Int J Theor Phys
DOI 10.1007/s10773-016-3018-7

Frequency Shift and Sub-band Effect in Pair-Production Process Under Adiabatic Closing the External Field

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Received: 14 December 2015 / Accepted: 15 April 2016

Abstract Oscillating electric field is chosen to investigate the electron-positron pair production process by using a quantum kinetic theory and the effective mass model [Phys. Rev. Lett. **112**, 050402 (2014)]. The particle yield exhibits a characteristic oscillatory structure which is related to the multi-photon thresholds. The true peak positions are typically slightly above the naive threshold estimate, which is defined as frequency shift. During the numerical calculations, we find the frequency shift can be affected by the system parameters under adiabatic closing the external field, it is worthwhile to study in detail. In this paper, we investigate the frequency shift and the sub-band effect in electron-positron pair production with oscillating electric field. First, a quantum kinetic theory and the effective mass are presented to obtain the frequency shift, the results are fitted very well. And we find the frequency shift and the sub-band effect can be influenced by pulse duration, photon number, and strength of the external field. The frequency shift becomes evident as increases of photon number and the external field strength. The sub-band width is relatively lower at longer pulse duration, higher photon number region, and weaker external field. The results shown in the paper are helpful for understanding multi-photon pair production process in the strong field.

Keywords Frequency shift · Sub-band effect · Quantum kinetic theory · Effective mass

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Published online: 21 April 2016

1 Introduction

The vacuum is unstable [1] in the presence of extreme strong electromagnetic field and may decay by emitting electron-positron pairs. The pair production rate was firstly calculated by the famous Schwinger's formula [2], which described the boson pair production in a static, homogeneous electric field. Then some theoretical studies on pair-production are started [3–6]. At the extreme laser intensity, many quantum electrodynamics(QED) [7] phenomena can be realized in the laboratory. The pair-production process, as an fascinating topic in strong-field QED [8–11], is reported through several different manners [12–16]. With the development of strong laser technology, observing the pair-production process by using only laser beams [17] can be expected. However, the theoretical investigations [18, 19] are much ahead of experimental efforts.

Many new issues on pair-production are introduced, such as the pair production created by the seed photon [20], the pair production of subcycle, cycle and supercycle laser pulses [21], and the application of low-density approximation [22] in pair production process. Recently, the pair production process is investigated by a quantum kinetic theory and the effective mass model [23]. The effective mass as a signature for multi-photon production could explain pair-production clearly. Particle yield exhibits a characteristic oscillatory structure which is related to the multi-photon thresholds. Results in Ref. [23] apparently show that the true peak positions are typically slightly above the naive threshold estimate, which is defined as frequency shift. Since the frequency shift can be affected by the system parameters when closing the external field adiabatically, a comprehensive treatment and investigation of pair-production process under adiabatic closing the external field is necessary.

In this paper, based on the effective mass model [23] and quantum kinetic theory (QKT), we study the electron-positron pair-production process under adiabatic closing the external field. The frequency shift and the sub-band effects when external field is closed adiabatically are found in the oscillating field. The sub-band width between the highest peak and the sub-highest peak is investigated. We find that the frequency shift and sub-band effect in the pair production can be influenced by the pulse duration, photon number, and the strength of external field. The results obtained from QKT and the effective mass are fitted very well.

This paper is organized as follows. First, the QKT and the effective mass are described. Then, frequency shift and sub-band effect are investigated. Finally, a brief discussion is given.

2 The Theoretical Method

The pair-production process from the vacuum is a time-dependent [1] and non-equilibrium process. The QKT can be stated as a quantum Vlasov equation (QVE), by which we can obtain the pair-production rate and the momentum information. With the help of auxiliary functions $G(q,t)$, $H(q,t)$ (q is the canonical momentum and $\mathbf{q} = -i\hbar\nabla$, t is the time), we could obtain the momentum distribution function $F(q,t)$ from the the reduced form of QVE as follows [6, 8]

$$\dot{F}(q,t) = W(q,t)G(q,t), \tag{1}$$

$$\dot{G}(q,t) = W(q,t)[1 - F(q,t)] - 2w(q,t)H(q,t), \tag{2}$$

$$\dot{H}(q,t) = 2w(q,t)G(q,t), \tag{3}$$

where

$$w^2(q,t) = m^2 + [q - eA(t)]^2, \quad W(q,t) = \frac{eE(t)m}{w^2(q,t)}. \tag{4}$$

