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Directed selective-tunneling of bosons with periodically modulated interaction

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ABSTRACT

We study the tunneling dynamics of bosons with periodically modulated interaction held in a triple-well potential. In high-frequency approximation, we derive a set of reduced coupled equations and the corresponding Floquet solutions are obtained. Based on the analytical results and their numerical correspondence, the directed selective-tunneling effect of a single atom is demonstrated when all bosons are prepared in middle well initially. A scheme for separating a single atom from N bosons is presented, in which the atom can be trapped in right or left well by adjusting the modulation strength.

1. Introduction

The coherent manipulation for a quantum system subjected to an external field has been an attractive subject in recent years in both theoretical and experimental physics [1,2]. Coherent control of quantum tunneling for a periodically driven system is one of the most important technologies due to its many applications [3], such as quantum device [4], artificial magnetic fields [5], and quantum information processing [6,7], etc. One of the recent topics in the quantum control of tunneling dynamics is the effect known as coherent destruction of tunneling (CDT) [8], namely, when the strength and frequency of driving force are chosen appropriately, a particle initially located in one of two wells never tunnels to the other. The CDT is based on the fast modulation of level unbalance and the corresponding effect has been verified experimentally [9,10]. Then, a selective CDT effect was found numerically in a driven quantum-dot array [11], in which the quantum tunneling between dots can be suppressed selectively. Such an effect has been demonstrated analytically in a driven tight-binding chain [12]. Further, the selective CDT effect has been introduced to realize a directed-motion scheme of atoms held in a driven one-dimensional bipartite lattice [13,14].

It is well-known that the sign and strength of the s-wave scattering length of interacting cold atoms can be adjusted by using magnetic or optical Feshbach resonances [15]. This technique has been used extensively [16] and many interesting phenomena were demonstrated in the framework of mean-field and Bose–Hubbard models. Such as stable Bloch oscillations [17], self-confinement of two- and three-dimensional Bose–Einstein condensates (BECs) without an external trap [18] and the generation of nonground-state BECs [19]. In a two-mode Bose–Hubbard model, a butterfly pattern of Floquet spectrum is displayed based on the double-kicked modulation of atomic interaction [20]. And in Ref. [21], Gong, Molina and Hänggi have proposed a many-body CDT effect by the periodic modulations of atomic interaction, in which only an arbitrarily, a priori prescribed atoms are allowed to participate in the tunneling process between double wells. An optical realization of the corresponding phenomenon was presented based on light transport in engineered waveguide arrays [22]. Further, the double-well model has been extended to an optical lattice system for the ultracold atoms with periodically modulated interaction [23]. An effective Hubbard-like model was presented, which includes a nonlinear hopping that depends on the difference of occupations at neighboring sites. The rich physics introduced by this hopping were discussed, such as pair superfluid phases, exactly defect-free Mott-insulator states, pure holon, doublon superfluids and quantum Peierls phase, etc.

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Recently, the tunneling dynamics of cold atoms held in a triple-well potential have attracted substantial interest and were investigated extensively. Such as stimulated Raman adiabatic transport [24], the transistor-like effect [25] and the effect of dipole–dipole interaction [26,27], etc. In this paper, we further consider the tunneling dynamics of bosons held in a triple-well potential and we are interested in the quantum manipulation of tunneling dynamics based on the periodical modulation of atomic interaction. In our work, we choose bias potential $\varepsilon_0 = 0.5\omega$ and time-independent interaction $U_0 = 0.5\omega/(N-1)$ with ω, N being the modulating frequency and number of bosons, respectively. Under high-frequency approximation, we obtain a set of truncated coupled equations that relate to the subspace spanned by Fock states $\{|0,N-1,1\rangle, |0,N,0\rangle, |1,N-1,0\rangle\}$. When initial state is located in this subspace, we obtain a set of analytical Floquet solutions and the corresponding superposition states. Based on these analytical results, the directed selective-tunneling effect of a single boson is demonstrated, in which the good correspondence is exhibited between analytical and numerical results. It is shown that a single atom can be separated from N bosons and trapped in right or left well by adjusting the modulation strength. The corresponding result presented in our work may be useful in the design of atomic devices [4,6,25,28].

