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Pair-production in polychromatic light oscillating electric fields

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Electron–positron pair-production process is observed in polychromatic light oscillating electric fields. A quantum kinetic theory is presented to obtain the pair-production rate and the momentum information. Oscillating structures of the particle yield and the odd–even structure of the momentum–frequency spectrum are depicted. The roles of the phase factor and the frequency multiplication are analyzed. We find the odd–even structures are remained in the triple-frequency field.

Keywords: Polychromatic; monochromatic; phase factor; frequency multiplication.

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1. Introduction

The vacuum is unstable[1] in the presence of extremely strong electromagnetic field and may decay by emitting electron–positron pairs. The pair-production rate was first calculated by the famous Schwinger's formula[1] which describes the boson pair-production in a static, homogeneous electric field. Then some theoretical studies on

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pair-production started.[2,3] At the extreme laser intensity, many quantum electrodynamics (QED)[4] phenomena can be realized in the laboratory. The theoretical studies on pair-production started at the early stage of QED,[5,6] then a quantum kinetic theory (QKT)[7,8] was introduced and widely used. With the development of strong laser technology, observing the pair-production process by using only laser beams[9] can be expected. The pair-production process, as a fascinating topic in strong-field QED, is reported by many experimental processes through several different manners. However, the theoretical investigations[10–13] are much ahead of experimental efforts.[14,15] Also, some works have speculated that the ultraintense laser could induce QED cascade,[16] which could be made as an abundant source of electrons, positrons and photons.[17–21]

In recent years, there arise some new topics on pair-production, such as the bondonic quasi-particle pair formation,[22] the pair-production created by the seed photon,[23] the delocalized quanta of the chemical field,[22] the effective mass model[24] and the chemical field contributions.[25]

Nevertheless, most of these theoretical researches are mainly focused on the monochromatic light field and there still lacks some physical descriptions and explanations of polychromatic light field. Therefore, a comprehensive treatment and investigation of electron–positron pair-production in polychromatic field is highly necessary, as we have done in this work.

This paper is organized as follows. First, the theoretical method is described. Second, the momentum–frequency spectrum of monochromatic field is given and the odd–even structures are defined. Third, particle yield and momentum–frequency spectrum of polychromatic field are shown and the roles of phase factor and frequency multiplication are analyzed. Finally, a brief summary is given.

2. Theory

The pair-production process from the vacuum is a time-dependent[1] and nonequilibrium process. A quantum Vlasov equation (QVE) can be used to obtain the pair-production rate and the momentum information. With the help of auxiliary functions $G(q,t)$ and $H(q,t)$ (q is the momentum and t is the time), we could obtain the momentum distribution function $F(q,t)$ from the the reduced form of QVE as follows[7,8]:

$$\dot{F}(q,t) = W(q,t)G(q,t)\,, \tag{1}$$

$$\dot{G}(q,t) = W(q,t)[1-F(q,t)] - 2w(q,t)H(q,t)\,, \tag{2}$$

$$\dot{H}(q,t) = 2w(q,t)G(q,t)\,, \tag{3}$$

where

$$w^2(q,t) = m^2 + [q - eA(t)]^2\,, \quad W(q,t) = \frac{eE(t)m}{w^2(q,t)}\,. \tag{4}$$

Here, we restrict our discussion in one dimension. $A(t)$, $E(t)$ and e are the vector potential, the strength of electric field and electron charge, respectively. In this work, natural units ($\hbar = e = c = 1$) are used and all quantities are scaled by the rest mass of electron m. The initial conditions are given by $F(q, -\infty) = G(q, -\infty) = H(q, -\infty) = 0$. The particle yield N per Compton wavelength is[24]

$$N = \int dq/(2\pi) F(q, \infty)\,. \tag{5}$$

In the following, particle yield and momentum distribution of the electron–positron pair-production process under the conditions of both monochromatic field and polychromatic field are studied.

3. Monochromatic Light Field

First, let us consider the homogeneous monochromatic electric field laser pulse[24]

$$E(t) = \varepsilon \exp\left(-\frac{t^2}{2\tau^2}\right) \cos(\omega t)\,, \tag{6}$$

where ε denotes the peak strength, τ is the pulse duration and ω is the frequency, by setting the parameters $\tau = 300/m$, $e\varepsilon/m^2 = 0.10$.