Here, we restrict our discussion in one dimension, $A(t)$, $E(t)$ and e are the vector potential, the strength of electric field and electron charge respectively. In this work, natural units ($\hbar = e = c = 1$) is used and all quantities are scaled by the rest mass of electron m. The initial conditions are given by $F(q, -\infty) = G(q, -\infty) = H(q, -\infty) = 0$. The particle yield N per Compton wavelength is [23]

$$N = \int dq/(2\pi)F(q, \infty), \tag{5}$$

We now introduce the effective mass model. The concept of a particle's effective mass is widely used in physics, for example the effective mass effects are used to study the pair-production process. The effective mass model [23] can be written as

$$m_* = m\sqrt{1+\xi^2} \simeq m\sqrt{\left(1 + \frac{e^2}{m^2}\frac{\varepsilon^2}{2\omega^2}\right)}. \tag{6}$$

The corresponding frequency is written as

$$\omega^* = \sqrt{\frac{2m^2}{n^2} + \sqrt{\frac{4m^4}{n^4} + \frac{2e^2\varepsilon^2 m^2}{n^2}}}. \tag{7}$$

Here, m_* is the effective mass, ω the frequency, ω^* the revised frequency, ε is the peak strength of field, n the corresponding photon number. For a multi-photon absorption pair production process, the energy conservation satisfies

$$n\hbar\omega = 2\sqrt{c^2 q_n^2 + m^2 c^4}. \tag{8}$$

We could obtain the threshold $\omega = 2m$ for the single photon from (8), under which the pair-production process occurs evidently. We will investigate the signatures of the frequency shift with QKT and the effective mass model in the following.

3 Frequency Shift and Sub-Band Effect in Pair-Production Process

In this section, we study frequency shift and sub-band effect during the electron-positron pair-production process under adiabatic closing the external field.

3.1 Frequency Shift

Oscillating electric field is selected to study the electron-positron pair production in the strong field, and two different pulses are applied as follows. One is Gaussian-type pulse [23], which is a homogeneous electric field pulse

$$E(t) = \varepsilon \exp\left(-\frac{t^2}{2\tau^2}\right)\cos(\omega t), \tag{9}$$

where τ is the duration of the pulse.

Another one is Half-Ladder shaped pulse,

$$E(t) = E_0(t)\cos(\omega t),$$

where

$$E_0(t) = \begin{cases} \varepsilon, & t \le t_1, \\ \varepsilon(t_1 + t_2 - t)/t_2, & t_1 < t \le t_1 + t_2. \end{cases} \tag{10}$$

Here, t_1 is the platform period, t_2 is the linearly decaying period, and $t_1 + t_2$ is the duration of the pulse.

In the following, the distribution near momentum $q = 0$ will be concerned, which is $F(q = 0, \infty)$. For Gaussian-type pulses, the particle yield exhibits a characteristic oscillatory structure which can be interpreted as a signature for multi-photon production [23]. For half-ladder shaped pulses, the same characteristic oscillatory structure is found in the process of pair production for multi-photon, as shown in Fig. 1. Figure 1a shows the log-plot of particle yield N below and near the pair threshold $\omega = 2m$ for $t_1 = 100/m$, $t_2 = 2000/m$, and $e\varepsilon/m^2 = 0.20$. We could see obvious oscillating structures in the pair-production process, and there are evident resonance peaks at some special frequency values, which are defined as the peak frequencies. The oscillating structures are related to the n-photon thresholds, which have been shown in Gaussian-type pulse [23]. To explain the resonance peaks in detail, we show one-particle distribution in momentum space $F(q, \infty)$ in Fig. 1b. The