2. Floquet solutions under high-frequency approximation

We consider a system described by the three-mode Bose–Hubbard Hamiltonian, which is realized physically by bosons trapped in a triple-well potential. We consider the interaction strength is modulated periodically in time and the system is described by corresponding Hamiltonian as [21–23]

$$\hat{H}(t) = -\Omega\sum_{\langle k,l\rangle}\left(\hat{c}_k^\dagger\hat{c}_l + \hat{c}_l^\dagger\hat{c}_k\right) + \frac{U(t)}{2}\sum_{k=1}^{3}\hat{c}_k^\dagger\hat{c}_k^\dagger\hat{c}_k\hat{c}_k + \varepsilon_0\left(\hat{c}_1^\dagger\hat{c}_1 - \hat{c}_3^\dagger\hat{c}_3\right), \quad (1)$$

where $\hat{c}_k^\dagger$ ($\hat{c}_k$) creates (annihilates) an atom in the well k. $\Omega > 0$ is the couplings between nearest-neighbor wells and ε_0 is the potential bias along the triple-well axis. The on-site interaction between atoms is characterized by $U(t) = U_0 + U_1\cos(\omega t)$, which can be controlled by using suitable Feshbach resonances [15].

In our paper, we have set $\hbar = 1$ and $U_0, U_1, \varepsilon_0, \omega$ and Ω are in units of reference frequency ω_0 on the order of $10^2\ \text{s}^{-1}$ [29], and the time t has been normalized in units of ω_0^{-1}. To study tunneling dynamics of bosons held in triple-well system, we introduce the Fock basis $|n_1, n_2, N-n_1-n_2\rangle$ with n_1, n_2 and $N-n_1-n_2$ being the number of atoms in the left, middle and right wells, respectively. In this paper, we consider the total number of atoms N is a constant. On the basis of Fock states, the corresponding quantum state $\Psi(t)$ can be expanded as $|\Psi(t)\rangle = \sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1} a_{n_1,n_2}(t)|n_1, n_2, N-n_1-n_2\rangle$, where $a_{n_1,n_2}(t)$ denote the time-dependent probability amplitudes that obey the normalization condition $\sum_{n_1=0}^{N}\sum_{n_2=0}^{N-n_1}|a_{n_1,n_2}(t)|^2 = 1$. Inserting Eq. (1) and the expanded expression of $|\Psi(t)\rangle$ into Schrödinger equation $i\frac{\partial\Psi(t)}{\partial t} = H(t)\Psi(t)$ results in a set of coupled equations of $a_{n_1,n_2}(t)$ with equation number $\zeta = (N+1)(N+2)/2$.

It is very difficult to obtain the exact solutions of all coupled equations because of the periodically varying coefficients. However, the coherent manipulation of tunneling dynamics can be investigated analytically in high-frequency approximation with $\omega \gg \Omega$. We introduce a set of slowly varying functions $b_{n_1,n_2}(t)$ through the transformation [30] $a_{n_1,n_2}(t) = b_{n_1,n_2}(t)\exp\{-i\int_0^t[0.5U(t)(n_1(n_1-1)+n_2(n_2-1)+(N-n_1-n_2)(N-n_1-n_2-1))+\varepsilon_0(2n_1+n_2-N)]dt\}$ with $|a_{n_1,n_2}(t)|^2 = |b_{n_1,n_2}(t)|^2$, which leads to that the high-frequency oscillating modulation will be contained in the phase factors. Resembling the fractional photon resonance effect [31], we set the parameters $\varepsilon_0 = 0.5\omega$, $U_0 = 0.5\omega/(N-1)$, and a set of coupled equations of $b_{n_1,n_2}(t)$ can be obtained as

