# Directed selective tunneling of dipolar bosons in a driven triple well

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We study the coherent control of quantum tunneling for dipolar bosons held in a driven triple-well potential. In the high-frequency region within the resonance case, based on the non-Floquet solutions of two dipolar bosons, the influence of dipolar interaction on tunneling is investigated analytically and numerically, in which the directed selective-tunneling of a single atom is demonstrated when the two bosons are located in the middle well initially. Further, the corresponding effect is exhibited numerically for more dipolar atoms $N > 2$ and the directed tunneling of 1 or $(N-1)$ atoms occurs by adjusting the driving parameters. These results may be useful in the design of atomic devices.

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## I. INTRODUCTION

The quantum manipulation via external field has been underway for a long time in both physics and chemistry [1]. In recent years, the coherent control of tunneling dynamics in a driven multi-well potential has become one of the most important subjects and attracted both theoretical [2] and experimental interest [3,4]. The time-periodic driving field is a powerful tool to realize the accurate control of quantum tunneling and many interesting tunneling phenomena have been demonstrated, such as dynamic localization (DL) [5], photon-assisted tunneling [6–8], coherent destruction of tunneling (CDT) [9], and selective CDT effect [10,11] of a single particle, etc. Further, the selective CDT effect has been extended to many-body bosons held in a double-well potential and an interesting scheme that an arbitrarily and *a priori* prescribed number of bosons are allowed to tunnel from one well to the other has been presented by modulating the self-interaction strength [12] or energy level unbalance [13]. In addition, the selective CDT effect was applied to realize the directed motion of atoms in a driven 1D bipartite lattice [14,15]

Recently, the dipolar quantum gases with long-range dipole-dipole interaction (DDI) have attracted a lot of interest theoretically and experimentally [16,17]. Some fascinating novel properties induced by DDI were exhibited, such as the supersolid phase and the insulating checkerboard phase [18], long-lived Bloch oscillations [19], etc. In recent discussions, the triple-well potential with one-dimensional configuration, a minimal system loaded with a dipolar gas for discussing the effect of DDI, was studied extensively. For example, a variety of possible ground-state phases were revealed and the dynamical creation of mesoscopic quantum superpositions was discussed [20]. Because of the intersite interaction, the nonlocal coherence was studied [21]. Depending on the strength of the contact and dipolar interaction, the stable and unstable regions in parameter space were depicted [22] and the role of excited and metastable states was clarified [23]. The formation of intrawell localization was demonstrated for a very strong dipolar interaction [24] and the long-range macroscopic Josephson oscillations and long-range coherent quantum transportation were shown [25]. Furthermore, the role of anisotropy for dipolar bosons [26] and entanglement entropy [27] was investigated in a three-well with equilateral triangular configuration.

In this paper, we investigate the coherent control of quantum tunneling for dipolar bosons held in driven triple-well with one-dimensional configuration and we are interested in the influence of the dipolar interaction on quantum tunneling. For two dipolar bosons, in the high-frequency region within resonance case, we obtain a set of analytical Floquet and non-Floquet solutions that depend on the relation between interaction and driving frequency. Based on the non-Floquet superposition states, the directed selective-tunneling of a single atom is exhibited by adjusting driving parameters, in which all bosons are located in the middle well initially and a good correspondence is demonstrated between analytical and numerical results. In the nonresonance case, we show that the directed tunneling scheme is robust in a proper reduced dipolar-interaction region. Further, when more bosons $N > 2$ are considered, the directed tunneling effect of 1 or $(N-1)$ bosons is found. It is shown that, for a set of proper parameters, only a single atom is allowed to tunnel along one direction and the atomic probabilities show a series of steps that composed of $N$ small oscillations. As the driving strength is adjusted to another proper value, we find that $(N-1)$ bosons participate in the tunneling process along another direction. These results presented in our paper may be employed in the design of a single-atom source and atomic transistor [14,15,28,29].

## II. FLOQUET AND NON-FLOQUET SOLUTIONS OF TWO DIPOLAR BOSONS

We consider the tunneling dynamics of dipolar bosons held in a driven triple-well potential. Under the single-band tight-binding approximation, the corresponding Hamilton reads as [20,21,30,31]

$$\hat{H}(t) = -\Omega \sum_{\langle k,l\rangle}(\hat{c}_k^\dagger \hat{c}_l + \hat{c}_l^\dagger \hat{c}_k) + \frac{U_0}{2}\sum_{k=1}^{3}\hat{c}_k^\dagger\hat{c}_k^\dagger\hat{c}_k\hat{c}_k + U_1[\hat{n}_1\hat{n}_2 + \hat{n}_2\hat{n}_3] + U_2\hat{n}_1\hat{n}_3 + \varepsilon(t)(\hat{c}_1^\dagger\hat{c}_1 - \hat{c}_3^\dagger\hat{c}_3), \tag{1}$$

where $\hat{c}_k^\dagger$ and $\hat{c}_k$ are, respectively, the atom creation and annihilation operator in the well $k$. $\Omega$ describes the nearest-neighbor couplings and the on-site interaction between atoms is denoted by $U_0$. $U_1$ is the nearest-neighbor dipole-dipole interaction and