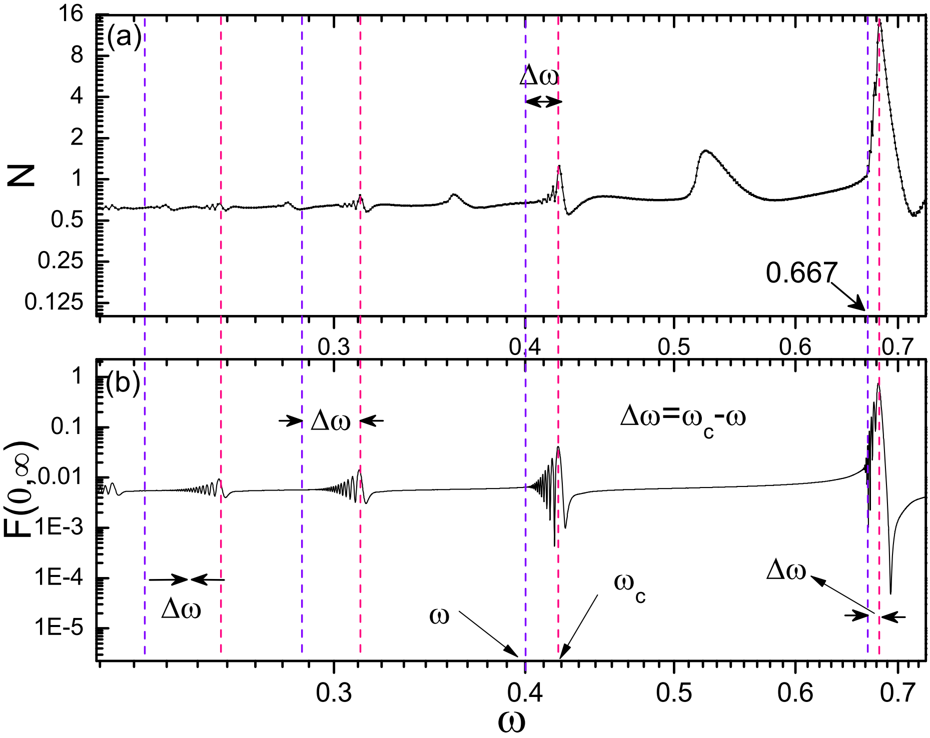

Fig. 1 (Color online) **a** Log-plot of the particle yield for Half-Ladder shaped pulse, $t_1 = 100/m$, $t_2 = 2000/m$, and $e\varepsilon/m^2 = 0.20$. **b** One-particle distribution in momentum space. The vertical dashed red lines denote positions of the true frequency ω_c, the vertical dashed purple lines are the estimated threshold frequency ω. $\Delta\omega = \omega_c - \omega$ is the frequency shift

positions of the true frequency ω_c(vertical dashed red lines) and the naive estimate threshold frequency ω (vertical dashed purple lines) are labeled in Fig. 1. $\Delta\omega = \omega_c - \omega$ is defined as the frequency shift, which describes the deviation between ω_c and ω. Obviously, the frequency shift $\Delta\omega$ becomes evident with higher photon number n.

Meanwhile, this deviation is a signature for the effective mass [23] of the electron and positron in the strong field. In Fig. 2a, the particle yield N as a function of momentum q and frequency ω is depicted, which is called the momentum-frequency spectrum.

For the even n the particle distribution has to vanish at $q = 0$ due to charge-conjugation invariance [19, 23], so we focus on the odd n in this paper. In Fig. 2b we describe the spectrum of one-particle distribution at $q = 0$.

Examples of $n = 3$, $t_1 = 100/m$, $t_2 = 2000/m$, and $e\varepsilon/m^2 = 0.20$ are shown in Fig. 2, where n is the naive threshold estimate $n\omega = 2m$ and n^* is the result of the revised value by the effective mass. The vertical dashed lines are ω_c (pink, the true peak positions), ω^* (orange, the revised result) and ω (green, the naive estimate value) respectively. $\Delta\omega$ is

