High-fidelity fast quantum driving in nonlinear systems

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We investigate high-fidelity quantum driving in a nonlinear two-level system and find that nonlinear atomic interaction can break the speed limit of the linear model. We show that repulsive atomic interaction can decrease the minimal time requested for reaching target state even to zero, while attractive atomic interaction tends to increase the minimal time. There exists a critical attractive interaction beyond which the target state cannot be reached with high fidelity. Possible experimental observation of the nonlinear effects using a Bose-Einstein condensate in an accelerating optical lattice is discussed.

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The accurate control of quantum state evolution is a fundamental requirement in many areas of modern physics [1], ranging from the coherent manipulation of molecular systems [2,3] and high-precision measurements [4,5], to the pronounced quantum information processing [6]. In the practical implementation of quantum computing, however, quantum decoherence is found to be a kernel obstacle for successful quantum information processing [6–8]. One feasible way to circumvent the dilemma is to drive quantum state to a target state in the shortest possible time, which can prominently minimize the decoherence effect. Nevertheless, quantum driving protocol not only needs to be fast, but also needs to be reliable, i.e., with a high fidelity.

Recently, following the recipe of optimal control at quantum speed limit [9], high-fidelity superfast quantum driving (also called the short-cut protocol [10,11]) has been experimentally implemented by Bose-Einstein condensates (BECs) in optical lattices [12]. The speed and fidelity of various protocols that the system takes between given starting and final quantum states are measured. In a race to achieve a fidelity of 0.9, a superfast (time-minimal) composite pulse (CP) protocol wins out as compared to the usual Landau-Zener (LZ) adiabatic control scheme and locally adiabatic Roland-Cerf (RC) protocol. The experimental explanation and associated theoretical discussion are limited to a linear two-level system, in which the interaction between atoms is ignored. In the ultracold BEC system, since atoms are coherent, interactions between atoms can significantly influence the quantum driving dynamics [13].

In this paper, we report our results of superfast quantum driving in a nonlinear two-level system, in which level energies depend on the occupation of the levels, representing a mean-field interaction between the coherent atoms. We show that repulsive atomic interaction can break the speed limit of the linear model with decreasing minimal time, while attractive interactions tend to increase the minimal time. In particular, we find a critical value of the attractive interaction strength, beyond which the minimal time becomes infinity. A possible experimental observation of the nonlinear effects is suggested.

Model. The nonlinear system is described by the following second-quantized two-mode model [14]:

$$\hat{H}(t) = \Gamma(\tau)(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}) + \omega(\tau)(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger) + \frac{g}{2}(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})^2, \tag{1}$$

where $\hat{a}^\dagger$ ($\hat{a}$) and $\hat{b}^\dagger$ ($\hat{b}$) denote the generators (annihilators) for diabatic states $|0\rangle$ and $|1\rangle$, and $\Gamma(\tau)$ and $\omega(\tau)$ are the energy bias and coupling strength between two diabatic states, respectively. $\tau = t/T \in [0,1]$ is the rescaled time and g describes the interaction strength between atoms. In this system the total number N of particles is conserved. Under the mean-field approximation ($N \to \infty$), the system dynamics is given by the following nonlinear two-level model [15,16]:

$$i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} = \{[\Gamma(\tau) + c(|b|^2 - |a|^2)]\hat{\sigma}_z + \omega(\tau)\hat{\sigma}_x\}\begin{pmatrix} a \\ b \end{pmatrix}, \tag{2}$$

where a and b are the probability amplitudes of diabatic state $|0\rangle$ and $|1\rangle$, $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are Pauli matrices, and $c = -Ng$ is the nonlinear parameter describing the atomic interaction. It is noted that the regime $c < 0$ describes the repulsive interaction while the regime $c > 0$ represents the attractive interaction in our model. Technically, to obtain the mean-field model, we focus on the Bloch coherent states $|\Psi\rangle = \frac{1}{\sqrt{N!}}(a\hat{a}^\dagger + b\hat{b}^\dagger)^N|\text{vac}\rangle$ (where $|\text{vac}\rangle$ denotes the vacuum) [17]. By computing the expectation value $\langle\hat{H}\rangle = \langle\Psi|\hat{H}|\Psi\rangle$, one obtains the mean-field Hamiltonian $H = \langle\hat{H}\rangle/N$ (up to a trivial constant) in the limit of $N \to \infty$ [18]. The Hamiltonian H leads to the dynamics in Eq. (2). The model not only has aroused great interest in theory but also has important applications in physics; for example, for describing a spin tunneling of nanomagnets [19], a BEC in a double-well potential or in an optical lattice [16,20], coupled waveguide arrays [21], etc.