 

$U_2$ describes the interact between nonadjacent atoms, where the ratio of the nearest-neighbor and next- nearest-neighbor interaction $4 \leqslant U_1/U_2 \leqslant 8$ [20,21]. We choose the driving field $\varepsilon(t) = \varepsilon_0 + \varepsilon_1 \cos(\omega t)$ with $\varepsilon_0$ being the strength of dc field, $\varepsilon_1$ and $\omega$ the strength and frequency of the ac field.

We have assumed that the tunneling of atoms only can occur between these vibrational ground states for proper depths of the three wells. Throughout this paper, $\hbar = 1$ is adopted such that $\varepsilon_0, \varepsilon_1, \omega$, and $\Omega$ are in units of $\omega_0$ with $\omega_0$ being the reference frequency on the order of $10^2$ s$^{-1}$[32], and time $t$ is normalized in units of $\omega_0^{-1}$. Here a Fock basis $|n_1, n_2, N - n_1 - n_2\rangle$ is used to describe the tunneling dynamics in the driven three-well, with $n_1$ atoms in the left well, $n_2$ atoms in the middle well, and $N - n_1 - n_2$ atoms in the right well. Here the total number of atoms $N$ is a constant.

First, we take into account the coherent control of tunneling dynamics for two bosons, which is the simplest quantum system to study the dipolar effect. Based on the Fock bases, the corresponding quantum state $\Psi(t)$ of system (1) can be expanded as

$$|\Psi(t)\rangle = a_1(t)|2,0,0\rangle + a_2(t)|0,2,0\rangle + a_3(t)|0,0,2\rangle + a_4(t)|1,1,0\rangle + a_5(t)|1,0,1\rangle + a_6(t)|0,1,1\rangle, \quad (2)$$

where $a_i(t)$ denote the time-dependent probability amplitudes that obey the normalization condition $\sum_{i=1}^{6} |a_i(t)|^2 = 1$. Inserting Eqs. (1) and (2) into Schrödinger equation $i\frac{\partial\Psi(t)}{\partial t} = H(t)\Psi(t)$ results in the coupled equations

$$\begin{aligned}
i\dot{a}_1 &= 2\varepsilon(t)a_1 + U_0a_1 - \sqrt{2}\Omega a_4,\\
i\dot{a}_2 &= U_0a_2 - \sqrt{2}\Omega a_4 - \sqrt{2}\Omega a_6,\\
i\dot{a}_3 &= -2\varepsilon(t)a_3 + U_0a_3 - \sqrt{2}\Omega a_6,\\
i\dot{a}_4 &= \varepsilon(t)a_4 + U_1a_4 - \sqrt{2}\Omega a_1 - \sqrt{2}\Omega a_2 - \Omega a_5,\\
i\dot{a}_5 &= U_2a_5 - \Omega a_4 - \Omega a_6,\\
i\dot{a}_6 &= -\varepsilon(t)a_6 + U_1a_6 - \sqrt{2}\Omega a_2 - \sqrt{2}\Omega a_3 - \Omega a_5.
\end{aligned} \quad (3)$$

It is difficult to obtain the exact solutions of Eq. (3), but the coherent control of tunneling dynamics can be investigated analytically in high-frequency approximation $\omega \gg \Omega$ and a strong interaction region [13,33]. We introduce the functions $b_j(t)$ through the transformation [34] $a_1(t) = b_1(t)e^{-i\int_0^t(2\varepsilon(t')+U_0)dt'}$, $a_2(t) = b_2(t)e^{-i\int_0^t U_0dt'}$, $a_3(t) = b_3(t)e^{i\int_0^t(2\varepsilon(t')-U_0)dt'}$, $a_4(t) = b_4(t)e^{-i\int_0^t(\varepsilon(t')+U_1)dt'}$, $a_5(t) = b_5(t)e^{-i\int_0^t U_2dt'}$, $a_6(t) = b_6(t)e^{i\int_0^t(\varepsilon(t')-U_1)dt'}$, where $b_j(t)$ are slowly varying functions of time. In our work, we set $\varepsilon_0 = m_1\omega$, $U_0 - U_1 = m_2\omega$, $U_1 - U_2 = m_3\omega + \beta$ with reduced dipolar interaction $|\beta| \leqslant \frac{\omega}{2}$ ($m_1$, $m_2$, $m_3$ integers). Under the high-frequency approximation, based on the Fourier expansion $e^{\pm i\int(\varepsilon(t)+U_0-U_1)dt} = \sum_{m'} \mathcal{J}_{m'}(\frac{\varepsilon_1}{\omega})e^{\pm i(m'+m_1+m_2)\omega t}$, $e^{\pm i\int(\varepsilon(t)-(U_0-U_1))dt} = \sum_{m''} \mathcal{J}_{m''}(\frac{\varepsilon_1}{\omega})e^{\pm i(m''+m_1-m_2)\omega t}$, $e^{\pm i\int(\varepsilon(t)+U_1-U_2)dt} = \sum_{m'''} \mathcal{J}_{m'''}(\frac{\varepsilon_1}{\omega})e^{\pm i(m'''+m_1+m_3)\omega t \pm i\beta t}$, and $e^{\pm i\int(\varepsilon(t)-(U_1-U_2))dt} = \sum_{m''''} \mathcal{J}_{m''''}(\frac{\varepsilon_1}{\omega})e^{\pm i(m''''+m_1-m_3)\omega t \mp i\beta t}$ ($m'$, $m''$, $m'''$, $m''''$ integers), the set of differential equations (3) is transformed to