Figure 1 gives the particle distribution function $F(q, t = \infty)$ as a function of ω for monochromatic field in plane (q, ω), which describes the function of $F(q, t = \infty)$

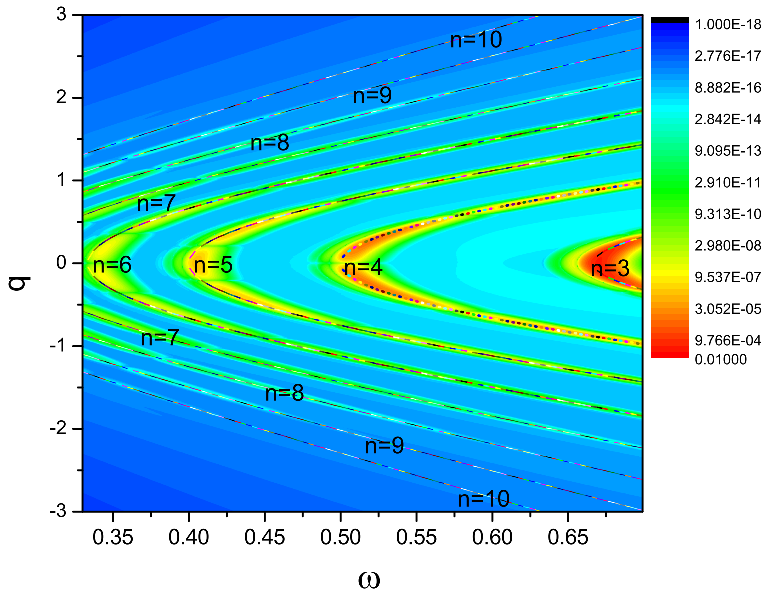

Fig. 1. (Color online) The particle distribution function $F(q, t = \infty)$ of ω for monochromatic field in plane (q, ω). Curves of q_n [dotted lines derived from Eq. (8)] for relevant n are given. The particle distribution disappears near $q = 0$ for even n and remains for odd n (the odd–even structure).

with the momentum q and frequency ω (the momentum–frequency spectrum). For a multi-photon absorption pair-production process, the energy conservation satisfies

$$n\hbar\omega = 2\sqrt{c^2 q_n^2 + m^2 c^4}\,, \tag{7}$$

where n is the photon number. Then we obtain the function of q_n as

$$q_n = \sqrt{\frac{1}{4} n^2 \omega^2 - m^2}\,. \tag{8}$$

The momentum curves [dotted lines derived from Eq. (8)] for corresponding photon numbers are depicted clearly in the map, which fitted well with the numerical results. Also, we could obtain the threshold $\omega = 2m$ for the single photon from Eq. (7), under which the pair-production process occurs evidently.

From Fig. 1, one can see that the relations between $F(q, t = \infty)$ and the photon numbers n are obviously depicted, where $n = 3, 5, 7, 9$ are the photon numbers for odd ones and $n = 4, 6, 8, 10$ are the photon numbers for even ones. It is shown that the difference between the odd and even photon numbers mainly manifests in zero-momentum region; for the even n the particle distribution has to vanish at $q = 0$ due to charge-conjugation invariance.[18,24]

In the case of monochromatic field, the momentum–frequency spectrum disappears (depicted by the interspace near $q = 0$) for the even n but remains for the odd n, which is defined as the odd–even structure. This takes a quantitative influence on the total particle yield. The odd–even structures and some similar findings are also observed in momentum–frequency spectrum of polychromatic field and we will describe these interesting findings as follows.