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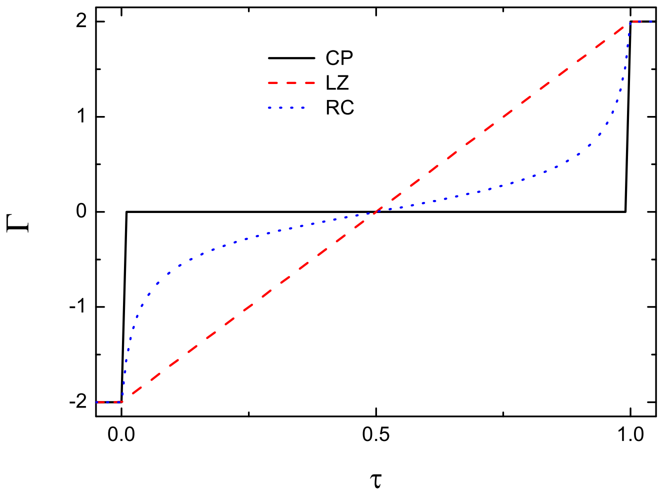

FIG. 1. (Color online) The time dependence of Γ is shown for the CP (solid line), LZ (dashed line), and RC (dotted line) protocols.

The instantaneous adiabatic eigenstates of system are $|\psi_{g,e}(\tau)\rangle$, where the subscripts g and e stand for the ground state and the excited state, respectively. Assume that the system is initially in the adiabatic ground state $|\psi_g(0)\rangle$ at time $t = 0$. The final state at time $t = T$ is the state $|\psi_{\rm fin}\rangle$. Our goal is to design a quantum control protocol [i.e., $\Gamma(\tau)$ and $\omega(\tau)$] that can drive the system from the starting state $|\psi_{\rm ini}\rangle = |\psi_g(0)\rangle$ to the final state $|\psi_{\rm fin}\rangle$ in the shortest possible time (i.e., T) and with high fidelity, i.e., the final state $|\psi_{\rm fin}\rangle$ is as close as possible to the adiabatic ground state $|\psi_g(1)\rangle$, realizing a fidelity close to unity. Here the fidelity function $F_{\rm fin}$ as follows can be used to measure how close the final quantum state is to the adiabatic ground state:

$$F_{\rm fin} = |\langle\psi_{\rm fin}|\psi_g(1)\rangle|^2. \tag{3}$$

In the linear model ($c = 0$), ω is set to be constant, and the different protocols examined in the following correspond to different time dependence $\Gamma(\tau)$ satisfying the bound conditions $\Gamma(0) = -2$ and $\Gamma(1) = 2$ [12,22]:

$$\Gamma(\tau) = \begin{cases} 0, & \text{CP} \\ 4(\tau - 1/2), & \text{LZ} \\ \frac{4\omega(\tau-1/2)}{\sqrt{4+\omega^2-16(\tau-1/2)^2}}, & \text{RC,} \end{cases} \tag{4}$$

as shown in Fig. 1.

The high-fidelity quantum driving has been experimentally implemented for a linear two-level quantum system comprising BECs in optical lattices [12]. At the fast end of quantum control, the CP protocol has been used and tested against the linear LZ scheme and the locally adiabatic RC protocol. It is found that an initial quantum state can be transformed into a desired final state with high fidelity in the shortest allowed time, reaching the quantum speed-limit bound given by [9,12]

$$T_{qsl} = \frac{\arccos|\langle\psi_{\rm fin}|\psi_{\rm ini}\rangle|}{\omega}. \tag{5}$$

The experimentally measured minimal time for the CP protocol approaches the quantum speed limit. Fixing a threshold value at $F_{\rm fin} = 0.9$, $T_{0.9}$ approaches the minimal time given