$$\begin{aligned}
i\dot{b}_1 &= -\sqrt{2}J_1b_4,\\
i\dot{b}_2 &= -\sqrt{2}J_2b_4 - \sqrt{2}J_1b_6,\\
i\dot{b}_3 &= -\sqrt{2}J_2b_6,\\
i\dot{b}_4 &= -\sqrt{2}J_1b_1 - \sqrt{2}J_2b_2 - J_3b_5e^{i\beta t},\\
i\dot{b}_5 &= -J_3b_4e^{-i\beta t} - J_4b_6e^{-i\beta t},\\
i\dot{b}_6 &= -\sqrt{2}J_1b_2 - \sqrt{2}J_2b_3 - J_4b_5e^{i\beta t},
\end{aligned} \quad (4)$$

where the couplings coefficient $\Omega$ has been renormalized by the effective ones $J_1 = \Omega\mathcal{J}_{-(m_1+m_2)}(\frac{\varepsilon_1}{\omega})$, $J_2 = \Omega\mathcal{J}_{-(m_1-m_2)}(\frac{\varepsilon_1}{\omega})$, $J_3 = \Omega\mathcal{J}_{-(m_1+m_3)}(\frac{\varepsilon_1}{\omega})$, and $J_4 = \Omega\mathcal{J}_{-(m_1-m_3)}(\frac{\varepsilon_1}{\omega})$ with $\mathcal{J}_m(x)$ being the $m$-order Bessel function of $x$. In the following, we are going to focus on coherent control of tunneling dynamics for dipolar bosons held in the driven triple-well.

### A. Resonance case

When the atomic interaction is an integer multiple of the frequency of the driving field with zero reduced interaction, the photon resonance effect occurs [33]. Here, we consider an analogous resonance case as the reduced dipolar interaction $\beta = 0$, which is dependent on the difference of dipolar interactions $U_1$ and $U_2$. When $\beta = 0$, Eqs. (4) are reduced as differential equations with constant coefficient and a set of analytical Floquet and non-Floquet solutions can be obtained based on Eqs. (4) and the transforming relation of functions $a_j(t)$ and $b_j(t)$ with $j = 1,2,\ldots,6$.

In Eqs. (4) with $\beta = 0$, we set $b_j = B_je^{-iEt}$ with $B_j$ being constants and $E$ is the eigenvalue. Thus the analytical solution of Eq. (1) can be constructed as $\Psi(t) = \phi(t)e^{-iEt}$, where $\phi(t) = B_1e^{-i\int_0^t(2\varepsilon(t')+U_0)dt'}|2,0,0\rangle + B_2e^{-i\int_0^t U_0dt'}|0,2,0\rangle + B_3e^{i\int_0^t(2\varepsilon(t')-U_0)dt'}|0,0,2\rangle + B_4e^{-i\int_0^t(\varepsilon(t')+U_1)dt'}|1,1,0\rangle + B_5e^{-i\int_0^t U_2dt'}|1,0,1\rangle + B_6e^{i\int_0^t(\varepsilon(t')-U_1)dt'}|0,1,1\rangle$. Inserting such a form of $b_j$ into Eq. (4), we obtain the eigenvalue $E$:

$$E_1 = E_2 = 0, \quad E_{3,4} = \pm\sqrt{\frac{\lambda-\kappa}{2}}, \quad E_{5,6} = \pm\sqrt{\frac{\lambda+\kappa}{2}}, \quad (5)$$

where $\lambda = 4J_1^2 + 4J_2^2 + J_3^2 + J_4^2$ and $\kappa = \sqrt{(J_3^2 + J_4^2)^2 + 16J_1J_2(J_1J_2 + J_3J_4)}$. By applying the MATHEMATICA 8.0 code to Eqs. (4), we can obtain $B_j$ and the correspondingly functions $\phi(t)$ can be given as