4. Polychromatic Light Field

Now, let us study the effects of phase factor and frequency multiplication on particle yield and momentum–frequency spectrum in polychromatic (double-frequency and triple-frequency) field. The monochromatic field relevant to the phase factor is

$$E_1(t) = \varepsilon \exp\left(-\frac{t^2}{2\tau^2}\right)\left[\cos(\omega t) + \cos(\omega t + \phi)\right], \tag{9}$$

while the polychromatic fields are given by

$$E_2(t) = \varepsilon \exp\left(-\frac{t^2}{2\tau^2}\right)\left[\cos(\omega t) + \cos(2\omega t + \phi)\right], \tag{10}$$

$$E_3(t) = \varepsilon \exp\left(-\frac{t^2}{2\tau^2}\right)\left[\cos(\omega t) + \cos(3\omega t + \phi)\right], \tag{11}$$

where ϕ is the phase factor and $k\omega t$ ($k = 1, 2, 3$) is the frequency multiplication. Here, ωt, $2\omega t$ and $3\omega t$ are defined as the single-frequency, the double-frequency and the triple-frequency, respectively. Parameters for $\tau = 300/\mathrm{m}$, $e\varepsilon = 0.10\ \mathrm{m}^2$ are given in the following numerical results.

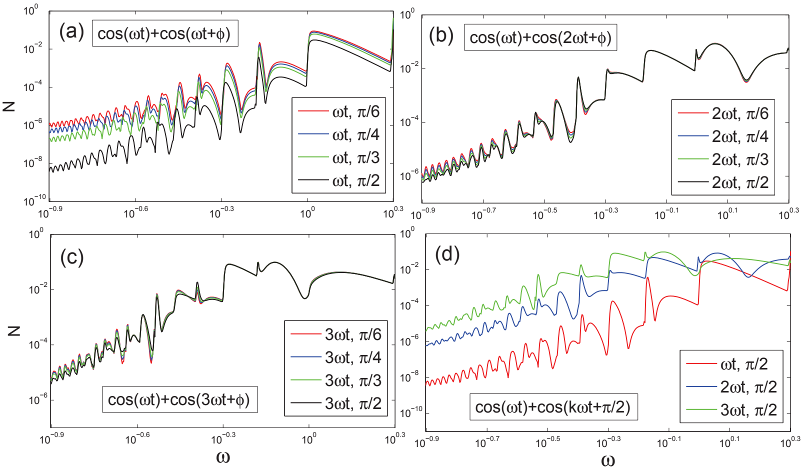

Fig. 2. (Color online) Particle yields near and below $\omega = 2m$. (a) The roles of ϕ in the single-frequency field. (b) The roles of ϕ in the double-frequency field. (c) The roles of ϕ in the triple-frequency field. (d) The roles of $k\omega t$ for $\phi = \pi/2$.

4.1. *Particle yield*

The corresponding particle yields of different phase factors and frequency multiplications are shown in Fig. 2. To give a credible and evident result, the particle yields are calculated numerically and the results are shown in double-log coordinate. The particle yield distributions near and below $\omega = 2m$ are given in different fields.

From the particle yield curves we could see obvious oscillating structures and resonance peaks in both monochromatic field and polychromatic field, which concerned the multi-photon process. This fact can be seen from Figs. 2(a)–2(c). As the duration of the envelope τ is much larger than $2\pi/\omega$, for higher frequency ω there are many oscillating periods in an envelope. It could be predicted that the effect of phase factor ϕ on particle yield is not obvious; from Figs. 2(b) and 2(c), we could hardly see any difference in the particle yields for different phase factors ϕ.

It is not surprising that the oscillating structures of the single-frequency field are influenced by the phase factor ϕ evidently, especially in lower frequency region as shown in Fig. 2(a). In this case, the phase factor ϕ affects the strength of electric field mainly. By comparing Figs. 2(b) and 2(c), we see that the oscillating structures of double-frequency and triple-frequency are not sensitive to the phase factor ϕ.

The roles of the frequency multiplication $k\omega t$ on the particle yield N could be seen from Fig. 2(d). It is shown that the particle yield is relatively larger in higher frequency multiplication region and that the same oscillating structures and resonance peaks are observed in the single-frequency, the double-frequency and the triple-frequency fields. However, the maximum values of particle yield are changed by the multiple-frequency (that occurs at more lower frequency positions) which is a modulation of the frequency multiplication.

