High-fidelity composite adiabatic passage in nonlinear two-level systems

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We investigate the composite adiabatic passage (CAP) reported by B. T. Torosov *et al.* [Phys. Rev. Lett. **106**, 233001 (2011)] in a nonlinear two-level system in which the level energies depend on the occupation of the levels, representing a mean-field type of interaction between the particles. A high-fidelity, fast, and robust quantum manipulation is achieved in the system. We consider the effect of interparticle interaction and find that it tends to increase the number of the pulse sequences. The CAP technique can suppress the nonadiabatic oscillations below the quantum-information benchmark 10^{-4}, as long as there exist sufficiently long composite sequences. We analyze the robustness against the variations in the field parameters. The difference between the nonlinear and linear systems on the CAP technique is also discussed.

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I. INTRODUCTION

Manipulating the state of a quantum system by external fields is crucial in atomic and molecular physics for applications such as metrology, interferometry, nuclear magnetic resonance, quantum-information processing, or driving of chemical reactions [1–4]. The practical implementation of quantum-information processing, however, requires time-dependent schemes featuring three important issues: The driving quantum state to a target state should be achieved (i) with a high fidelity, typically with an admissible error lower than 10^{-4}, (ii) in the shortest possible time in order to prominently minimize decoherence effects, and (iii) in a robust way with respect to the imperfect knowledge of the system or to variations in experimental parameters [5–7].

Adiabatic passage (AP) techniques are a popular tool for quantum state manipulation. Various AP techniques have been proposed and demonstrated, including rapid adiabatic passage, Stark-chirped rapid adiabatic passage, piecewise adiabatic passage, stimulated Raman adiabatic passage, and their variations [8]. The techniques are robust, but in nearly all of them transition probability is incomplete. Another basic approach to robust coherent control of quantum systems is the technique of composite pulses, which is widely used in nuclear magnetic resonance [9] and, more recently, in quantum optics and quantum-information processing [10,11]. This technique replaces the single pulse used traditionally for driving a two-state transition by a sequence of pulses with appropriately chosen phases, which are used as a control tool for shaping the excitation profile in a desired manner, e.g., to make it more robust to variations in the experimental parameters of intensity and frequency. The imperfections may be caused by an imprecise pulse area, an undesirable frequency offset, or an unwanted frequency chirp [12].

To combine the advantages of adiabatic passage and composite pulse techniques, and to achieve robust and high-fidelity quantum state control, some optimal control approaches have been proposed [13–16]. Among them, the composite adiabatic passage (CAP) technique is a powerful and flexible control tool [16], which can deliver extremely high fidelity of population transfer, far beyond the fault-tolerant quantum computing benchmark. Recently, the CAP technique has been widely studied [17] and demonstrated experimentally in a rare-earth ion-doped solid [18]. The experimental explanation and associated theoretical discussion are limited to a linear two-level system, in which the interaction between particles is ignored. In recent years, however, an increasing interest has been devoted to studying the nonlinear quantum system with interparticle interaction. The interaction between the particles can significantly influence the quantum transition dynamics [19,20]. Moreover, a single-pulse duration is very long (i.e., at infinitely slow sweep speeds) to satisfy the adiabaticity criteria. However, a CAP sequence contains N pulses. Hence, for CAP the total pulse duration is N times the single-pulse duration. Under adiabatic conditions, the total pulse duration will be infinite, which is impractical (and indeed, unphysical).

In this paper, we study the CAP and achieve high-fidelity, fast, and robust quantum state manipulation in a nonlinear two-level system in which the level energies depend on the occupation of the levels, representing a mean field of interaction between the particles. The influence of interparticle interaction on CAP is investigated. We show that the interaction tends to increase the numbers of pulse sequences, i.e., the high-fidelity transition probability can still be achieved in nonlinear systems as long as there exist sufficiently long composite sequences. Different from the linear quantum system, no matter how many pulse sequences, the total pulse duration is fixed for nonlinear quantum systems.

