams [23]. The ultrahigh fidelity, good robustness against external field parameters, and fast quantum driving features show that the superadiabatic protocol is a potentially important tool for quantum information science such as quantum computing, quantum communication and quantum metrology [5,38].

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The work is supported by the National Basic Research Program of China (973 Program) (Grants No. 2013CBA01502 and No. 2013CB834100), the National Natural Science Foundation of China (Grants No. 11665020, No. 11547046, No. 11374040, No. 11475027, No. 11575027, and No. 11274051), the China Postdoctoral Science Foundation (Grant No. 2015M580068), and the Natural Science Foundation of Gansu Province, China (Grant No. 1606RJZA081).

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