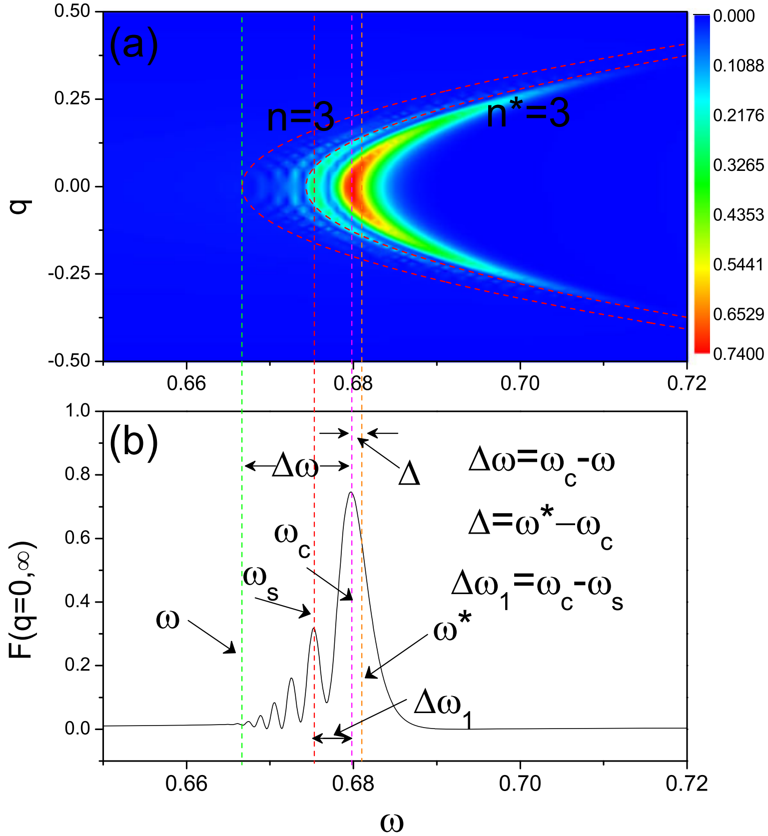

Fig. 2 (Color online) **a** The momentum-frequency spectrum during the pair-production process, where $n = 3$, $t_1 = 100/m$, $t_2 = 2000/m$, and $e\varepsilon/m^2 = 0.20$ for Half-Ladder shaped pulse. n is the naive threshold estimate from $n\omega = 2m$, n^* is the revised value from the effective mass model. **b** One-particle distribution in momentum space $F(q = 0, \infty)$. The vertical dashed lines are ω_c (pink, the true peak positions), ω^* (orange, the revised result) and ω (green, the naive estimate value) respectively. $\Delta\omega$ is the frequency shift, Δ gives the difference between ω_c and ω^*. $\Delta\omega_1$ is the sub-band width

defined as the frequency shift, which describes the deviation between w_c and w. Δ gives the difference between the true peak position frequency ω_c and the revised value [23] ω^* from the revised value. Obviously, the revised value is close to the true peak frequency positions for $n = 3$, ω^* and the peak position frequency ω_c fit well.

In order to investigate the roles of the electric field $e\varepsilon$ and photon numbers n. Figure 3a gives the frequency shift $\Delta\omega$ as a function of the electric field $e\varepsilon$ (scaled by m). Figure 3b shows the frequency shift $\Delta\omega$ as a function of photon numbers n, examples of curves for $e\varepsilon/m^2 = 0.15, 0.20, 0.25$ are given for two different pulses, and the dashed lines in Fig. 3b are results of the revised formula from the effective mass [23].

Figure 3 shows the same frequency shift $\Delta\omega$ for two different pulses. The numerical results and the revised results fit well in lower photon region and relatively smaller electric field regions. Since the spectrum of one-particle distribution at $q = 0$ shows a well oscillating structure. Actually, the highest peak and the sub-highest peak in frequency positions may be more interesting, so we investigate the sub-band effect in the flowing section.

3.2 Sub-Band Effect

The above results show a well oscillating frequency structure, the structure of the highest peak and the sub-highest peak in frequency positions contains rich sub-band effect. The sub-band width $\Delta\omega_1$ between the highest peak and the sub-highest peak in frequency positions