$$\begin{aligned}
\phi_{1,2} &= \mp\xi J_2J_3e^{-i2\alpha(t)-iU_0t}|2,0,0\rangle \mp \xi J_1J_4e^{i2\alpha(t)-iU_0t}|0,0,2\rangle \pm \sqrt{2}\xi J_1J_2e^{-iU_2t}|1,0,1\rangle,\\
\phi_{3,4} &= -\frac{J_1E_3M_+(\lambda+\kappa)}{8\eta}e^{-i2\alpha(t)-iU_0t}|2,0,0\rangle + \frac{E_3M_+\rho_+}{8\eta}e^{-iU_0t}|0,2,0\rangle + \frac{J_2E_3M_+\sigma_+}{8\eta}e^{i2\alpha(t)-iU_0t}|0,0,2\rangle\\
&\quad \pm \frac{M_+}{2}e^{-i\alpha(t)-iU_1t}|1,1,0\rangle - \frac{E_3M_+\upsilon_+}{4\sqrt{2}\eta}e^{-iU_2t}|1,0,1\rangle \mp \frac{M_+(J_3^2 - J_4^2 + \kappa)}{4(2J_1J_2 + J_3J_4)}e^{i\alpha(t)-iU_1t}|0,1,1\rangle,
\end{aligned}$$

$$\phi_{5,6} = -\frac{J_1 E_5 M_-(\lambda+\kappa)}{8\eta} e^{-i2\alpha(t)-iU_0 t}|2,0,0\rangle + \frac{E_5 M_- \rho_-}{8\eta} e^{-iU_0 t}|0,2,0\rangle + \frac{J_2 E_5 M_- \sigma_-}{8\eta} e^{i2\alpha(t)-iU_0 t}|0,0,2\rangle$$
$$\pm \frac{M_-}{2} e^{-i\alpha(t)-iU_1 t}|1,1,0\rangle - \frac{E_5 M_- \upsilon_-}{4\sqrt{2}\eta} e^{-iU_2 t}|1,0,1\rangle \mp \frac{M_-\left(-J_3^2+J_4^2+\kappa\right)}{4(2J_1J_2+J_3J_4)} e^{i\alpha(t)-iU_1 t}|0,1,1\rangle, \qquad (6)$$

where $\xi = 1/\sqrt{2J_1^2J_2^2+J_2^2J_3^2+J_1^2J_4^2}$, $M_\pm = \sqrt{2(\kappa \mp J_3^2 \pm J_4^2)/\kappa}$, $\eta = 2J_1^4 - 2J_1J_2J_3J_4 + (J_1^2+J_2^2)(2J_2^2+J_3^2+J_4^2)$, $\rho_\pm = \{J_2J_3J_4(\pm 8J_1^2 \mp \lambda - \kappa) + 2J_1^3(\pm J_3^2 \mp J_4^2 + \kappa) + J_1[(\pm J_3^2(J_3^2+J_4^2 \pm \kappa) \mp 4J_2^2(J_2^2+J_4^2)]\}/(2J_1J_2+J_3J_4)$, $\sigma_\pm = [\pm 8J_1J_2J_3J_4 + 2J_1^2(\pm 4J_2^2 \pm J_3^2 \mp J_4^2 + \kappa) + 2J_2^2(\pm J_3^2 \mp J_4^2 + \kappa) \pm J_3^2(J_3^2 + J_4^2 \pm \kappa)]/(2J_1J_2 + J_3J_4)$ and $\upsilon_\pm = [J_1J_2J_3(\lambda \pm \kappa - 4J_4^2) + J_2^2J_4(J_3^2 + J_4^2 \mp \kappa) + J_1^2J_4(-4J_4^2 + J_3^2 + J_4^2 \mp \kappa)]/(2J_1J_2 + J_3J_4)$, and $\alpha(t) = \varepsilon_1 \sin(\omega t)/\omega + \varepsilon_0 t$.

From the Floquet theorem [35], we know that a Floquet state must have the same period as the Hamiltonian (1). When $\varepsilon_0$, $U_0$, $U_1$, and $U_1$ are integer multiple of frequency of the driving field, we find that $\phi(t+T) = \phi(t)$ from Eqs. (6) with $T$ being the period of Eq. (1). The result implies that the solution $\phi(t)$ are the Floquet states and the corresponding eigenvalues $E$ in Eqs. (5) are the analytical quasienergies. It is clear that the interaction can be a non-integer multiple of frequency of the driving field in the resonance case. In this case, the analytical solutions in Eqs. (6) are the non-Floquet solutions with $\phi(t+T) \neq \phi(t)$. Thus, in the high-frequency region within resonance case, we obtain a set of analytical Floquet and non-Floquet solutions that depend on the relation between interaction and driving frequency.