$$\begin{aligned}
i\dot{b}_{0,N-1}(t) &= -\sqrt{N}\Omega b_{0,N}(t)e^{-i[\omega t+U_1(N-1)\sin(\omega t)/\omega]} \\
&\quad - \sqrt{N-1}\Omega b_{1,N-2}(t)e^{-i[\frac{\omega t}{2(N-1)}-U_1(N-2)\sin(\omega t)/\omega]} \\
&\quad - \sqrt{2(N-1)}\Omega b_{0,N-2}(t)e^{i[\omega t-\frac{\omega t}{N-1}+U_1(N-3)\sin(\omega t)/\omega]}, \\
i\dot{b}_{0,N}(t) &= -\sqrt{N}\Omega b_{0,N-1}(t)e^{i[\omega t+U_1(N-1)\sin(\omega t)/\omega]} \\
&\quad - \sqrt{N}\Omega b_{1,N-1}(t)e^{i[U_1(N-1)\sin(\omega t)/\omega]}, \\
i\dot{b}_{1,N-1}(t) &= -\sqrt{N}\Omega b_{0,N}(t)e^{-i[U_1(N-1)\sin(\omega t)/\omega]} \\
&\quad - \sqrt{N-1}\Omega b_{1,N-2}(t)e^{i[\omega t-\frac{\omega t}{2(N-1)}+U_1(N-2)\sin(\omega t)/\omega]} \\
&\quad - \sqrt{2(N-1)}\Omega b_{2,N-2}(t)e^{-i[\frac{\omega t}{N-1}-U_1(N-3)\sin(\omega t)/\omega]},
\end{aligned} \quad (2)$$

where only three coupled equations are presented, in which the probability-amplitude functions $b_{0,N-1}(t)$, $b_{0,N}(t)$, $b_{1,N-1}(t)$, $b_{0,N-2}(t)$, $b_{1,N-2}(t)$ and $b_{2,N-2}(t)$ correspond to states $|0,N-1,1\rangle$, $|0,N,0\rangle$, $|1,N-1,0\rangle$, $|0,N-2,2\rangle$, $|1,N-2,1\rangle$ and $|2,N-2,0\rangle$, respectively. By using Fourier expansion $\exp[\pm i(n\omega t + x\sin(\omega t))] = \Sigma_{n'=-\infty}^{\infty}\mathcal{J}_{n'}(x)\exp[\pm i(n+n')\omega t]$ with $n = 0, 1$ and under the high-frequency approximation, we can neglect these rapidly oscillating terms of the Fourier expansion with $n \pm n' \neq 0$. Simultaneity, these functions oscillating rapidly such as $e^{-i\frac{\omega t}{N-1}}$ and $e^{-i\frac{\omega t}{2(N-1)}}$ in differential equations (2) can be replaced by their average value of zero in the short time interval $2\pi/\omega$ when $\omega \gg 2(N-1)$ [32]. Thus, in high-frequency approximation, the set of coupled equations of $b_{n_1,n_2}(t)$ can be effectively truncated as

$$\begin{aligned}
i\dot{b}_{0,N-1}(t) &= -J_1 b_{0,N}(t), \\
i\dot{b}_{0,N}(t) &= -J_1 b_{0,N-1}(t) - J_2 b_{1,N-1}(t), \\
i\dot{b}_{1,N-1}(t) &= -J_2 b_{0,N}(t).
\end{aligned} \quad (3)$$

In Eq. (3), the effective couplings are given as $J_1 = \sqrt{N}\Omega\mathcal{J}_{-1}\times[(N-1)U_1/\omega]$ and $J_2 = \sqrt{N}\Omega\mathcal{J}_0[(N-1)U_1/\omega]$ with $\mathcal{J}_n(x)$ being the n-order Bessel function of x. Here the effective couplings depend on the modulating parameters, number of atom. And the zeroth- and first-order Bessel functions emerge in the effective couplings resulting from appropriate bias ε_0 and interaction U_0. The different order Bessel function will result in asymmetric tunneling dynamics in the subspace spanned by Fock states $\{|0,N-1,1\rangle, |0,N,0\rangle, |1,N-1,0\rangle\}$ when initial state is prepared in this subspace.