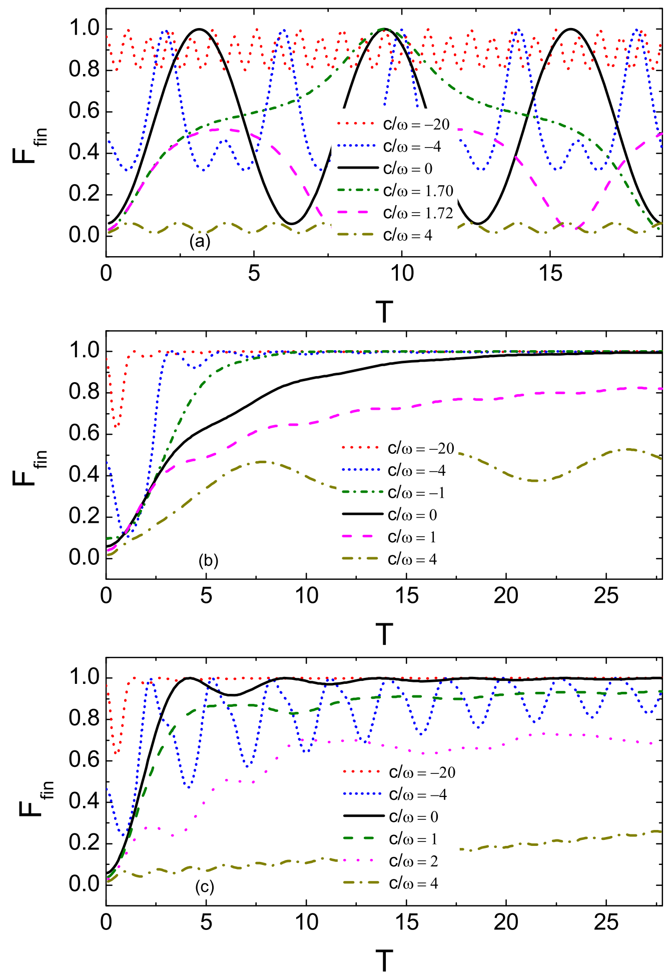

FIG. 2. (Color online) The fidelity of the final state as a function of the duration in different nonlinear interactions for (a) the CP protocol, (b) the LZ protocol and (c) the RC protocol.

by the quantum speed limit for the parameters of the system $T_{qsl} = 2.75$.

Nonlinear superfast quantum driving. We want to investigate the superfast quantum driving in the presence of nonlinear atomic interaction and consider how the nonlinear interaction would affect the quantum speed limit. With the emergence of nonlinearity, the Schrödinger equation (2) is no longer analytically solvable. We therefore exploit a fourth to fifth-order Runge-Kutta algorithm to trace the quantum evolution numerically and calculate the fidelity and the minimal time to arrive at the desired final state.

For comparison with Ref. [12], in all numerical simulations, we take the coupling strength $\omega = 0.5$. Figure 2(a) shows the fidelity of the final state for the CP protocol in different nonlinear interactions. We see that the minimal time arriving to the desired final state with high fidelity is strongly dependent on the nonlinear interaction. For repulsive and small attractive interaction, the fidelity oscillates between a certain value corresponding to the initial ground state and 1. The period of the oscillation decreases when c takes larger and larger negative values (repulsive interaction) and increases when $c > 0$ is increased (attractive interaction). In other words, when c is increased (from $c/\omega = -20$ to $c/\omega = 1.70$) the minimal time increases as well and, apparently, reaches its maximal value for $c/\omega \approx 1.70$. Interestingly, the maximal amplitude of fidelity can stay at 1 for all the repulsive interactions, while not always for the attractive interactions. The fidelity value 1 is not reached for sufficiently strong attractive interaction (e.g., $c/\omega = 1.72,4$). More strikingly, the minimal time between given starting and final states can tend to be zero for the strong repulsive interaction. This means that the quantum speed limit of the linear model breaks in the presence of nonlinearity.

We compare the CP protocol to the LZ protocol and to the locally RC protocol for the nonlinear model. The results of the comparison are shown in Fig. 2 for different interactions. It is evident that the nonlinear repulsive interaction inclines to decrease the minimal time between given starting and final states, while the nonlinear attractive interaction tends to increase the minimal time. The LZ protocol is more than an order of magnitude slower than the CP pulse protocol, whereas the RC protocol reaches the final state in a time that is in between the CP and the LZ protocols. Similarly, the minimal time to arrive at the final state tends to zero for the LZ and RC protocols for the sufficiently large repulsive interaction strength. However, the superfast quantum control with high fidelity will no longer be achieved for the sufficiently large attractive interaction strength.