II. MODEL

The nonlinear two-state system we consider is described by the following dimensionless Schrödinger

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equation [21]:

$$i\frac{\partial}{\partial t}\mathbf{a}(t) = \mathbf{H}(t)\mathbf{a}(t), \tag{1}$$

with the Hamiltonian given by

$$\mathbf{H}(t) = \frac{v(t)}{2}\hat{\sigma}_x + \left[\frac{\gamma(t)}{2} + \frac{c}{2}(|a_2|^2 - |a_1|^2)\right]\hat{\sigma}_z, \tag{2}$$

where $\mathbf{a}(t) = [a_1(t), a_2(t)]^T$ is the probability amplitudes of the two states, $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are Pauli matrices, and $\gamma(t)$ and $v(t)$ are the energy bias and coupling strength between two states, respectively. c is the nonlinear parameter describing the interparticle interaction. The total probability $|a_1|^2 + |a_2|^2$ is conserved and set to be 1. The model not only has aroused great interest in theory but also has important applications in physics, for example, for describing a spin tunneling of nanomagnets [22], a BEC in a double-well potential or in an optical lattice [23], coupled waveguide arrays [24], etc.

In the linear model ($c = 0$), the CAP method has been proposed [16] in which the propagator of a two-state system can be parametrized by the Cayley-Klein parameters, and the single pulse driving the quantum transition is replaced by a sequence of pulses with appropriately chosen phases. The technique allows one to suppress the nonadiabatic oscillations in the transition probability and to reduce the error below the 10^{-4} quantum computation benchmark, even with simple three- and five-pulse composite sequences. Besides, the composite phases do not depend on the specific pulse shape and chirp as long as the latter satisfies the symmetry property.

The success of the CAP in linear systems has demonstrated its great ability to realize quantum manipulation [17,18]. Keeping this in mind, for system (1) with Hamiltonian (2), we employ a sequence of N ($N = 2n+1$, n is an integer) pulses, each with a phase $\phi_k (k = 1,2,\cdots N)$, to achieve high-fidelity quantum transition. The phase ϕ_k is imposed upon the driving field Rabi frequency (coupling strength), $v(t) \to v(t)e^{i\phi_k}$. For simplicity and as the very first try toward a CAP protocol for nonlinear system, the composite control phase in the linear systems [16] is used here. Assuming the coupling strength $v(t)$ is an even function of time and the detuning $\gamma(t)$ is odd, the composite phase is

$$\phi_k = \left(N + 1 - 2\left\lfloor\frac{k+1}{2}\right\rfloor\right)\left\lfloor\frac{k}{2}\right\rfloor\frac{\pi}{N}, \tag{3}$$

where the symbol $\lfloor x \rfloor$ denotes the floor function. The phase sequence is symmetric, i.e., $\phi_k = \phi_{N+1-k}$, and $\phi_1 = \phi_N = 0$.

III. NONLINEAR EFFECT

Our goal is to investigate the CAP with nonlinear interparticle interaction and to consider how the nonlinear interaction would affect the CAP technique. With the emergence of nonlinearity, the transition dynamics dramatically changes. In this case, the Schrödinger equation (1) is no longer analytically solvable. We therefore exploit a 4th–5th-order Runge-Kutta algorithm to trace the quantum evolution numerically and calculate the transition probability of system.

As an example, we consider the Allen-Eberly (AE) model assuming a sech couple strength (pulse) and a tanh frequency energy bias (chirp) [16],

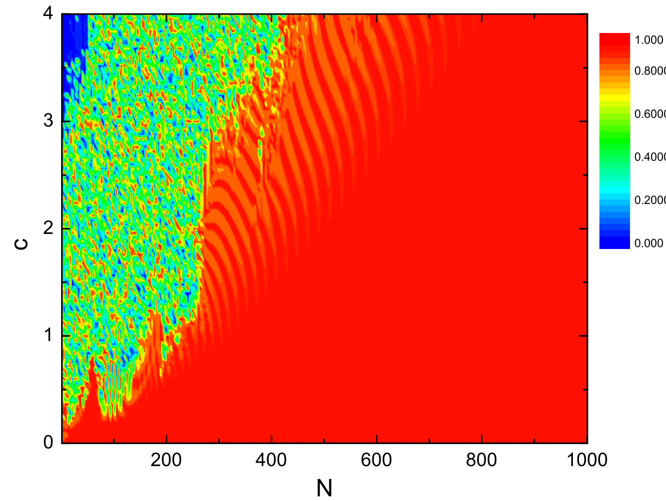

FIG. 1. Contour plots of transition probability as the function of composite sequence pulses N and interaction c for the AE pulse with $\alpha = 1$ and $v_0 = 1.2$.

$$v(t) = v_0 \text{sech}(t/T), \quad \gamma(t) = \alpha \tanh(t/T), \tag{4}$$

where v_0 and α are constant parameters with the dimension of frequency, and T is the pulse width.