Setting $b_{n_1,n_2} = B_{n_1,n_2}e^{-irt}$ with B_{n_1,n_2} and r being constants and inserting such a form of b_{n_1,n_2} into Eq. (3), the constant r can be obtained as $r_1 = 0$, $r_{2,3} = \pm\sqrt{J_1^2 + J_2^2}$. It is well-known that a quantum state of periodically driven system can be described by $\Psi(t) = \phi(t)e^{-iEt}$ based on the Floquet theorem [33], in which the Floquet state $\phi(t+T) = \phi(t)$ with T and E being the period of Eq. (1) and Floquet quasienergies, respectively. Based on the transformation relation between functions $a_{n_1,n_2}(t)$ and $b_{n_1,n_2}(t)$ and the expression $b_{n_1,n_2} = B_{n_1,n_2}e^{-irt}$, the Floquet energies can be constructed as $E_1 = k$, $E_{2,3} = \pm\sqrt{J_1^2 + J_2^2} + k$ with $0 \le k = (N/4 - m)\omega < \omega$ and $m = 0, 1, 2, \ldots$. The constant B_{n_1,n_2} can be obtained easily from Eq. (3) and the corresponding Floquet states $\phi(t)$ are constructed as

$$\phi_1(t) = \frac{1}{\sqrt{J_1^2 + J_2^2}} \times\left[-J_2 e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t) - i(m-1)\omega t}|0, N-1, 1\rangle\right.$$

$$+ J_1 e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-im\omega t}|1, N-1, 0\rangle\Big]$$

$$\phi_{2,3}(t) = \frac{1}{\sqrt{2(J_1^2+J_2^2)}} \times \Big[J_1 e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-i(m-1)\omega t}|0, N-1, 1\rangle \mp \sqrt{J_1^2+J_2^2} e^{-i\frac{N(N-1)U_1}{2\omega}\sin(\omega t)-im\omega t}|0, N, 0\rangle + J_2 e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-im\omega t}|1, N-1, 0\rangle\Big]. \quad (4)$$

Based on the analytical Floquet solutions obtained above, the superposition state of modulating system can be constructed [32] as

$$\Psi(t) = \Sigma_{j=1}^{3} \Lambda_j \phi_j(t) e^{-iE_j t}, \quad (5)$$

where the superposition coefficient Λ_j depends on the initial conditions. It can be seen that the superposition state in Eq. (5) implies quantum interference effect among Floquet states. The corresponding interference may cause the coherent enhancement or suppression of tunneling, whose degree depends on the value of the effective couplings. Firstly, we consider all atom are prepared in middle well initially and the superposition coefficient can be obtained as $\Lambda_1 = 0$, $\Lambda_{2,3} = \mp\frac{\sqrt{2}}{2}$. Correspondingly, the superposition state is given as

$$\Psi(t) = \frac{iJ_1 \sin(\sqrt{J_1^2+J_2^2}t)}{\sqrt{J_1^2+J_2^2}} \times e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-i(m-1)\omega t-ikt}|0, N-1, 1\rangle + \cos\Big(\sqrt{J_1^2+J_2^2}t\Big) e^{-i\frac{N(N-1)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|0, N, 0\rangle + \frac{iJ_2 \sin(\sqrt{J_1^2+J_2^2}t)}{\sqrt{J_1^2+J_2^2}} \times e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|1, N-1, 0\rangle. \quad (6)$$