In the above-mentioned case, $b_j(t) = B_j e^{-iEt}$ with constant $B_j$ denotes a kind of simple solutions of the driving system. Generally, the system with $\beta = 0$ contains some complicated periodic and quasiperiodic general solutions. Here we consider general non-Floquet solutions obtained by a linear superposition of the above solutions as

$$\Psi(t) = \Sigma_{j=1}^{6} \Lambda_j \phi_j(t) e^{-iE_j t}, \qquad (7)$$

where $\Lambda_j$ is superposition coefficient that depends on the initial conditions.

In what follows, we only focus on the special superposition state that describes the selective tunneling effect of dipolar atoms. Note that the dipolar interaction can be adjusted [20] and if the interaction relation $U_0 - U_1 = U_1 - U_2$ is satisfied, we obtain $m_2 = m_3$, which leads to the effective couplings $J_1 = J_3$ and $J_2 = J_4$. By choosing proper driving parameters, there exist several roots of the Bessel function with $\mathcal{J}_n(\varepsilon_1/\omega) = 0$ that implies the zero effective couplings $J_1 = J_3 = 0$ or $J_2 = J_4 = 0$.

In our whole work, it is assumed that the initial state is $|0,N,0\rangle$, which means that all atoms are located in the middle well initially. By adjusting the dipolar interaction $U_0 - U_1 = U_1 - U_2$ and driving parameters with $\mathcal{J}_n(\varepsilon_1/\omega) = 0$, the zero effective couplings $J_2 = J_4 = 0$ can be obtained. From Eqs. (4), the results lead to the probability function $b_2(t)$ being only related to function $b_6(t)$ and the analytical solutions of Eqs. (4) are given as $b_2(t) = \cos(\sqrt{2}J_1 t)$, $b_6(t) = i\sin(\sqrt{2}J_1 t)$, and other functions $b_i(t) = 0$, which imply that the quantum state of the two-boson system can be given as $|\Psi(t)\rangle = \cos(\sqrt{2}J_1 t)e^{-iU_0 t}|0,2,0\rangle + i\sin(\sqrt{2}J_1 t)e^{i(\alpha - U_1 t)}|0,1,1\rangle$. Clearly, the state $|\Psi(t)\rangle$ exhibits the directed selective-tunneling effect, in which one of two bosons oscillates periodically between the middle and right wells.

As an example, the atomic interactions are set as $U_0 = 60$, $U_1 = 32$, and $U_2 = 4$. The driving parameters are taken as $\varepsilon_0 = \omega = 28$ and $\varepsilon_1 = 2.405\omega (m_1 = m_2 = m_3 = 1)$. The set of parameters leads to the effective coupling $J_2 = J_4 = 0$ and the time evolutions of the occupation probabilities in each well are plotted in Fig. 1(a). It is shown that the directed selective-tunneling effect occurs, in which only a single atom is allowed to tunnel from the initial well to right one $|020\rangle \rightleftarrows |011\rangle$. The tunneling passage between well 1 and 2 is shut off and one of two bosons performs a Rabi oscillation along this pathway between wells 2 and 3 with tunneling time $\Delta t = \frac{\pi}{2\sqrt{2}\mathcal{J}_2(2.405)} \simeq 2.57$. The analytical results (circles) are confirmed numerically from Eq. (1), as shown by the solid lines of Fig. 1(a), and good agreement is found between both.

In this paper, we keep all parameters as in Fig. 1(a) except for the driving strength $\varepsilon_1 = 5.135\omega$, which leads to the effective couplings $J_1 = J_3 = 0$. Thus, from Eqs. (4), the tunneling dynamics of the two dipolar bosons can be described by the quantum state $|\Psi(t)\rangle = \cos(\sqrt{2}J_2 t)e^{-iU_0 t}|0,2,0\rangle + i\sin(\sqrt{2}J_2 t)e^{-i(\alpha + U_1 t)}|1,1,0\rangle$. The result means the occurrence of a directed selective-tunneling of a single atom as shown in Fig. 1(b), in which one of two bosons performs a Rabi oscillation along another pathway between the middle and left wells with tunneling time $\Delta t = \frac{\pi}{2\sqrt{2}\mathcal{J}_0(5.135)} \simeq 8.4$. It can be seen that the analytical results (circles) are in good agreement with the numerical ones from Eqs. (1).

In the resonance case, we show that the tunneling direction of a single boson can be manipulated by adjusting driving parameters $\varepsilon_1/\omega$ with $U_0 - U_1 = U_1 - U_2 = \varepsilon_0 = \omega$. Here the resonance condition is dependent on the difference of the interaction when the dipolar interaction is considered. If the dipolar interaction is not considered, the similar tunneling scheme only can be realized approximately in the nonresonance case with reduced parameters $\gamma = \delta = \omega/2$ [31].