For linear case, the transition probability $p = |a_2|^2$ is [16]

$$p = \frac{\cosh(\pi\alpha T) - \cos\left(\pi T\sqrt{v_0^2 - \alpha^2}\right)}{1 + \cosh(\pi v_0 T)} = 1 - \frac{\cos^2\left(\frac{1}{2}\pi T\sqrt{v_0^2 - \alpha^2}\right)}{\cosh^2\left(\frac{\pi\alpha T}{2}\right)}. \tag{5}$$

For $v_0 < \alpha$, the cosine in Eq. (5) has to be replaced by a hyperbolic cosine. A transition probability $p = 1$ (complete population inversion) is obtained for $\sqrt{v_0^2 - \alpha^2}T = 2n + 1$, with $n = 0,1,2,\ldots$ (integer). In the adiabatic limit ($v_0 > \alpha \gg 2/T$), the transition probability also tends to unity. If α is not large enough, nonadiabatic oscillations versus v_0 appear and the probability is reduced. These oscillations can be suppressed to any order by the CAP technique, even with simple three- and five-pulse composite sequences. Note also that all the variables here should be understood as scaled dimensionless variables. Throughout, we use T to scale. Then, $T = 1$, v_0,α, and c are in units of $1/T$, respectively.

Figure 1 shows the final transition probability as a function of both the number N of composite sequence pulses and particle interaction c for the AE model with $\alpha = 1$ and $v_0 = 1.2$. The blue zones correspond to low transition probability whereas red areas indicate high transition probability. We see that the transition dynamics is strongly dependent on the nonlinear interaction. For very weak interaction, a high transition probability can be achieved, even with simple three- and five-pulse composite sequences. As the nonlinear interaction grows, nonadiabatic oscillations are significantly strengthened and the probability is reduced dramatically. This means that high transition probability will no longer be achieved with a

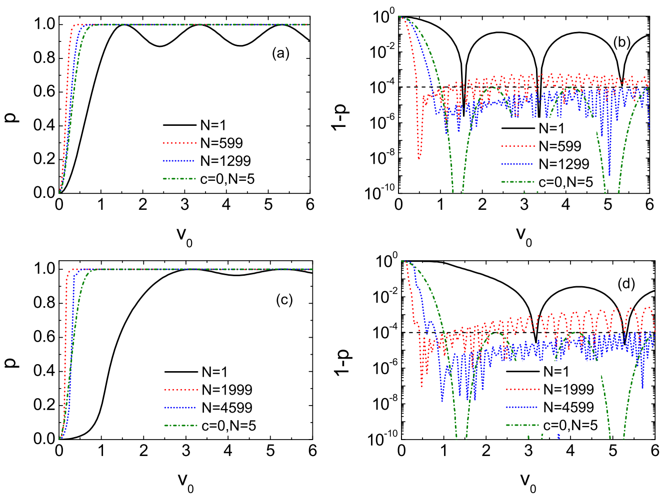

FIG. 2. Transition probability as the function of peak Rabi frequency v_0 for different interactions $c = 0.2$ (top) and $c = 2.0$ (bottom) and composite sequence pulses N. Frames (b) and (d) show the infidelities of the respective (a) and (c) profiles. The green dash-dotted cure is for five-pulse sequences $N = 5$ with $c = 0.0$. The value of parameter $\alpha = 1$.