When the initial state is $|0, 4, 1\rangle$, from Eq. (5) the superposition coefficient can be derived as $\Lambda_1 = -J_2/\sqrt{J_1^2+J_2^2}$, $\Lambda_2 = \Lambda_3 = J_1/\sqrt{2(J_1^2+J_2^2)}$ and the corresponding superposition state is

$$\Psi(t) = \frac{J_2^2 + J_1^2\cos(\sqrt{J_1^2+J_2^2}t)}{J_1^2+J_2^2} \times e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-i(m-1)\omega t-ikt}|0, N-1, 1\rangle + \frac{iJ_1 \sin(\sqrt{J_1^2+J_2^2}t)}{\sqrt{J_1^2+J_2^2}} e^{-i\frac{N(N-1)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|0, N, 0\rangle + \frac{J_1 J_2[\cos(\sqrt{J_1^2+J_2^2}t)-1]}{J_1^2+J_2^2} \times e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|1, N-1, 0\rangle. \quad (7)$$

If the initial state is changed as $|1, 4, 0\rangle$, we can get the superposition coefficient $\Lambda_1 = J_1/\sqrt{J_1^2+J_2^2}$, $\Lambda_2 = \Lambda_3 = J_2/\sqrt{2(J_1^2+J_2^2)}$ and the corresponding superposition state

$$\Psi(t) = \frac{J_1 J_2[\cos(\sqrt{J_1^2+J_2^2}t)-1]}{J_1^2+J_2^2} \times e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-i(m-1)\omega t-ikt}|0, N-1, 1\rangle + \frac{iJ_2 \sin(\sqrt{J_1^2+J_2^2}t)}{\sqrt{J_1^2+J_2^2}} e^{-i\frac{N(N-1)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|0, N, 0\rangle + \frac{J_1^2 + J_2^2\cos(\sqrt{J_1^2+J_2^2}t)}{J_1^2+J_2^2} \times e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|1, N-1, 0\rangle. \quad (8)$$

In the following, we are going to focus on coherent control of tunneling dynamics of bosons based on above analytical superposition state by adjusting the modulation strength.

3. Directed selective-tunneling effect in triple-well system

By adjusting the modulation strength U_1, there exists several roots of Bessel function, which leads to the effective coupling $J_n = \Omega\sqrt{N}\mathcal{J}_{n-2}[(N-1)U_1/\omega] = 0$ with $n = 1, 2$. Firstly, we consider all atom are prepared in middle well initially and the modulating interaction $U_1 = 2.405\omega/(N-1)$, which leads to the effective coupling $J_2 = 0$. Correspondingly, Eq. (6) is reduced as $|\Psi(t)\rangle = \cos(J_1 t)e^{-i\frac{N(N-1)U_1}{2\omega}\sin(\omega t)-im\omega t-ikt}|0, N, 0\rangle + i\sin(J_1 t)e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-i(m-1)\omega t-ikt}|0, N-1, 1\rangle$. Clearly, the tunneling only occur between states $|0, N, 0\rangle$ and $|0, N-1, 1\rangle$. The result means that the directed selective-tunneling effect occurs, in which a single atom perform Rabi oscillation between middle and right wells and the tunneling period can be obtained as $T = |\frac{\pi}{\Omega\sqrt{N}\mathcal{J}_{-1}(x_0)}|$ with x_0 satisfying $\mathcal{J}_0(x_0) = 0$.

As an example, we consider the tunneling dynamics for $N = 5$ ($m = 1$, $k = \omega/4$). Fixing $\omega = 80$ and $\Omega = 1$, $U_0 = 10$, $\varepsilon_0 = 40$, the modulation strength $U_1 = 2.405\omega/4$ leads to effective coupling $J_2 = 0$ and the corresponding time evolutions of atomic probabilities are exhibited as in Fig. 1(a) with $P_{n_1,n_2}(t) = |a_{n_1,n_2}(t)|^2 = |b_{n_1,n_2}(t)|^2$. It can be seen that all probabilities $P_{n_1,n_2}(t) \simeq 0$ except for the probabilities of states $|0, 5, 0\rangle$ and $|0, 4, 1\rangle$. The result implies that the directed selective-tunneling occurs, in which only one of five bosons is allowed to tunnel from the initial well to right one. Here the pathway between wells 1 and 2 is shut off and only a single boson performs Rabi oscillation along this pathway between wells 2 and 3. The directed tunneling effect implies that left and right symmetry is broken, which results from bias potential and atomic interaction. The maximal probability of state $|0, 4, 1\rangle$ $P_{\max} \simeq 1$ at $t' = T/2 \simeq 1.35$, which means that a single atom has completely tunnels into right well at this time. The analytical results (circles) are confirmed numerically from Eq. (1), as shown by the solid lines, and good agreement is found between both.