### B. Nonresonance case

Now we consider the nonresonance case with the reduced dipolar interaction $\beta \neq 0$. Clearly, it is difficult to obtain the analytical solutions of Eqs. (4) as $\beta \neq 0$. Here we only pay attention to the influence of $\beta$ on the directed selective-tunneling effect numerically. As an example, we consider the tunneling process of Fig. 1(a) for different $\beta$. The other parameters are the same as that in Fig. 1(a) except for $U_2$. Based on Eq. (3), we perform the numerical calculations for a different dipolar interaction $U_2 = 4.9$ $(\beta = -0.9)$, $U_2 = 7$

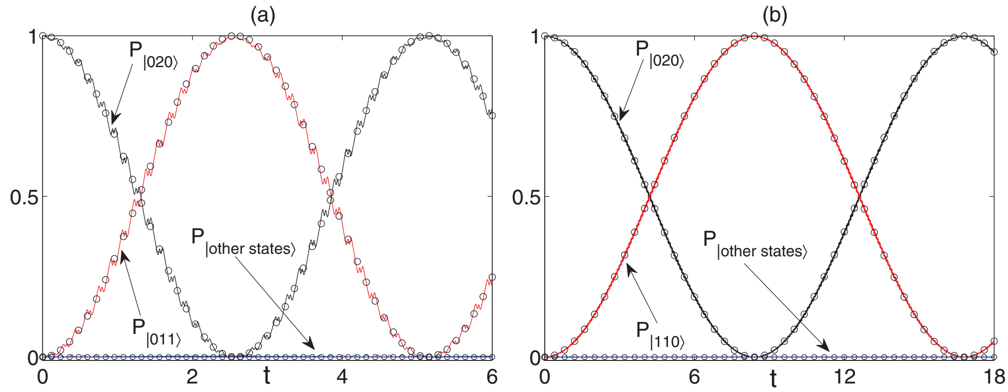


FIG. 1. (Color online) Time evolutions of the atomic probabilities for a different driving strength with $\Omega = 1$, $\omega = \varepsilon_0 = 28$, $U_0 = 60$, $U_1 = 32$, $U_2 = 4$. The two dipolar atoms are located in the middle well initially. (a) $\varepsilon_1 = 2.405\omega$; (b) $\varepsilon_1 = 5.135\omega$. Circles indicate the analytical results and solid lines the numerical correspondences. Here the time $t = 1(\omega_0^{-1})$ is on the order of $10^{-2}$ s.

$(\beta = -3)$, and $U_2 = 9$ $(\beta = -5)$ as in Fig. 2 with $P_j(t) = |a_j(t)|^2$ $(j = 1,2,\ldots,6)$. We find that $P_2 \simeq |\cos(\sqrt{2}J_1 t)|^2$, $P_6 \simeq |\sin(\sqrt{2}J_1 t)|^2$, and other probability functions $P_j(t) \simeq 0$ when $\beta \leqslant 0.9$. But for strong reduced dipolar interaction $\beta$, the directed selective-tunneling process of Fig. 1(a) is destroyed as in Figs. 2(b) and 2(c). The results imply that the directed selective-tunneling effect is insensitive to small reduced dipolar-interaction $\beta$.

For other parameter groups of directed selective-tunneling, we also perform many numerical calculations by choosing $\beta$ randomly, and we also find that the directed selective-tunneling effect is insensitive to small reduced interaction $\beta$. The result shows that there exists a safe reduced dipolar-interaction region and the directed tunneling scheme is robust in the region $-\beta_0 \leqslant \beta \leqslant \beta_0$, which $\beta_0$ is the maximal safe value. Thus, the rigorous interaction condition of the directed selective-tunneling effect is extended to $U_0 - U_1 = U_1 - U_2 + \beta$, which implies that the manipulation of the interaction becomes easier for realizing the directed selective-tunneling effect experimentally.

## III. DIRECTED SELECTIVE-TUNNELING EFFECT IN MANY-BOSON SYSTEM

Further, we consider the coherent control of tunneling dynamics for more dipolar bosons $N > 2$ in the driven triple-well. Fixing the relation of interaction $U_0 - U_1 = U_1 - U_2 = \Delta U$ and choosing the external field parameters $\varepsilon_0 = \omega = \Delta U$. Based on the numerical calculation from Eq. (1), we find that the selective tunneling between states $|0,N,0\rangle$ and $|0,N-1,1\rangle$ occurs when the relation $\mathcal{J}_{-(N-2)}(\frac{\varepsilon_1}{\omega}) = 0$ is satisfied. This means that only one of $N$ dipolar bosons participates in the tunneling process along the path between wells 2 and 3. Correspondingly, the tunneling time between these states is $\Delta t = \frac{\pi}{(2\Omega\sqrt{N})\mathcal{J}_N(x_0)}$, in which $x_0$ satisfying $\mathcal{J}_{-(N-2)}(x_0) = 0$.