The above studies show that the minimal time arriving to the desired final state with high fidelity is strongly dependent on the nonlinear interaction. We shall study the minimal time to achieve a fidelity of $F_{\rm fin} = 0.9$. In Fig. 3, we plot the minimal time $T_{0.9}$ as a function of the nonlinear interaction. It is clear that the minimal time to reach $F_{\rm fin} = 0.9$ is gradually increasing with the nonlinear interaction from repulsion to attraction and tends to zero for the sufficiently large repulsive nonlinear interaction while it diverges for the sufficiently strong attractive interaction. This phenomenon can be well understood from an analysis of the evolution behavior of nonlinear systems. Since the RC protocol reaches the final state in a time between that of the CP and the LZ protocols,

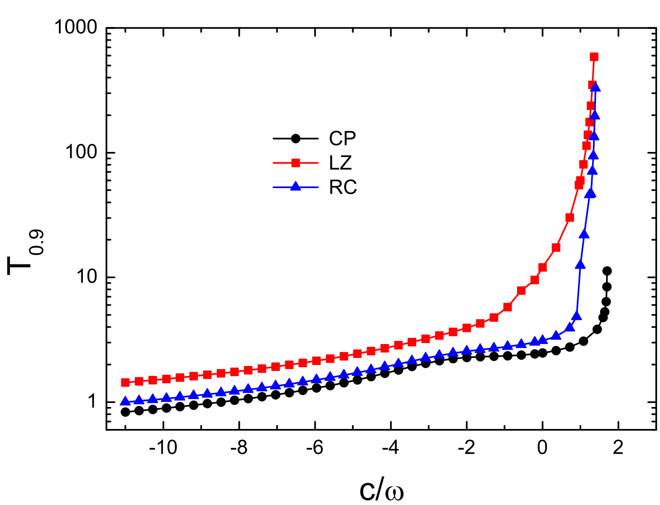

FIG. 3. (Color online) The minimal time to achieve fidelity $F_{\rm fin} = 0.9$ as a function of the nonlinear interaction for the CP protocol (black circles), the LZ protocol (red squares), and the RC protocol (blue triangles).

here we shall analyze the evolution behavior of two extreme cases, i.e., the CP and the LZ protocols.

For the nonlinear CP protocol, in the weak nonlinear interaction region, Josephson oscillation will be observed, and the period of the oscillation decreases as c becomes more and more negative (and thus the repulsive interaction increases) while the period increases as c becomes more and more positive (i.e., the attractive interaction increases). When the interaction is large enough, the system becomes self-trapped. For the system with repulsive interaction, the self-trapping emerges at the ground state while it appears in the excited state of the system with the attractive interaction. Consequently, the quantum speed limit of the linear model breaks in the strong repulsive interaction, while the superfast quantum control with high fidelity will no longer be achieved for the sufficiently large attractive interaction.

For the nonlinear LZ protocol, in the weak nonlinear interaction case, the LZ transition probability from the ground state to the excited state as a function of sweeping rates $1/T$ follows an exponential law, $F_{\rm fin} = 1 - \exp(-q\pi T\omega^2/4)$ [23], which reduces to the linear LZ formula when $q = 1$. Here q represents the atomic-interaction-induced modification factor, which increases (decreases) monotonically with the nonlinear repulsive (attractive) parameters, indicating a decreasing (increasing) transition probability. As a result, the fidelity increases or decreases as the repulsive or attractive interaction is increased, leading to the fact that the minimal time to reach $F_{\rm fin} = 0.9$ decreases or increases accordingly.

For a strong nonlinear repulsive interaction, the LZ transition probability tends to zero even in the fast-speed sweep limit. Therefore, the minimal time with high fidelity tends to zero. However, for the sufficiently strong attractive interaction, the LZ transition probability is a nonzero value even in the adiabatic limit [23–25]. Similar to the CP protocol, this implies that the high-fidelity quantum driving will no longer be achieved when the nonlinear parameter exceeds a critical value [see also Fig. 4(b)].