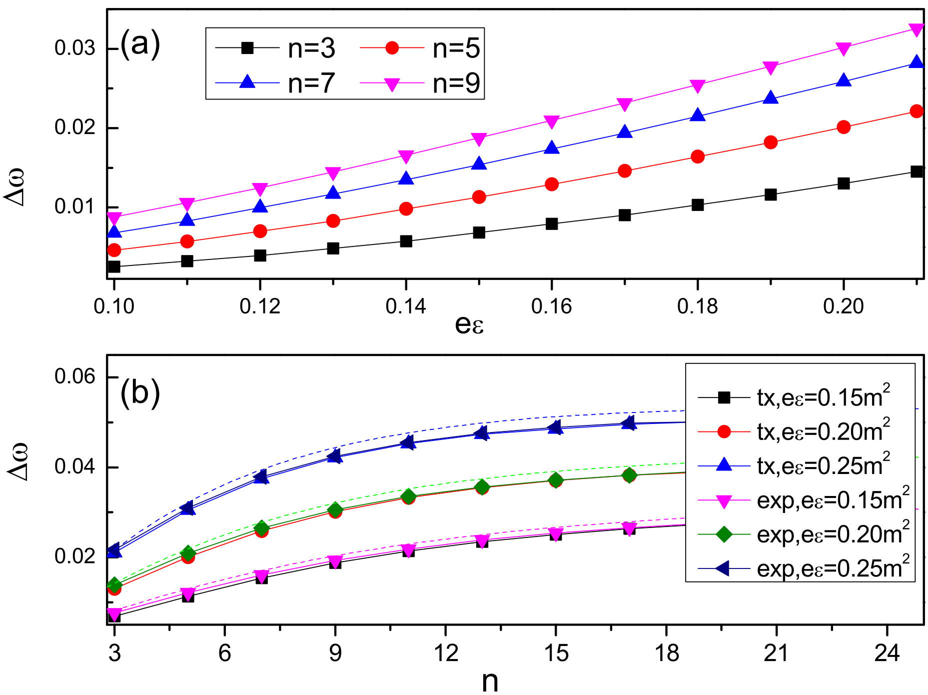

Fig. 3 (Color online) **a** The frequency shift $\Delta\omega$ as a function of the electric field $e\varepsilon$ (scaled by m). **b** The frequency shift $\Delta\omega$ as a function of photon numbers n, where $e\varepsilon/m^2 = 0.15, 0.20, 0.25$ for two different pulses, and the dashed lines are results of the effective mass model. Here, *exp* denotes the Gaussian-type pulse and *tx* is the Half-Ladder shaped pulse

can be affected by the system parameters. To analyze the roles of the pulse duration, photon number and the strength of external field on the sub-band, we show the sub-band width in the following.

We show the influence of pulse duration on sub-band width in Fig. 4. We have shown the pulse duration t_2 growth of the the sub-band width $\Delta\omega_1$ in the pair-production process for two different external electric field strength: (a) $e\varepsilon = 0.20m^2$, and (b) $e\varepsilon = 0.21m^2$. It is shown that the sub-band width is relatively larger at the lower photon number region. With fixed photon number, as t_2 is increasing, the sub-band width decreases. Obviously, $\Delta\omega_1$ is relatively larger at lower n region.

Then, we analyze the effect of the photon number on the sub-band width. The sub-band width $\Delta\omega_1$ as a function of the photon number n is shown in Fig. 5. Figure 5a-d with different external field strength: (a) $e\varepsilon = 0.21m^2$, (b) $e\varepsilon = 0.22m^2$, (c) $e\varepsilon = 0.23m^2$, and (d) $e\varepsilon = 0.24m^2$. We can see the sub-band width $\Delta\omega_1$ becomes smaller as the photon number n is increasing. Figures 4 and 5 show that the sub-band width $\Delta\omega_1$ can be affected by the external field strength, therefore, the detailed studies are shown in the following.

Finally, we study the effect of the external field strength on the sub-band width, we extract the sub-band $\Delta\omega_1$ as a function of the strength of external field $e\varepsilon$ in Fig. 6, with (a) $t_2 = 1500/m$ and (b) $t_2 = 1500/m$. We can see that the sub-band width increases linearly