As an example, we consider the tunneling dynamics for $N = 5$. Setting $U_0 = 60, U_1 = 32, U_2 = 4$, and driving parameters $\varepsilon_0 = \omega = 28$, the ratio $\varepsilon_1/\omega = 6.3802$ leads to Bessel function $\mathcal{J}_{-3}(\frac{\varepsilon_1}{\omega}) = 0$ and the corresponding time evolutions of the atomic probabilities are exhibited as shown in Fig. 3(a). It can be seen that all probabilities $P_i \simeq 0$ except for the probabilities of states $|0,5,0\rangle$ and $|0,4,1\rangle$. The result implies that the directed selective-tunneling occurs, in which only one of five bosons is allowed to tunnel from the initial well to the right one. Here the pathway between wells 1 and 2 is shut off and only a single atom performs Rabi oscillation along this pathway between wells 2 and 3 with tunneling time $\Delta t = \frac{\pi}{(2\Omega\sqrt{5})\mathcal{J}_5(6.3802)} \simeq 1.88$. The schematic diagram of such a directed selective-tunneling effect is displayed by Fig. 3(c). Besides, from Fig. 3(a), we find that the time evolution of probabilities exhibit a step as that in Ref. [36]. Furthermore, based on many numerical experiments for different $N$, we find that the platform is composed of rapid

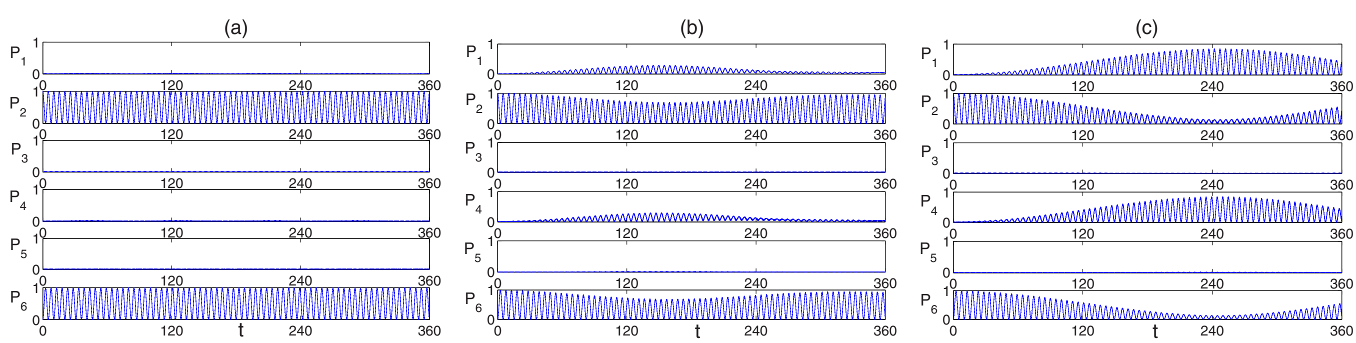


FIG. 2. (Color online) Time evolutions of the atomic probabilities with $P_j(t) = |a_j(t)|^2$ $(j = 1,2,\ldots,6)$. The parameters are same as that in Fig. 1(a) except for nonadjacent interaction $U_2$: (a) $U_2 = 4.9$ $(\beta = -0.9)$; (b) $U_2 = 7$ $(\beta = -3)$; (c) $U_2 = 9$ $(\beta = -5)$.