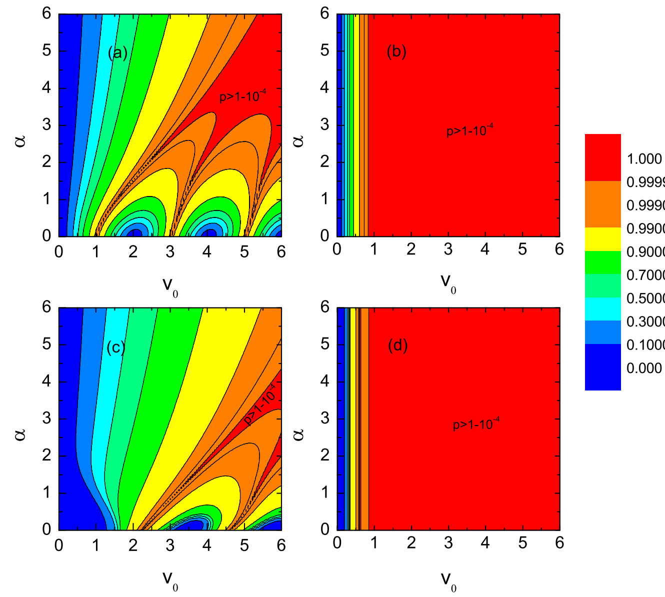

FIG. 3. Contour plots of transition probability as the function of peak Rabi frequency v_0 and chirp rate α at different interactions $c = 0.2$ (top) and $c = 2.0$ (bottom) with composite sequence pulses (a) $N = 1$, (b) $N = 1399$, (c) $N = 1$, and (d) $N = 4799$.

small number of composite sequences. However, interestingly, these oscillations can be suppressed by the CAP technique with sufficiently long composite sequences, i.e., as long as there exist sufficiently long composite sequences, the CAP technique can still be applied to a nonlinear two-level system. It is noted that for linear systems the total pulse duration of the CAP is N times the single-pulse duration. However, the total pulse duration is fixed in our nonlinear system. In all numerical simulations, the numerical time was performed from times -100 to 100.

We calculate the transition probability as the function of peak Rabi frequency v_0 for different interactions and composite sequence pulses N. The results are shown in Fig. 2. Figures 2(a) and 2(b) show that a 1299-pulse CAP with interaction $c = 0.2$ suffices to suppress the nonadiabatic oscillations below the quantum-information benchmark 10^{-4}. Figures 2(c) and 2(d) depict the transition probability vs peak Rabi frequency v_0 for the interaction $c = 2.0$. For comparison with Ref. [16], we also plot transition probability as the function of peak Rabi frequency v_0 for a five-pulse sequences $N = 5$ with $c = 0.0$ (green dash-dotted cure). We see that, for a linear system, even a sequence of five pulses is enough to achieve extremely high fidelity with an error below 10^{-4}. However, for a nonlinear system, the 10^{-4} error benchmark can still be reached, albeit with longer sequences.

To test the robustness against the variations in the field parameters, we vary both α and v_0 around their optimal values and calculate the fidelity. The results are summarized in Fig. 3, which shows clearly that the CAP is extremely robust with respect to an increase in α and v_0. Furthermore, the high-fidelity region with an error below 10^{-4} of CAP is hugely expanded compared to a single pulse. Different from the linear quantum system, in a nonlinear quantum system, to achieve ultrahigh fidelity, a large number of composite pulses is need in CAP, and the number of pulses gradually increases as the nonlinear interaction grows.

To further explore the above peculiar phenomena, we introduce the relative phase $\theta = \theta_2 - \theta_1$ and the transition probability $p = |a_2|^2$ as two canonical conjugate variables with $a_1 = \sqrt{1-p}\exp(i\theta_1)$ and $a_2 = \sqrt{p}\exp(i\theta_2)$, then we can obtain an effective classical Hamiltonian and satisfy the canonical equations, i.e., $dp/dt = -\partial H/\partial\theta$, $d\theta/dt = \partial H/\partial p$,

$$H(t) = \frac{\gamma}{2}(1-2p) - \frac{c}{4}(1-2p)^2 + v\sqrt{p(1-p)}\cos(\theta + \phi_k). \tag{6}$$