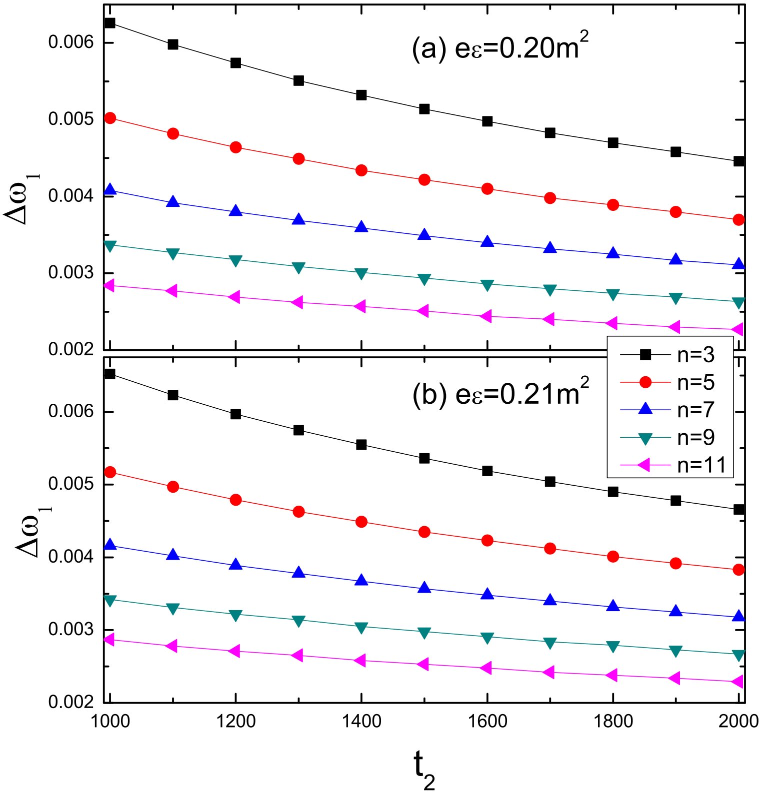

Fig. 4 (Color online) With different $e\varepsilon$, we extract the sub-band width $\Delta\omega_1$ as a function of the pulse duration t_2 (scaled by m) for different photon number, (**a**) for $e\varepsilon = 0.20m^2$, (**b**) for $e\varepsilon = 0.21m^2$. It is found that the sub-band width is decreases as t_2 increasing and it is relatively larger for the lower photon number

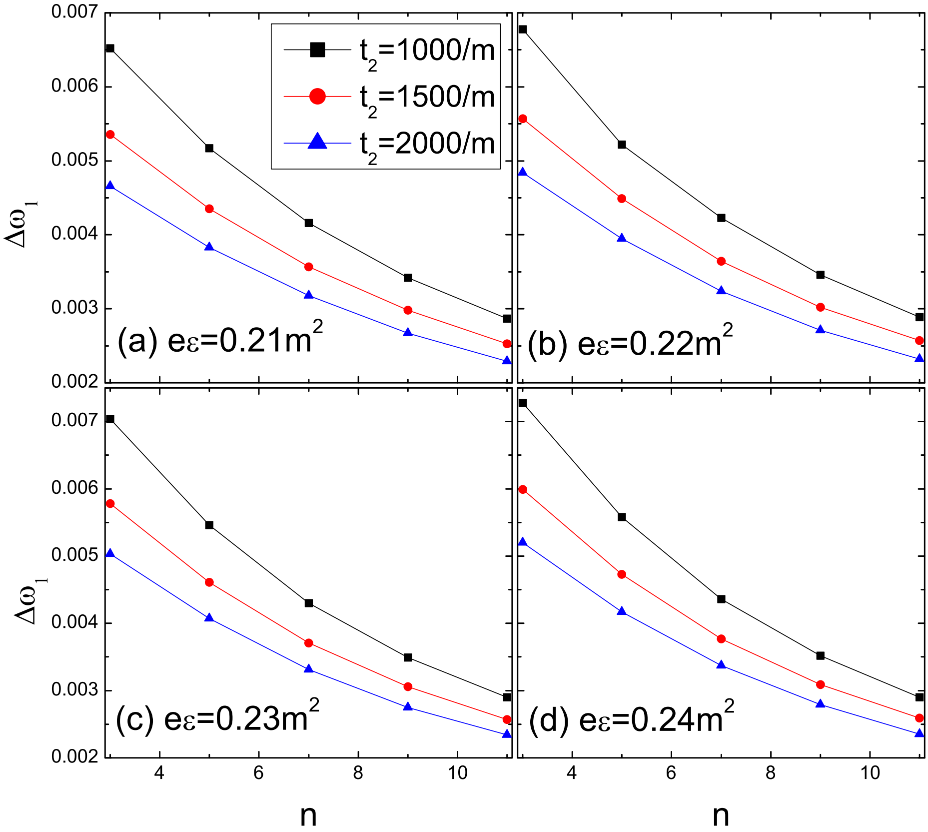

Fig. 5 (Color online) With different strength of external field, we extract the sub-band width $\Delta\omega_1$ as a function of the photon number n, (**a**) $e\varepsilon = 0.21m^2$, (**b**) $e\varepsilon = 0.22m^2$, (**c**) $e\varepsilon = 0.23m^2$, and (**d**) $e\varepsilon = 0.24m^2$. We can see the sub-band width decreases as photon number is increasing