Now we consider the initial state is changed as $|0, N-1, 1\rangle$ and $U_1 = 3.832\omega/(N-1)$ that leads to the effective coupling $J_1 = 0$. Correspondingly, Eq. (7) is reduced as $\Psi(t) = e^{-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t)-i(m-1)\omega t-ikt}|0, N-1, 1\rangle$. The result implies that the quantum tunneling of all bosons has been suppressed completely and the CDT effect occurs as in Fig. 1(b) for $N = 5$, in which the analytical results (circles) is good agreeable with numerical one.

Quantum manipulation is an interesting and important research area and possesses potential applications in quantum devices and information technologies [32,34]. Based on above analytical results, we propose a manipulation scheme for separating a single atom

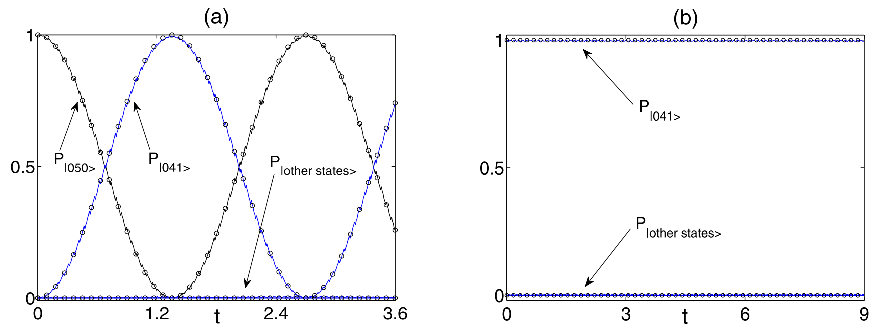

Fig. 1. The time evolutions of the atomic probabilities for $U_1 = 2.405\omega/4$ in (a) and $U_1 = 3.832\omega/4$ in (b) with $N = 5$. In (a) and (b), the initial states are $|0, N, 0\rangle$ and $|0, N-1, 1\rangle$, respectively. The other parameters are set as $\Omega = 1$, $\omega = 80$, $\varepsilon_0 = 40$, $U_0 = 10$. Circles indicate the analytical results and solid lines the numerical correspondences. Here and in other figures all variables are dimensionless.

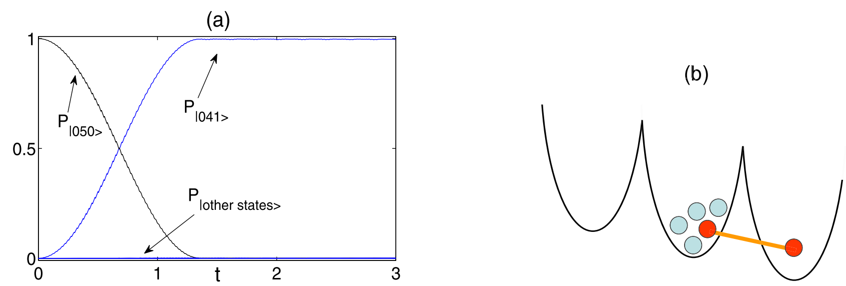

Fig. 2. (a) shows the coherent manipulation for separating a single atom from N bosons numerically with $N = 5$. The initial state is $|0, N, 0\rangle$ and the modulation strength are chosen as $U_1 = 2.405\omega/4$ $(0 \le t < 1.35)$ and $U_1 = 3.832\omega/4$ $(1.35 \le t \le 3)$. The other parameters are the same as in Fig. 1(a). The (b) exhibits the schematic diagrams of directed selective-tunneling of a single atom $|0, 5, 0\rangle \to |0, 4, 1\rangle$.