Critical atomic interaction. In order to further investigate the critical behavior, we define a maximal fidelity $F_{\rm max}$, which is assumed to be the maximal value of the fidelity reached by $F_{\rm fin}$ during the time evolution T from 0 to ∞. We find that the maximal fidelity is also strongly dependent on the nonlinear interaction. The results are summarized in Fig. 4(a). The numerical calculations show that the value of $F_{\rm max}$ is maintained at 1 in repulsive and weak attractive interaction regions and drops suddenly when attractive interaction is over the critical point. For the LZ and the RC protocols, the critical point is approximately at $c/\omega = 1$ and the maximal fidelity slowly decreases when $c/\omega > 1$, which corresponds to the appearance of a loop at the ground-state level for the nonlinear LZ protocol, leading to a breakdown of adiabaticity [23]. However, for the CP protocol, the critical point is about $c/\omega = 1.7044$ (i.e., $c = 0.8522$) for the parameters of our system. When the attractive interaction is beyond the critical value, the maximal fidelity $F_{\rm max}$ drops sharply, which corresponds to the emergence of self-trapping at the excited state of the system. At the moment, the energy of a system described by a classical Hamiltonian $H_{\rm eff}(c,\omega) = -c(1-2|a|^2)^2/2 - \omega[1-(1-2|a|^2)^2]^{1/2}$ is smaller than $-\omega$ [26]. Therefore,

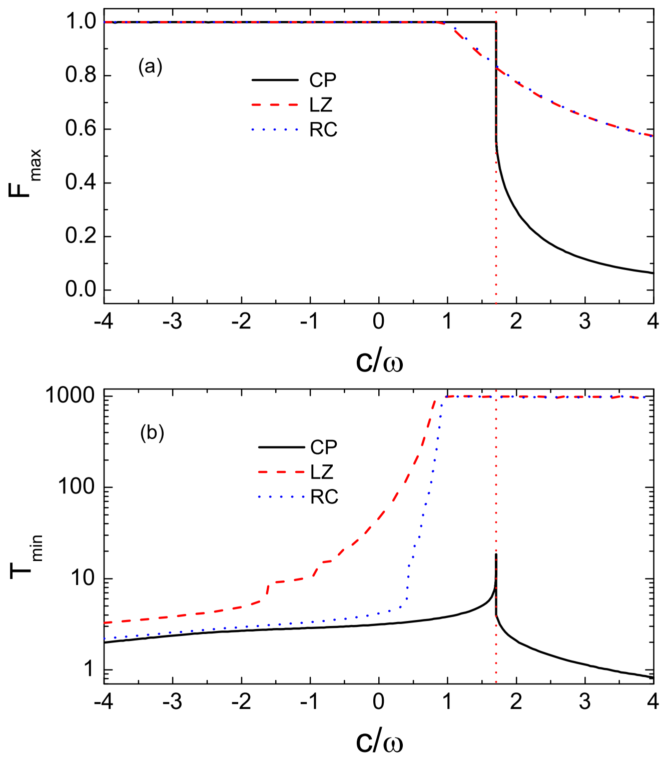

FIG. 4. (Color online) The maximal fidelity and the corresponding minimal time as a function of the nonlinear interaction for the CP protocol (solid line), the LZ protocol (red dashed line), and the RC protocol (blue dotted line). The vertical dotted line represents the critical point of theoretical prediction of Eq. (6) for the CP protocol. All results are for evolution time $T \in [0,1000]$.

we can obtain the general criterion for the occurrence of the critical behavior, i.e., the classical Hamiltonian of the system $H_{\rm eff}(c,\omega) < -\omega$. Then the critical point is expressed as

$$\left(\frac{c}{\omega}\right)_{cr} = \frac{2[1-\sqrt{1-(1-2|a_i|^2)^2}]}{(1-2|a_i|^2)^2}, \quad (6)$$

where $|a_i|^2$ is the corresponding population of the initial state. In Fig. 4(b), we also plot the corresponding minimal time $T_{\rm min}$ of the maximal fidelity as a function of the nonlinear interaction. It is noted that in both the linear LZ and RC protocols the time to achieve complete adiabaticity; that is, $F_{\rm max} = 1$, tends to infinity. Therefore, in our numerical simulations, the values of the maximal fidelity take $F_{\rm max} \approx 0.9999$ (i.e., infidelity $1 - F_{\rm max}$ is at the 10^{-4} level) for LZ and RC protocols when $c/\omega < 1$. We also find the same critical behavior: the high-fidelity quantum driving will no longer be achieved when the attractive interaction strength exceeds the critical value.