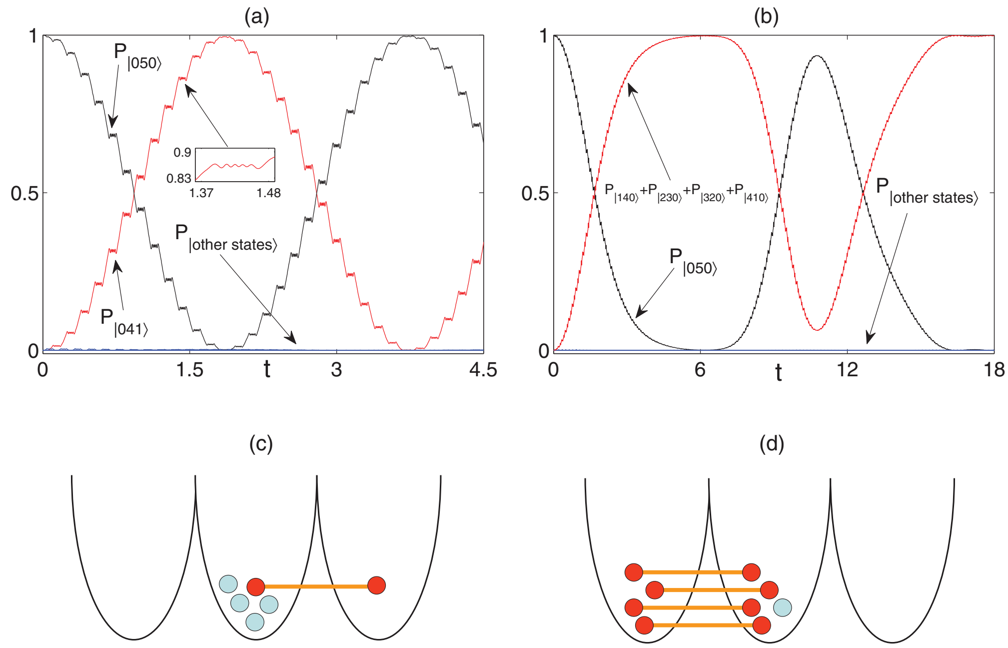


FIG. 3. (Color online) The time evolutions of the atomic probabilities are exhibited as in (a) and (b) for different driving strengths. All dipolar atoms are located in the middle well initially and the parameters are set as $N = 5$, $\Omega = 1$, $\omega = \varepsilon_0 = 28$, $U_0 = 60$, $U_1 = 32$, $U_2 = 4$, and in (a) $\varepsilon_1 = 6.3802\omega$; (b) $\varepsilon_1 = 8.7715\omega$. The schematic diagrams of directed selective-tunneling are shown as in (c) $|0,5,0\rangle \rightarrow |0,4,1\rangle$ and (d) $|0,5,0\rangle \rightarrow c_1|1,4,0\rangle + c_2|2,3,0\rangle + c_3|3,2,0\rangle + c_4|4,1,0\rangle$.

oscillations with small amplitude and the number of oscillations in every platform is consistent with the total number $N$ of bosons.

Adjusting the driving strength and keeping other parameters, from Eq. (1) we find that when the relation $\mathcal{J}_{-N}(\frac{\varepsilon_1}{\omega}) = 0$ is satisfied, the selective tunneling between states $|0,N,0\rangle$ and superposition state $\sum_{i=1}^{N-1} c_i|i,N-i,0\rangle$ occurs, in which $(N-1)$ bosons participate in the tunneling process along the path between wells 1 and 2. As an example, we consider the tunneling dynamics for $N = 5$ and all parameters are the same as that in Fig. 3(a) except for the driving strength $\varepsilon_1 = 8.7715\omega[\mathcal{J}_{-5}(8.7715) = 0]$. The time evolutions of the atomic probabilities are shown as in Fig. 3(b). It can be seen that the directed selective-tunneling occurs, in which only the pathway between wells 1 and 2 is switched on and four of five bosons participate in the tunneling process along this tunneling path. The corresponding schematic diagram is displayed by Fig. 3(d).

Here by adjusting the driving strength, the directed selective-tunneling effect of 1 or $(N-1)$ bosons occurs as the corresponding relation $\mathcal{J}_{-(N-2)}(\frac{\varepsilon_1}{\omega}) = 0$ or $\mathcal{J}_{-N}(\frac{\varepsilon_1}{\omega}) = 0$ is satisfied. Note that the transistorlike effects in a triple-well have been demonstrated [28], in which the occupation of the middle well controls the tunneling between the outer wells. But in our work, we exhibit another scheme that the directed tunneling of 1 or $(N-1)$ bosons between the middle well and right or left well can be manipulated by a time-dependent driving field.

## IV. SUMMARY AND DISCUSSION

In summary, we have investigated the coherent control of quantum tunneling for dipolar bosons held in a driven triple-well potential. In the high-frequency region within resonance case, based on the non-Floquet solutions of two dipolar bosons, the influence of a dipolar interaction on tunneling has been investigated analytically and numerically. When two bosons were located in the middle well initially, the directed selective-tunneling of a single atom was exhibited by adjusting driving parameters and the robustness of the directed tunneling scheme was shown in a proper reduced dipolar-interaction region. Further, we considered more dipolar bosons $N > 2$ and the corresponding directed selective-tunneling effect has been found. For a set of proper parameters and when the relation $\mathcal{J}_{-(N-2)}(\frac{\varepsilon_1}{\omega}) = 0$ was satisfied, only a single atom was allowed to tunnel along one direction and the atomic probabilities show a series of steplike changes. As the driving strength was adjusted to $\mathcal{J}_{-N}(\frac{\varepsilon_1}{\omega}) = 0$, the directed tunneling of $(N-1)$ atoms occurs along another direction. The directed selective-tunneling effect of dipolar bosons can be verified under the presently accessible experimental conditions [3,4,17] and further can also give us an insight into the tunneling dynamics of dipolar bosons trapped in optical lattice.

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