from N bosons through controlling the modulation strength. Initially, we consider that all bosons are prepared in middle well with $P_{0,N}(t_0) = 1$ and the parameters $\varepsilon_0 = 0.5\omega$, $U_0 = 0.5\omega/(N-1)$ and $U_1 = 2.405\omega/(N-1)$. The set of parameters leads to the effective coupling $J_2 = 0$ and the selective-tunneling effect of a single atom occurs between middle and right wells. At $t' = T/2$, one atom of N bosons tunnels completely into right well and the quantum state can be described by Fock state $|0, N-1, 1\rangle$. Just now, the modulating interaction is changed as $U_1 = 3.832\omega/(N-1)$ that leads to $J_1 = 0$. Subsequently, the CDT effect occurs based on Eq. (7), in which the tunneling of all bosons among three-well system will be suppressed completely and one atom will be trapped in right well at all times. Thus, we successfully separate single atom from N bosons by adjusting the modulation interaction. The coherent manipulation scheme for separating a single atom from N bosons is demonstrated numerically as in Fig. 2(a) for $N = 5$ and the corresponding schematic diagram is displayed by Fig. 2(b).

Further, fixing the parameters relations $\varepsilon_0 = 0.5\omega$, $U_0 = 0.5\omega/(N-1)$ and $U_1 = 2.405\omega/(N-1)$, we calculate numerically all probabilities $P_{n_1,n_2}(t)$ from Eq. (1) for different frequency ω and atom number N in the time interval $t \in [0, 200 \times \frac{2\pi}{\omega}]$. We take the maximal value of probability for the state $|0, N-1, 1\rangle$ and the maximal probability $P_{\max}$ versus the modulating frequency ω is exhibited as in Fig. 3, in which the $P_{\max} \simeq 1$ means the occurrence of directed selective-tunneling between states $|0, N, 0\rangle$ and $|0, N-1, 1\rangle$. It is shown that the selective-tunneling effect of a single boson only occur in high-frequency region where $P_{\max} \simeq 1$. When $N = 2$, it can be seen that the directed selective-tunneling of a single boson can be realized for $\omega \simeq 40$. But with N increasing, more higher modulating frequency ω is required to realize the directed selective-tunneling effect of a single boson.

When all bosons are located in middle well initially and the relation $J_1 = 0$ is satisfied, Eq. (6) is reduced as $|\Psi(t)\rangle = \cos[J_2 t]\exp[-i\frac{N(N-1)U_1}{2\omega}\sin(\omega t) - im\omega t - ikt]|0, N, 0\rangle + i\sin[J_2 t] \times \exp[-i\frac{(N-1)(N-2)U_1}{2\omega}\sin(\omega t) - im\omega t - ikt]|1, N-1, 0\rangle$. The result means the tunneling only occur between states $|0, N, 0\rangle$ and $|1, N-1, 0\rangle$ and the tunneling period of single atom $T' = |\frac{\pi}{\Omega\sqrt{N}\mathcal{J}_0(x'_0)}|$ with x'_0 satisfying $\mathcal{J}_{-1}(x'_0) = 0$.

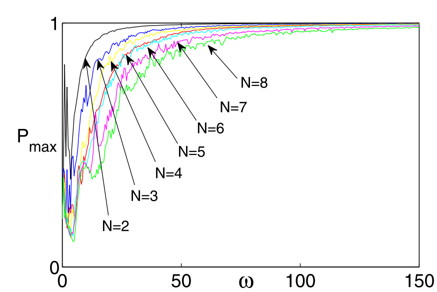

Fig. 3. (Color online.) The maximal probability of the state $|0, N-1, 1\rangle$ in the time evolution $t \in [0, 200 \times \frac{2\pi}{\omega}]$ for different frequency ω and atomic number N. The initial state is $|0, N, 0\rangle$ and other parameters $\Omega = 1$, $\varepsilon_0 = 0.5\omega$, $U_0 = 0.5\omega/(N-1)$ and $U_1 = 2.405\omega/(N-1)$.