Experimental realization. The superfast quantum driving in a nonlinear two-level system can be realized experimentally using BECs between Bloch bands in an accelerated optical lattice, where sufficiently high densities of the atoms and Feshbach resonances can be achieved so that the nonlinear effect discussed above should be readily detectable. The mathematical model for this system can be described by Eq. (2), where $\Gamma(\tau)$ and $\omega(\tau)$ can be controlled through the quasimomentum q and the depth V_0 of the optical lattice, respectively. The system is initially prepared in the lowest-energy band of the lattice with $q = 0$ (corresponding to $|\psi_{\rm ini}\rangle$), and the target is to reach $|\psi_{\rm fin}\rangle$ after an evolution of duration T. BECs can be loaded into optical lattice potentials of the form $V = (V_0/2)\cos[2\pi x/d_L + \phi(t)]$ with lattice spacing d_L and acousto-optic-modulator factor $\phi(t)$. The ratio $c/\omega = 8\pi\hbar^2 n_0 a_s/(mV_0)$ [16], where n_0 is the average density of the condensate, m is the mass of the atoms, and a_s is the s-wave scattering length between atoms, determining the nonlinearity of the system. In typical experiments [12], we have $\omega = 0.5$ corresponding to $V_0 = 2E_{\rm rec} = 2\hbar\omega_{\rm rec}$ ($\omega_{\rm rec} = 2\pi \times 3.15$ kHz defines the natural units of energy $\hbar\omega_{\rm rec}$ and time $1/\omega_{\rm rec}$), $a_s = \pm 5.4$ nm for rubidium [27], which gives us $c/\omega = \pm 0.025 \sim \pm 2.5$ for $n_0 = 1 \times 10^{19}$ m$^{-3} \sim 1 \times 10^{21}$ m^{-3}, with higher density corresponding to larger nonlinearity c. Besides, Feshbach resonances can be used to tune the nonlinear interaction c from strong repulsion to strong attraction via an external magnetic field [28]. In the experiments of Ref. [12], the parameters $|c| \leqslant 0.05$ and therefore the interactions are negligible [29]. However, the small nonlinearity leads to the inconsistency between the theories and experimental data. To observe the nonlinear effects, we need the ratio c/ω to be changed from a negative value to a positive value. By comparing population $|a_i|^2$ in Eq. (6) with the average density value of the condensate n_0, we find that the high-fidelity superfast quantum driving cannot be achieved for rubidium with attractive interaction when the experimental value $n_0 > 3.41 \times 10^{20}$ (corresponding the initial-state population satisfying $|a_i|^2 > 0.676$) under the above experimental parameters. In addition, the natural units of time is of order 10^{-5} s for our system, which is far less than the life time of condensates (typical value 1 to 100 s). It is noted that the nonlinear LZ tunneling between two energy bands of a BEC in a periodic potential has been observed [26,30], indicating that superfast quantum driving in nonlinear two-level systems can be realized experimentally.

Conclusions. In conclusion, we have investigated the high-fidelity superfast quantum driving in a nonlinear two-level system and explored the influence of atomic interactions on high-fidelity superfast quantum driving. We have found that a repulsive interaction between atoms inclines to decrease the minimal time for reaching the target state even to zero for sufficiently large interaction strength, resulting in a breakdown of the quantum speed limit of the linear model, and the attractive interaction tends to increase the minimal time between given starting and final states. There exists a critical value of the interaction strength beyond which the superfast quantum driving cannot be achieved with high fidelity. For the same interaction value, the minimal time to achieve high fidelity in the CP protocol is the shortest compared to the nonlinear LZ protocol and to the nonlinear RC protocol, i.e., the CP protocol can achieve superfast quantum driving. The optimal protocol which allows one to implement the desired transformation from the initial state to the final state with fidelity equal to 1 in the minimal time but does not require any adiabatic following during the protocol. Therefore, the minimal time T can be decreased without conflicting with the adiabatic character of the model. Possible experimental observation and the significant values of experimental parameters for the nonlinear high-fidelity superfast quantum driving comprising

BECs between Bloch bands in an accelerating optical lattice have been suggested.

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