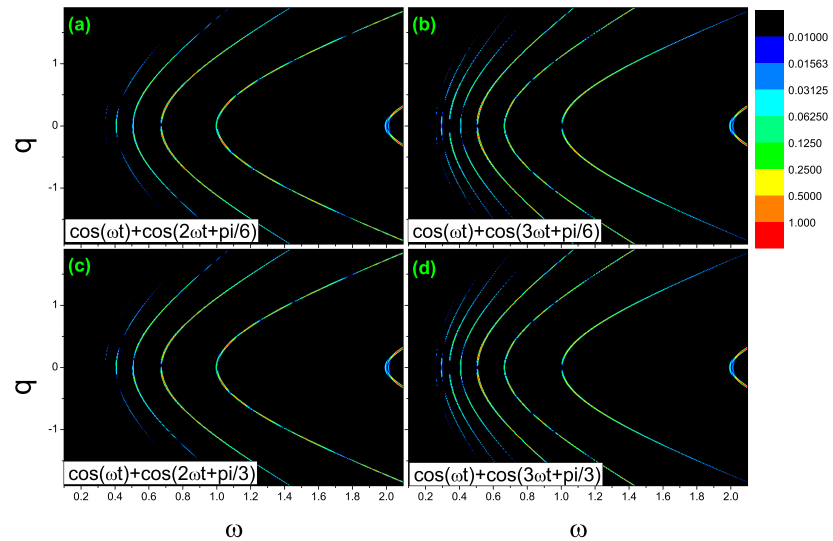

Fig. 3. (Color online) The particle distribution function $F(q, t = \infty)$ of ω for polychromatic field in plane (q, ω). Influences of ϕ and $k\omega t$ on the odd–even structures near $q = 0$. (a) The double-frequency for $\phi = \pi/6$. (b) The triple-frequency for $\phi = \pi/6$. (c) The double-frequency for $\phi = \pi/3$. (d) The triple-frequency for $\phi = \pi/3$. In the triple-frequency fields, for the odd–even structures remain, whereas in the double-frequency ones, they do not.

4.2. *The momentum–frequency spectrum*

Furthermore, influences of ϕ and $k\omega t$ on the odd–even structures near $q = 0$ are studied, as shown in Fig. 3. The particle distribution functions $F(q, t = \infty)$ of ω for multiple-frequency and triple-frequency fields in plane (q, ω) are given with $\phi = \pi/3$ and $\phi = \pi/6$.

The stripe structures could still be seen in multiple-frequency and the triple-frequency fields, however some details are modulated. For single-frequency ωt or $2\omega t$, in $q = 0$ region, we could see odd–even structures obviously. Suppose that the effects of ωt and $2\omega t$ are superimposed together with ignoring the nonlinear dynamics, then we could see that a vacancy of single-frequency ωt at even n is filled with the peak of single-frequency $2\omega t$ at odd n.

For example, at $\omega = m$, $q = 0$: for ωt, $n = 2$ corresponds to a vacancy; for $2\omega t$, $n = 1$ correspond to a peak. Therefore, we find that the odd–even structures disappear for multiple-frequency, which could be seen in Figs. 3(a) and 3(c). For triple-frequency, however, there is no fill effect. Hence, in the triple-frequency field remain the odd–even structures in momentum–frequency spectrum, as shown in Figs. 3(b) and 3(d).

It is shown that the particle yield is not influenced by the phase factor ϕ evidently, which could be seen by comparing Figs. 3(a) and 3(c) [or Figs. 3(b) and 3(d)] and also could be concluded from Fig. 2. It fitted our prediction well. Therefore, it can be concluded that in the triple-frequency fields remain the odd–even structures and the double-frequency ones destroy the odd–even structures. The multiple-frequency plays an important role in the momentum–frequency spectrum in the pair-production process.

5. Conclusions

With the help of QVE, oscillating signatures of the particle yield and the odd–even structure of the momentum–frequency spectrum are described. The pair-production process in different oscillating electric fields, such as the double-frequency field and the triple-frequency field is studied. The roles of the phase factor and the frequency multiplication are analyzed in detail. For different frequency multiplications, there are both oscillating structures and evident resonance peaks, which correlated to the multi-photon process. We find the phase factor affects the strength of electric field mainly; in the triple-frequency field the odd–even structures remain and in the double-frequency field, they do not. These interesting physical structures manifest some significant features of the pair-production process relevant to multi-photon process, which could directly or indirectly reveal some physical effect of pair-production in oscillating electric fields. These interesting physical images of our simulations would be vital to the multi-photon pair-production process.

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