The classical Hamiltonian can describe completely the dynamic properties of system (1) [19]. In Fig. 4, we show the transition probability and relative phase evolution for the AE model during CAP with and without the control phase under different interactions and pulse sequences. In Figs. 4(a)–4(c), we plot the trajectories in phase space, the transition probability, and the relative phase evolution for the different parameters. The black triangle and the star represent the initial state and the final state, respectively. We see that the control phases play an important role in the CAP technique, which can significantly influence the quantum transition dynamics. It allows one to suppress the nonadiabatic oscillations in the transition probability and to ensure an ultrahigh fidelity in the CAP process. In the linear case, the CAP works for a small number of pulses and each constituent pulse produces a large population change but not complete inversion; the destructive interference of the deviations drives the system to complete inversion in the end. Whereas in the nonlinear quantum system, the CAP requires a large number of pulses, each of which produces a small change in population. However, the universal composite phases are

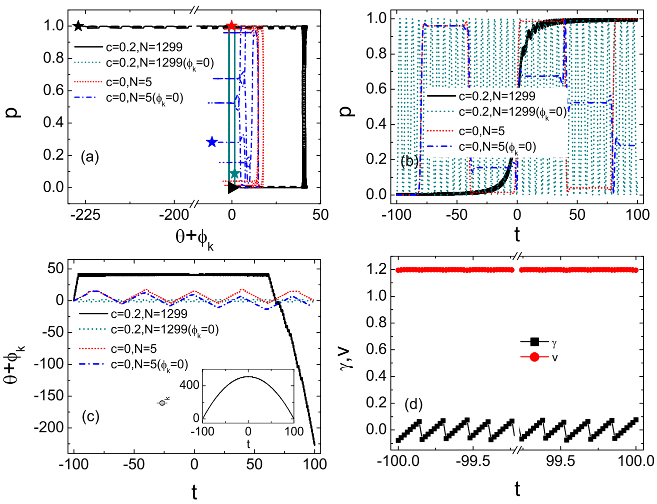

FIG. 4. (a) Trajectories in phase space. (b) Transition probability as function of time (inset: control phase as the function of time). (c) Relative phase as function of time. (d) 1299-pulse sequences as function of time.

derived from the condition to cancel the deviations from unit transfer efficiency due to nonadiabatic effects by enforcing destructive interference of these deviations. By directly solving the Schrödinger equation (1) using the same approach, we can reproduce the above results. In contrast to the CAP technique of Ref. [16], no matter the number of pulse sequences, the total pulse duration is fixed. Thus, the profiles of pulse and chirp are different for different pulse sequences. For example, for the 1299-pulse system, the AE model becomes the Landau-Zener model of finite duration [Fig. 4(d)], which generates the oscillations of the transition probability without the control phase.

IV. CONCLUSIONS

In conclusion, we have investigated the high-fidelity CAP technique in a nonlinear two-level system and explored the influence of interparticle interaction on high-fidelity CAP. Similar to linear cases, the CAP protocol can achieve high-fidelity population complete inversion in a nonlinear quantum system. We have found that interparticle interaction tends to increase the number of pulse sequences. However, the CAP technique can still suppress the nonadiabatic oscillations below the quantum-information benchmark 10^{-4} as long as there exist sufficiently long composite sequences. Different from the linear quantum system, the total pulse duration is fixed for the nonlinear quantum system. These features make the CAP technique a potentially important tool in applications requiring ultrahigh fidelity and superfast control, such as quantum-information processing and quantum optics. The high-fidelity quantum control in a nonlinear two-level system can be realized experimentally using a Bose-Einstein condensate (BEC) between Bloch bands in an accelerated optical lattice, where high enough densities of the atoms and Feshbach resonance can be achieved so that the nonlinear effect discussed above should be readily detectable [25]. The nonlinear Landau-Zener tunneling between two energy bands of a BEC in a periodic potential has been observed [26,27], and ultrashort laser pulse sequences have been used in quantum control [28], indicating that the high-fidelity CAP protocol in a nonlinear two-level system can be realized experimentally. We also should point out that the present control phase may not be the optimal one. However, the choice is a simple and convenient one for achieving high-fidelity quantum manipulation of nonlinear two-level systems. It remains a challenging and open problem on how to obtain the optimal control phase by the propagator method of nonlinear systems.

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