with $e\varepsilon$. With larger photon number n, the sub-band width is relatively lower, which are fitted well with the results shown in Figs. 4 and 5.

4 Discussions

With the help of QKT and the effective mass concept, the pair-production process under adiabatic closing the external field is described. The obvious oscillated structures of particle yield are shown, and the frequency shift and the sub-band effect are defined. The frequency shift and the sub-band effect are mainly focused in the paper. In the numerical process, we find that the frequency shift and sub-band effect in the pair production can be influenced by system parameters, such as the pulse duration, photon number, and the strength of external field. Also, the frequency shift obtained from QKT and the effective mass are fitted very well. The frequency shift becomes evident as increases of photon number and the external field strength. Then, we show the effect of the pulse duration, photon number, and the strength of external field on the sub-band width. We find that the sub-band width is relatively lower at longer pulse duration, higher photon number region, and weaker external field.

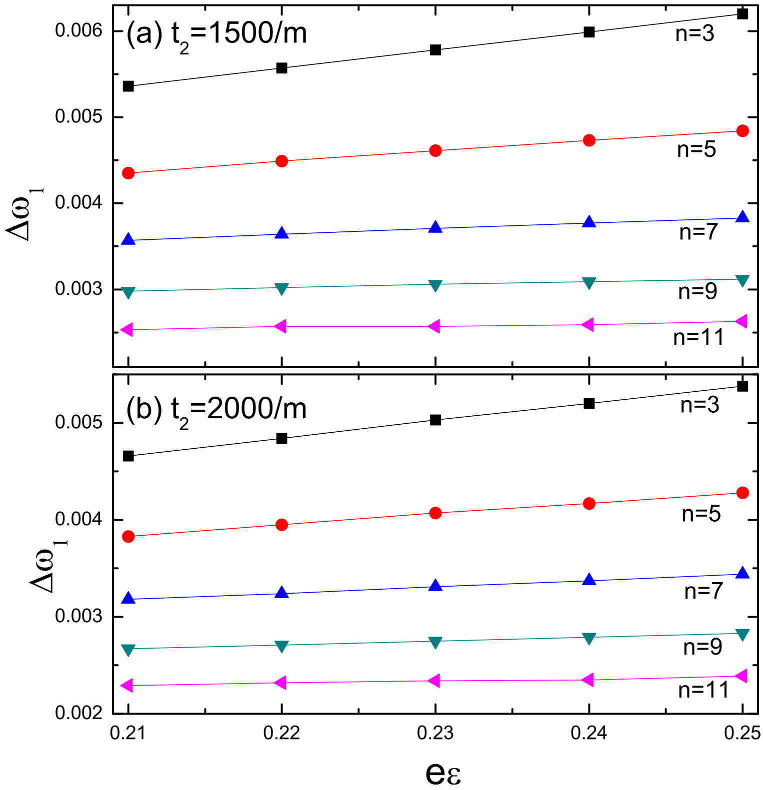

Fig. 6 (Color online) With different pulse duration t_2, we extract the sub-band width $\Delta\omega_1$ as a function of the strength of external field $e\varepsilon$, (**a**) $t_2 = 1500/m$ and (**b**) $t_2 = 2000/m$. It is shown that the sub-band width increases as strength of external field is increasing

The results shown in the paper are interesting and helpful for understanding multi-photon pair production process. The numerical results provide more fundament theory for the pair-production process in the strong field.

Acknowledgments The authors thank Jie Liu in Institute of Applied Physics and Computational Mathematics for his helpful suggestions. Project supported by the National Fundamental Research Program of China(973)(Grant Nos. 2013CBA01502, and 2013CB834100) and the National Natural Science Foundation of China(Grant Nos. 11374040, 11475027, 11274051 and 11125417)

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