In Fig. 4(a), the time evolutions of atomic probabilities are shown for the modulating strength $U_1 = 3.832\omega/4$ with $N = 5$. The other parameters are same as that in Fig. 1(a). It can be seen that the directed selective-tunneling occurs between states $|0, 5, 0\rangle$ and $|1, 4, 0\rangle$, in which only the pathway between wells 1 and 2 is switched on and only one of five bosons participates in the tunneling process along this tunneling path with tunneling time $t = T'/2 = 1.744$. It is shown that the analytical results (circles) are in a good agreement with the numerical ones (solid lines) from Eq. (1). Similarly, from the analytical solutions (6) and (8), we know that a single atom can be separated from N bosons and trapped in left well when we adjust the modulation strength to $U_1 = 2.405\omega/(N-1)$ at $t = T'/2$. In Fig. 4(b), we exhibit the corresponding schematic diagram of directed tunneling of a single boson.

In our paper, when all bosons are prepared in middle well initially, the selective-tunneling effect is demonstrated, in which a single atom can be separated from N bosons and trapped in right or left well through controlling the modulation strength. It

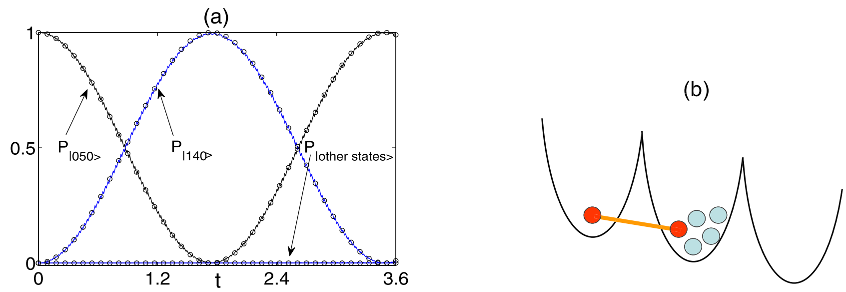

Fig. 4. The time evolutions of the atomic probabilities for $U_1 = 3.832\omega/4$ with $N = 5$. The initial condition and the other parameters are the same as in Fig. 1(a). The (b) exhibits the schematic diagrams of directed selective-tunneling of a single atom $|0,5,0\rangle \to |1,4,0\rangle$.

is well-known that the transistorlike effects of bosons held in a triple well has been demonstrated, in which the atomic population in the middle well controls the tunneling dynamics between the left and right wells [25]. In Ref. [27], the scheme of directed selective-tunneling has been presented for dipolar bosons held in a triple-well and the directed tunneling of 1 or $(N-1)$ bosons depend on the dipolar interaction and the fast modulation of level unbalance. But in our work, we exhibit another a manipulating scheme of the selective tunneling based on the periodical modulation of atomic interaction. However, it is a pity that the directed tunneling of a prescribed number of bosons can't be realized in our presented triple-well system because we can't find the appropriate ε_0 and U_0 to realize the photon-assisted resonance tunneling effect between $|0,N,0\rangle$ and other Fock states except for $|0,N-1,1\rangle$ and $|1,N-1,0\rangle$.

4. Summary and discussion

In summary, we have investigated the tunneling dynamics of bosons with periodically modulated interaction held in a triple-well potential. By choosing proper parameters ε_0, U_0 and under high-frequency approximation, we obtain a set of reduced coupled equations of probability amplitudes and the selective-tunneling effect is demonstrated analytically and numerically. By adjusting the modulation strength and frequency, we found that only one atom is allowed to tunnel into left or right well and a manipulation scheme is demonstrated for separating a single atom from N bosons. The coherent control of selective-tunneling effect is interesting for potential atomic devices and the manipulation scheme can be realized under the presently accessible experimental conditions [10,15,22,35]. Further, the result obtained in our work can help us to understand the tunneling dynamics of bosons trapped in an optical lattice [23].

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